

# Higgs Self-Coupling and Electroweak Baryogenesis

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# Outline

## §1. Introduction

-Connection between collider physics and cosmology

## §2. Conditions of baryogenesis

-Electroweak phase transition in the THDM

## §3. Radiative corrections to $hhh$ coupling constant

-Collider signal of electroweak baryogenesis?

## §4. Summary

# Introduction

- **Higgs physics at colliders**

- Discovery of the Higgs boson(s) (@Tevatron, LHC)

- Measurements of the Higgs couplings with  $\begin{cases} \text{gauge bosons} \\ \text{fermions} \end{cases}$  (mass generation)  
 $\mathcal{O}(1)\%$  accuracy (@ILC) ACFA Rep. TESLA TDR

- Measurements of the Higgs self-couplings (reconstruction of the Higgs potential)

- $\mathcal{O}(10 - 20)\%$  accuracy (@ILC) ACFA Higgs WG, Battaglia et al

- **Connection between collider physics and cosmology**

- Baryon Asymmetry of the Universe

- (1). B-L-gen. above EW phase transition (Leptogenesis, etc)
- (2). B-gen. during EW phase transition (EW baryogenesis)

- dark matter

- ▷ Since the EW baryogenesis depends on the dynamics of the phase transition, we can naively expect that a collider signal of it can appear in the Higgs self-coupling.

- ▷ We investigate the region where the EW baryogenesis is possible in the THDM, and calculate the deviation of the self-coupling constant from SM prediction in such a region.

# Conditions of Baryogenesis

Evidence of the BAU

$$\frac{n_B}{s} \equiv \frac{n_b - n_{\bar{b}}}{s} \simeq (8.7^{+0.4}_{-0.3}) \times 10^{-11}$$

- 3 requirements for generation of the BAU (Sakharov conditions)

- 1. baryon number violation**
- 2.  $C$  and  $CP$  violation**
- 3. out of equilibrium**

2 scenarios

- (1) B-L-generation above EW phase transition. (Leptogenesis, etc)
- (2) B-generation at the electroweak phase transition. (Electroweak baryogenesis)
  - based on a testable model

# Baryogenesis in the electroweak theory

- baryon number violation sphaleron process
- $C$  violation chiral interaction
- $CP$  violation KM-phase or other sources in the extension of the SM
- out of equilibrium 1st order phase transition with expanding bubble walls

In principle, SM fulfills the Sakharov conditions, *BUT*

- Phase transition is **not** 1st order for the current Higgs mass bound ( $m_h > 114$  GeV)
- KM-phase is **too small** to generate the sufficient baryon asymmetry

⇒ Extension of the minimal Higgs sector

THDM, MSSM, Next-to-MSSM, etc.

- ▷ THDM is a simple viable model **not so constrained**

# Two Higgs Doublet Model (THDM)

## Higgs potential

$$\begin{aligned}
 V_{\text{THDM}} = & m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - (m_3^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}) \\
 & + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 \\
 & + \left[ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right], \\
 \Phi_i(x) = & \begin{pmatrix} \phi_i^+(x) \\ \frac{1}{\sqrt{2}} (\mathcal{V}_i + h_i(x) + i a_i(x)) \end{pmatrix}. \quad (i = 1, 2)
 \end{aligned}$$

discrete sym. ( $\Phi_1 \rightarrow \Phi_1$ ,  $\Phi_2 \rightarrow -\Phi_2$ )  $\rightarrow$  FCNC suppression

## Yukawa interaction

Type I :  $\mathcal{L}_{\text{Yukawa}}^I = \bar{q}_L f_1^{(d)} \Phi_1 d_R + \bar{q}_L f_1^{(u)} \tilde{\Phi}_1 u_R + \bar{l}_L f_1^{(e)} \Phi_1 e_R + \text{h.c.}$ ,

Type II :  $\mathcal{L}_{\text{Yukawa}}^{II} = \bar{q}_L f_1^{(d)} \Phi_1 d_R + \bar{q}_L f_2^{(u)} \tilde{\Phi}_2 u_R + \bar{l}_L f_1^{(e)} \Phi_1 e_R + \text{h.c.}$ .

