

# Neutral Higgs Boson Production at LC and yet Another Source of CP Violation

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# Introduction

- Higgs Boson in the Standard Model

- Only one neutral scalar exists? (Maybe no Higgs at all?)
- Remnant of the symmetry breaking (Maybe yet another source of CPV?)
- SM Higgs boson production at the LC

$$\begin{aligned} e^-e^+ &\rightarrow ZH, \\ e^-e^+ &\rightarrow W^-W^+H, \quad e^-e^+ \rightarrow ZZH, \\ e^-e^+ &\rightarrow t^-t^+H, \end{aligned}$$

- Higgs boson branching ratios

$$\begin{aligned} H &\rightarrow b\bar{b}, \\ H &\rightarrow W^-W^+ \\ H &\rightarrow ZZ \\ H &\rightarrow t\bar{t}, \end{aligned}$$

- How about extension to 2 Higgs-Doublet (2HD)?
  - The simplest extension of Higgs sector (Minimal extension of SM)
  - 3 neutral Higgs boson + 1 pair of charged Higgs boson
  - Preserves  $\rho \equiv m_W/(m_Z \cos \theta_W) = 1$  up to finite radiative correction
  - Dangerous Higgs-mediated flavour-changing neutral currents (FCNS) exist at tree-level in general
    - ⇐ By imposing a discrete symmetry
    - ⇒ 3 types of Models in Yukawa couplings have been suggested
  - type I : Only one Higgs doublet couples to all the fermions
  - type II : One Higgs doublet couples only to up-type quarks (& leptons), the other Higgs doublet couples only to down-type quarks (& leptons). This model arises in the MSSM
  - type III : Tree level Higgs-mediated FCNC are present and suppressed (Phenomenologically)....
    - Spontaneous and explicit CP violation in the Higgs sector are possible.

# Two Higgs Doublet Model with CP Violation

- Analysis on the most general Higgs potential of 2HD model

– The Higgs potential

$$\begin{aligned} V = & \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ & + \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + H.c.] + [\lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + H.c.] \\ & - m_{11}^2((\phi_1^\dagger\phi_1) - m_{22}^2((\phi_2^\dagger\phi_2) - [m_{12}^2((\phi_1^\dagger\phi_2) + H.c.] \end{aligned}$$

– By imposing a discrete symmetry ( $Z_2$  Symmetry)

$$(\Rightarrow \quad \phi_1 \rightarrow -\phi_1, \quad \phi_2 \rightarrow \phi_2)$$

$$\rightarrow \lambda_5 = \lambda_6 = \lambda_7 = m_{12}^2 = 0$$

→ Tree level FCNC and CP violation are absent.

– How about soft violation of  $Z_2$  symmetry? *i.e.* allow  $\lambda_5, m_{12}^2 \neq 0$

– **By minimizing the potential**

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix},$$

**parametrized by**

$$\tan \beta = \frac{v_2}{v_1}, \quad v^2 = v_1^2 + v_2^2$$

– **we get**

$$\text{Im}(m_{12}^2 e^{i\xi}) = v_1 v_2 \text{Im}(\lambda_5 e^{2i\xi})$$

– **The global transform  $\phi_i \rightarrow \phi_i e^{i\varphi_i}$  with the rephasing;**

$$\lambda_5 \rightarrow \lambda_5 e^{-2i(\varphi_2 - \varphi_1)}, \quad m_{12}^2 \rightarrow m_{12}^2 e^{-i(\varphi_2 - \varphi_1)},$$

$$\xi \rightarrow \xi + \varphi_2 - \varphi_1,$$

**with  $\lambda_i, i = 1, 2, 3, 4$  and  $m_{11,22}^2$  invariant.**

→ **We can choose  $\xi = 0$  from the rephasing invariance.**

⇒ **Indicating no spontaneous CP violation but wholly explicit CP violation.**

- **Neutral Higgs bosons**

- The neutral states are defined by

$$\begin{aligned}
 G^0 &= \sqrt{2}(\text{Im } \phi_1^0 \cos \beta + \text{Im } \phi_2^0 \sin \beta), \\
 \varphi_1 &= \sqrt{2}\text{Re } \phi_1^0, \\
 \varphi_2 &= \sqrt{2}\text{Re } \phi_2^0. \\
 A^0 &= \sqrt{2}(-\text{Im } \phi_1^0 \sin \beta + \text{Im } \phi_2^0 \cos \beta),
 \end{aligned}$$