To avoid complication, we consider [Cline et al PRD54 '96]

$$m_1 = m_2 \equiv m, \quad \lambda_1 = \lambda_2 = \lambda, \quad \left( \sin(\beta - \alpha) = \tan \beta = 1 \right)$$

- Higgs VEVs:  $\langle \Phi_1 \rangle = \langle \Phi_2 \rangle = \frac{1}{2} \begin{pmatrix} 0 \\ \varphi \end{pmatrix}$

### • Tree-level potential

$$V_{\text{tree}}(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda_{\text{eff}}}{4}\varphi^4, \quad \mu^2 = m_3^2 - m^2, \quad \lambda_{\text{eff}} = \frac{1}{4}(\lambda + \underbrace{\lambda_3 + \lambda_4 + \lambda_5}_{\equiv \lambda_{345}})$$

### ▷ Mass formulae of the Higgs bosons

$$m_h^2 = \frac{1}{2}(\lambda + \lambda_{345})v^2,$$

$$\left. \begin{array}{l} m_H^2 = \frac{1}{2}(\lambda - \lambda_{345})v^2 + M^2, \\ m_A^2 = -\lambda_5 v^2 + M^2, \\ m_{H^\pm}^2 = -\frac{1}{2}(\lambda_4 + \lambda_5)v^2 + M^2 \end{array} \right\}$$

Two origins of the masses :  $\sum_i c_i \lambda_i v^2$  and  $M^2$ .

where  $M^2 = \frac{m_3^2}{\sin \beta \cos \beta}$  (soft breaking scale of the discrete symmetry)

# 1-loop effective potential

- Zero temperature

$$V_1(\varphi) = n_i \frac{m_i^4(\varphi)}{64\pi^2} \left( \log \frac{m_i^2(\varphi)}{Q^2} - \frac{3}{2} \right)$$

$$(n_W = 6, \ n_Z = 3, \ n_t = -12, \ n_h = n_H = n_A = 1, \ n_{H^\pm} = 2)$$

- Finite temperature

$$V_1(\varphi, T) = \frac{T^4}{2\pi^2} \left[ \sum_{i=\text{bosons}} n_i I_B(a^2) + n_t I_F(a) \right]$$

where  $I_{B,F}(a^2) = \int_0^\infty dx \ x^2 \log(1 \mp e^{-\sqrt{x^2+a^2}}), \quad \left( a(\varphi) = \frac{m(\varphi)}{T} \right)$

▷ High temperature expansion  $(a^2 \ll 1)$

$$I_B(a^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}a^2 - \frac{\pi}{6}(a^2)^{3/2} - \frac{a^4}{32} \left( \log \frac{a^2}{\alpha_B} - \frac{3}{2} \right) + \mathcal{O}(a^6),$$

$$I_F(a^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}a^2 - \frac{a^4}{32} \left( \log \frac{a^2}{\alpha_F} - \frac{3}{2} \right) + \mathcal{O}(a^6), \quad \left( \log \alpha_{F(B)} = 2 \log(4)\pi - 2\gamma_E \right)$$

$\varphi^3$ -term comes from the “bosonic” loop

# Finite temperature Higgs potential

For  $m_\Phi^2(v) \gg M^2, m_h^2(v)$        $m_\Phi^2(\varphi) \simeq m_\Phi^2(v) \frac{\varphi^2}{v^2}$ ,   ( $\Phi = H, A, H^\pm$ )

$$V_{\text{eff}} \simeq D(T^2 - T_0^2)\varphi^2 - ET\varphi^3 + \frac{\lambda_T}{4}\varphi^4$$

where

$$E = \frac{1}{12\pi v^3} (6m_W^3 + 3m_Z^3 + \underbrace{m_H^3 + m_A^3 + 2m_{H^\pm}^3}_{\text{additional contributions}})$$