- The mass matrix of neutral Higgs bosons

$$\mathcal{M}^2 = \begin{pmatrix} \mathcal{M}_{11}^2 & \mathcal{M}_{12}^2 & -\frac{1}{2}\text{Im}(\lambda_5) \sin \beta \\ \mathcal{M}_{21}^2 & \mathcal{M}_{22}^2 & -\frac{1}{2}\text{Im}(\lambda_5) \cos \beta \\ -\frac{1}{2}\text{Im}(\lambda_5) \sin \beta & -\frac{1}{2}\text{Im}(\lambda_5) \cos \beta & \mathcal{M}_{33}^2 \end{pmatrix} v^2$$

where

$$\begin{aligned}
 \mathcal{M}_{11}^2 &= R \sin^2 \beta + \lambda_1 \cos^2 \beta, \\
 \mathcal{M}_{22}^2 &= R \cos^2 \beta + \lambda_2 \sin^2 \beta, \\
 \mathcal{M}_{12}^2 &= (\lambda_3 + \lambda_4 + \text{Re}\lambda_5 - R) \frac{\sin 2\beta}{2}, \\
 \mathcal{M}_{33}^2 &= R - \text{Re}\lambda_5,
 \end{aligned}$$

with

$$R = \frac{\text{Re}(m_{12}^2)}{v_1 v_2}$$

- Diagonalization of the mass matrix

$$\mathcal{M}_{\text{diag}}^2 = \mathcal{R}\mathcal{M}^2\mathcal{R}^\dagger,$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \mathcal{R} \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ A \end{pmatrix}$$

- Parametrization of the rotation matrix

$$\begin{aligned} \mathcal{R} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_c & s_c \\ 0 & -s_c & c_c \end{pmatrix} \begin{pmatrix} c_b & 0 & s_b \\ 0 & 1 & 0 \\ -s_b & 0 & c_b \end{pmatrix} \begin{pmatrix} -s_a & c_a & 0 \\ c_a & s_a & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -c_b s_a & c_a c_b & s_b \\ c_a c_c + s_a s_b s_c & s_a c_c - c_a s_b s_c & c_b s_c \\ -c_a s_c + s_a s_b c_c & -s_a s_c - c_a s_b c_c & c_b c_c \end{pmatrix}, \end{aligned}$$

where  $s_{a,b,c} = \sin \theta_{a,b,c}$  and  $c_{a,b,c} = \cos \theta_{a,b,c}$ .

- The CP-odd state  $A$  is mixed with CP-even states  $\varphi_1, \varphi_2$   
 → manifest CP violation in the neutral Higgs sector.

• **Neutral Higgs Boson Production**  $e^+e^- \rightarrow Zh_i$  **and**  $e^+e^- \rightarrow h_i h_j$

– **Generalized  $h_i ZZ$  vertices**

$$h_1 ZZ \sim \sin(\beta - \alpha) \cos \theta_b,$$

$$h_2 ZZ \sim \cos(\beta - \alpha) \cos \theta_c - \sin(\beta - \alpha) \sin \theta_b \sin \theta_c,$$

$$h_3 ZZ \sim -\cos(\beta - \alpha) \sin \theta_c - \sin(\beta - \alpha) \sin \theta_b \cos \theta_c.$$

– **The cross sections for  $e^+e^- \rightarrow h_i Z$  processes**

$$\sigma(e^+e^- \rightarrow h_i Z) = \frac{f_i^2 \pi \alpha^2 \lambda^{1/2} (\lambda + 12sm_Z^2) [1 + (1 - 4\sin^2 \theta_W)^2]}{192s^2 \sin^4 \theta_W \cos^4 \theta_W (s - m_Z^2)^2}$$

where where  $f_i$  are the  $h_i ZZ$  coupling given above, and

$$\lambda = \lambda(s, m_h^2, m_Z^2)$$

with

$$\lambda(a, b, c) = (a + b - c)^2 - 4ab$$

– **CP violating coupling**

$$\mathcal{L} = \frac{gm_Z}{2 \cos \theta_W} \frac{\eta}{4} \epsilon_{\mu\nu\alpha\beta} Z^{\mu\nu} Z^{\alpha\beta}$$

induces the CP violation in this process.