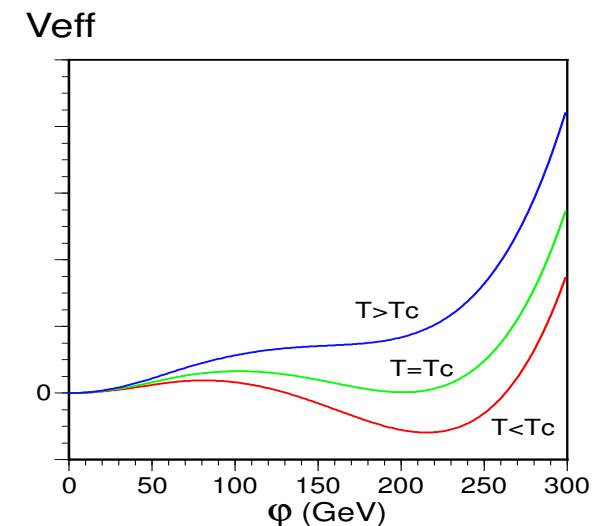
At  $T_c$ , degenerate minima:       $\varphi_c = \frac{2ET_c}{\lambda_{T_c}}$

- The magnitude of  $E$  is relevant for the strongly 1st order phase transition

- **Strongly 1st order phase transition:**       $\frac{\varphi_c}{T_c} \gtrsim 1$

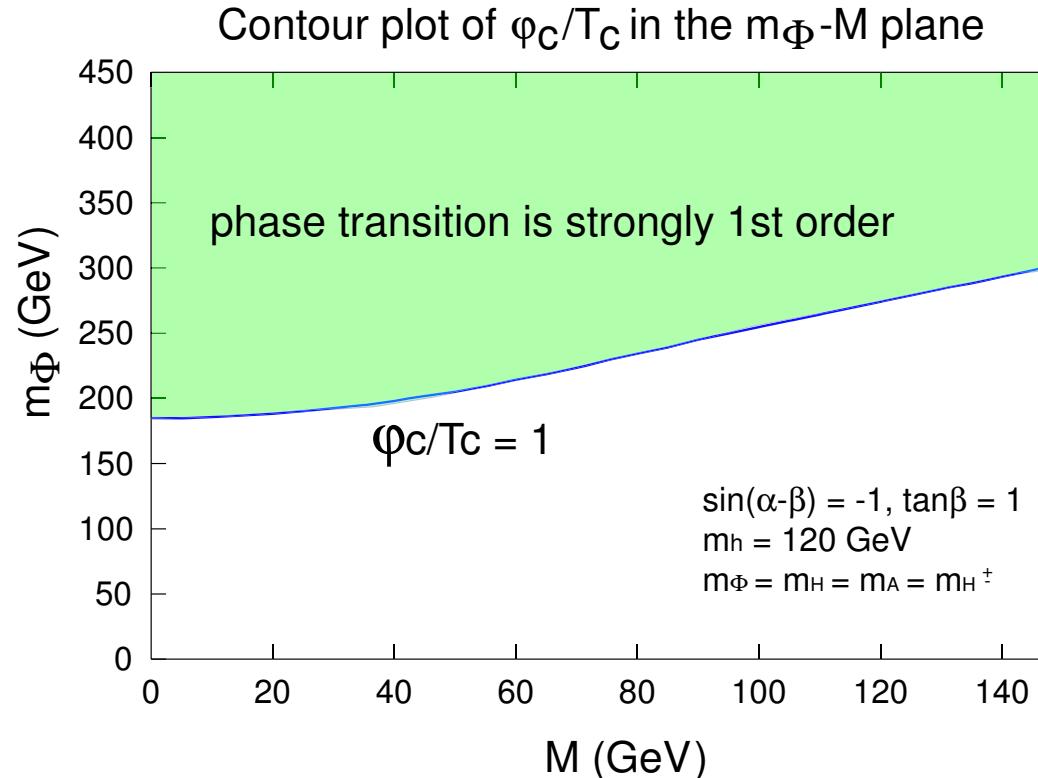
$\Rightarrow$  Not wash out the baryon density after EW phase transition

$\triangleright CP$  violation at the bubble wall       $\Rightarrow$  Asymmetry of the charge flow



## Contour plot of $\varphi_c/T_c$ in the $m_\Phi$ - $M$ plane

$$\sin^2(\alpha - \beta) = \tan \beta = 1, \ m_h = 120 \text{ GeV}, \ m_\Phi \equiv m_A = m_H = m_{H^\pm}$$



- For  $m_\Phi^2 \gg M^2, m_h^2$ ,

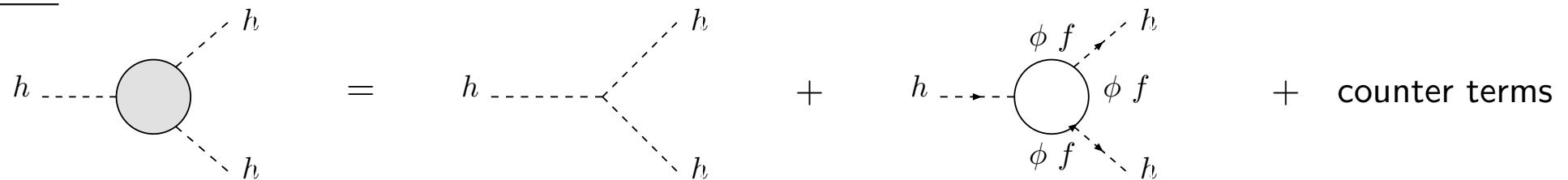
Strongly 1st order phase transition is possible due to the loop effect of the heavy Higgs bosons (**non-decoupling effect**). ( $\varphi^3$ -term is effectively large)

- What the magnitude of the  $\lambda_{hhh}$  coupling at  $T=0$  in such a region?

# Radiative corrections to $\lambda_{hhh}$

[S. Kanemura, S. Kiyoura, Y. Okada, E.S., C.-P. Yuan PL '03]

- $hhh$



$$(\phi = h, H, A, H^\pm, \quad f = t, b)$$

- For  $\sin(\beta - \alpha) = 1$ ,

$$\lambda_{hhh}^{\text{tree}} = -\frac{3m_h^2}{v}, \quad (\text{same form as in the SM})$$

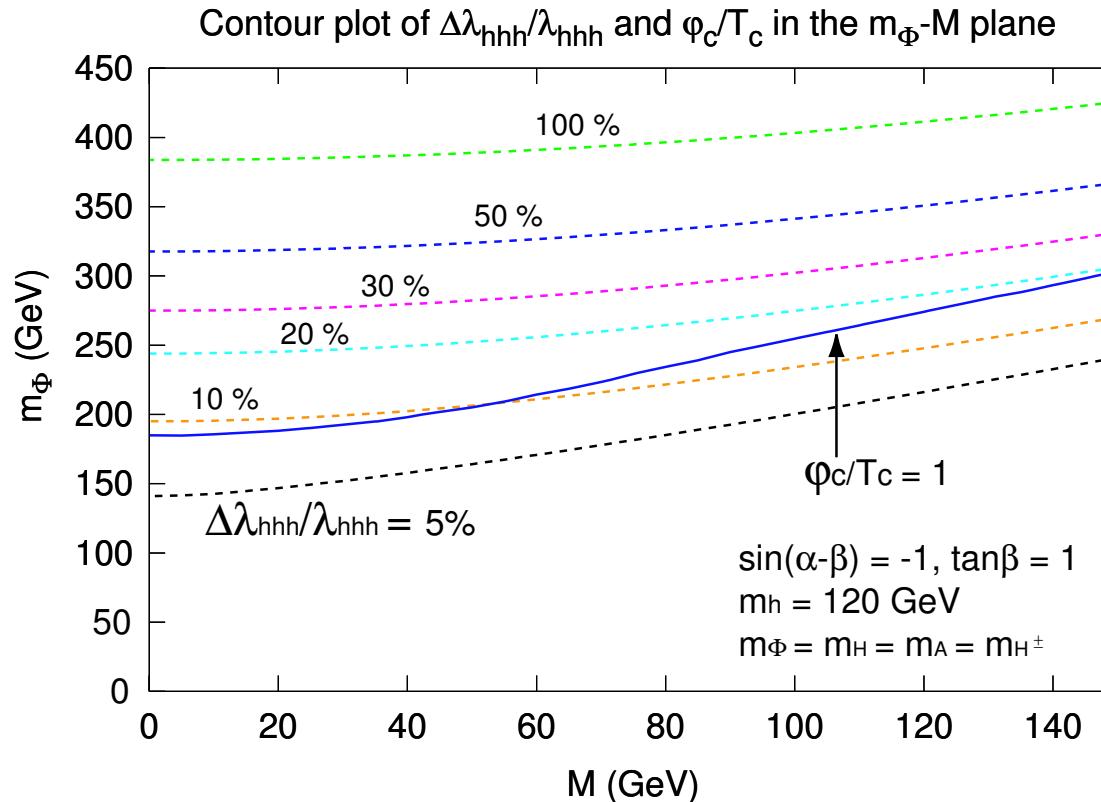
$$\lambda_{hhh} \sim -\frac{3m_h^2}{v} \left[ 1 + \frac{c}{12\pi^2} \frac{m_\Phi^4}{m_h^2 v^2} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^3 \right] \quad (\Phi = H, A, H^\pm)$$

$(c = 1 \text{ for neutral Higgs, } c = 2 \text{ for charged Higgs})$

For  $m_\Phi^2 \gg M^2, m_h^2$ , the loop effect of the heavy Higgs bosons is enhanced by  $m_\Phi^4$ , which does not decouple in the large mass limit. (**non-decoupling effect**)

## Contour plots of $\Delta\lambda_{hhh}/\lambda_{hhh}$ and $\varphi_c/T_c$ in the $m_\Phi$ - $M$ plane

$\sin^2(\alpha - \beta) = \tan \beta = 1$ ,  $m_h = 120$  GeV,  $m_\Phi \equiv m_A = m_H = m_{H^\pm}$



For  $m_\Phi^2 \gg M^2, m_h^2$ ,

- Phase transition is strongly 1st order, *AND*
- Deviation of hhh coupling from SM value becomes **large**. ( $\Delta\lambda_{hhh}/\lambda_{hhh} \gtrsim 10\%$ )

# Summary

We have investigated the region where the electroweak phase transition is strongly 1st order, and calculate the radiative correction to the triple Higgs self-coupling constant in the THDM.

For  $m_\Phi^2 \gg M^2, m_h^2$

- Phase transition is strongly 1st order.
- Deviation of  $hhh$  coupling from SM value becomes **large**. ( $\Delta\lambda_{hhh}/\lambda_{hhh} \gtrsim 10\%$ )

**due to the non-decoupling effect of the heavy Higgs bosons**

Such deviation can be testable at a Linear Collider.

EW baryogenesis



Strongly 1st order phase transition

$$V_{\text{eff}}(\varphi, T)$$



Large loop correction to  $\lambda_{hhh}$

$$V_{\text{eff}}(\varphi, 0)$$



Measurement of  $\lambda_{hhh}$  @ILC

## Definitions of the mass eigenstates

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}, \quad \left( -\frac{\pi}{2} \leq \alpha \leq 0 \right),$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} G^0 \\ A \end{pmatrix}, \quad \left( 0 < \beta < \frac{\pi}{2} \right)$$

$$\begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}$$

## Renormalized $h$ and $H$

$$\begin{aligned}
\begin{pmatrix} H_B \\ h_B \end{pmatrix} &= \underbrace{\begin{pmatrix} \cos \alpha_B & \sin \alpha_B \\ -\sin \alpha_B & \cos \alpha_B \end{pmatrix}}_{\equiv R(-\alpha_B)} \begin{pmatrix} h_{1B} \\ h_{2B} \end{pmatrix} = R(-\delta\alpha)R(-\alpha_R) \begin{pmatrix} h_{1B} \\ h_{2B} \end{pmatrix} \\
&= R(-\delta\alpha)R(-\alpha_R)\tilde{Z} \begin{pmatrix} h_{1R} \\ h_{2R} \end{pmatrix} \\
&= R(-\delta\alpha)ZR(-\alpha_R) \begin{pmatrix} h_{1R} \\ h_{2R} \end{pmatrix} \quad (Z \equiv R(-\alpha_R)\tilde{Z}R(\alpha_R)) \\
&= R(-\delta\alpha)Z \begin{pmatrix} H_R \\ h_R \end{pmatrix} \\
&\equiv \begin{pmatrix} 1 & \delta\alpha \\ -\delta\alpha & 1 \end{pmatrix} \begin{pmatrix} Z_H^{1/2} & \delta A \\ \delta A & Z_h^{1/2} \end{pmatrix} \begin{pmatrix} H_R \\ h_R \end{pmatrix} \\
&= \begin{pmatrix} 1 + \frac{1}{2}\delta Z_H & \delta A + \delta\alpha \\ \delta A - \delta\alpha & 1 + \frac{1}{2}\delta Z_h \end{pmatrix} \begin{pmatrix} H_R \\ h_R \end{pmatrix},
\end{aligned}$$

## On-shell renormalization in the Electroweak Theory

- Gauge sector

$$\begin{array}{ccc}
 g_2, \ g_1, \ v & \Leftarrow & \alpha_{\text{em}}, \ m_Z, \ G_F \quad \alpha_{\text{em}}, \ m_Z, \ m_W \\
 & & m_Z, \ m_W, \ G_F \quad \alpha_{\text{em}}, \ m_Z, \ \sin \theta_W \ \text{etc...}
 \end{array}$$

- Higgs sector

Renormalization conditions for  $\alpha$ ,  $\beta$  and  $M_{\text{soft}}$  (our scheme)

$$\begin{aligned}
 \text{Re}\Gamma_{hH}(m_h^2) = \text{Re}\Gamma_{hH}(m_H^2) = 0 &\Rightarrow Z_{Hh}^{1/2}, \ Z_{hH}^{1/2}, \\
 \text{where } Z_{Hh}^{1/2} = \delta A + \delta\alpha, \ Z_{hH}^{1/2} = \delta A - \delta\alpha
 \end{aligned}$$

$$\begin{aligned}
 \text{Re}\Gamma_{AG^0}(m_A^2) = \text{Re}\Gamma_{AZ}(m_A^2) = 0 &\Rightarrow Z_{G^0A}^{1/2}, \ Z_{AG^0}^{1/2}, \\
 \text{where } Z_{G^0A}^{1/2} = \delta B + \delta\beta, \ Z_{AG^0}^{1/2} = \delta B - \delta\beta
 \end{aligned}$$

$$\delta M_{\text{soft}} \Leftarrow \overline{MS} \text{ scheme}$$

# Ring-improved Higgs boson masses

$$\begin{aligned}
m_h^2(\varphi, T) &= \frac{3}{2}m_h^2(v_0)\frac{\varphi^2}{v_0^2} - \frac{1}{2}m_h^2(v_0) + aT^2, \\
m_H^2(\varphi, T) &= \left[m_H^2(v_0) + \frac{1}{2}m_h^2(v_0) - 2m_3^2\right]\frac{\varphi^2}{v_0^2} - \frac{1}{2}m_h^2(v_0) + 2m_3^2 + aT^2, \\
m_A^2(\varphi, T) &= \left[m_A^2(v_0) + \frac{1}{2}m_h^2(v_0) - 2m_3^2\right]\frac{\varphi^2}{v_0^2} - \frac{1}{2}m_h^2(v_0) + 2m_3^2 + aT^2, \\
m_{H^\pm}^2(\varphi, T) &= \left[m_{H^\pm}^2(v_0) + \frac{1}{2}m_h^2(v_0) - 2m_3^2\right]\frac{\varphi^2}{v_0^2} - \frac{1}{2}m_h^2(v_0) + 2m_3^2 + aT^2, \\
m_{G^0}^2(\varphi, T) &= m_{G^\pm}^2(\varphi, T) = \frac{1}{2}m_h^2(v_0)\frac{\varphi^2}{v_0^2} - \frac{1}{2}m_h^2(v_0) + aT^2.
\end{aligned}$$

where

$$a = \frac{1}{12v_0^2} \left[ 6m_W^2(v_0) + 3m_Z^2(v_0) + 5m_h^2(v_0) + m_H^2(v_0) + m_A^2(v_0) + 2m_{H^\pm}^2(v_0) - 8m_3^2 \right].$$