→ suppressed by loop



– **Generalized  $Zh_ih_j$  vertices**

$$\begin{aligned} Zh_1h_3 &\sim \cos(\beta - \alpha) \cos \theta_c - \sin(\beta - \alpha) \sin \theta_b \sin \theta_c, \\ Zh_2h_3 &\sim -\sin(\beta - \alpha) \cos \theta_b, \\ Zh_1h_2 &\sim \cos(\beta - \alpha) \sin \theta_c + \sin(\beta - \alpha) \sin \theta_b \cos \theta_c. \end{aligned}$$

– **The cross sections for  $e^+e^- \rightarrow h_ih_j$  processes**

$$\begin{aligned} \sigma(e^+e^- \rightarrow h_ih_j) &= \frac{g^4}{196\pi \cos^2 \theta_W} f_{ij}^2 \left( \frac{8 \sin^4 \theta_W - 4 \sin^2 \theta_W + 1}{\cos^2 \theta_W} \right) \\ &\quad \times \frac{\kappa^3}{\sqrt{s} [(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2]} \end{aligned}$$

where  $f_{ij}$  are the  $h_ih_jZ$  coupling given above, and the kinematic factor

$$\kappa^2 = \frac{\lambda(s, m_{h_i}^2, m_{h_j}^2)}{4s}$$

– **Numerical constraints**

$$\begin{aligned} &: \text{ordering, } m_1 < m_2 < m_3, \\ &: \text{perturbativity, } \frac{\lambda}{4\pi} < 1 \end{aligned}$$

- Discussion on a few limiting cases

- **If  $\theta_b = \theta_c = 0$  :**

- **CP conserving case**

- $h_1, h_2 \sim$  **CP-even**,  $h_3 \sim$  **CP-odd**

$\sigma(e^+e^- \rightarrow Zh_3), \sigma(e^+e^- \rightarrow h_1h_2)$  are suppressed.

- **If  $\sin \theta_b \sim \sin \theta_c \sim 1$  :**

- $h_1 \sim$  **CP-odd**,  $h_2, h_3 \sim$  **CP-even**

- **If  $\sin \theta_b \sim 0, \sin \theta_c \sim 1$  :**

- $h_2 \sim$  **CP-odd**,  $h_1, h_3 \sim$  **CP-even**

- **If  $\sin \theta_c \sim 0$  :**

$$\begin{aligned}\mathcal{M}_{13}^2 &= s_a c_b s_b (m_3^2 - m_1^2), \\ \mathcal{M}_{23}^2 &= -c_a c_b s_b (m_3^2 - m_1^2).\end{aligned}$$

- $\tan \beta \approx -\tan \theta_a,$

- $\beta \approx -\theta.$

$$\text{Im } \lambda_5 = \sin 2\theta_b \frac{m_3^2 - m_1^2}{v^2}.$$

- **Additionally**  $\sin \theta_b \sim 1$  :  
 $\rightarrow h_1 \sim \text{CP-odd}, h_2, h_3 \sim \text{CP-even}$

**but**

$$\begin{aligned} h_2 ZZ &\sim \cos(\beta - \alpha), \\ h_3 ZZ &\sim -\sin(\beta - \alpha), \\ h_1 h_3 Z &\sim \cos(\beta - \alpha), \\ h_1 h_2 Z &\sim \sin(\beta - \alpha), \end{aligned}$$

$h_2, h_3$  couplings are exchanged!

$$\frac{g_{h_2 ZZ}}{g_{h_3 ZZ}} = \frac{1}{\tan(\beta - \alpha)}$$

**while**

$$\frac{g_{hZZ}}{g_{HZZ}} = \tan(\beta - \alpha)$$

**in the CP conserving case.**

*ZZZZZZ*

$$m_1 = 100 \text{ GeV} \quad m_2 = 300 \text{ GeV} \quad m_{H^+} = 350 \text{ GeV}$$

$$\tan \beta = 0.7 \quad \sqrt{s} = 500 \text{ GeV}$$

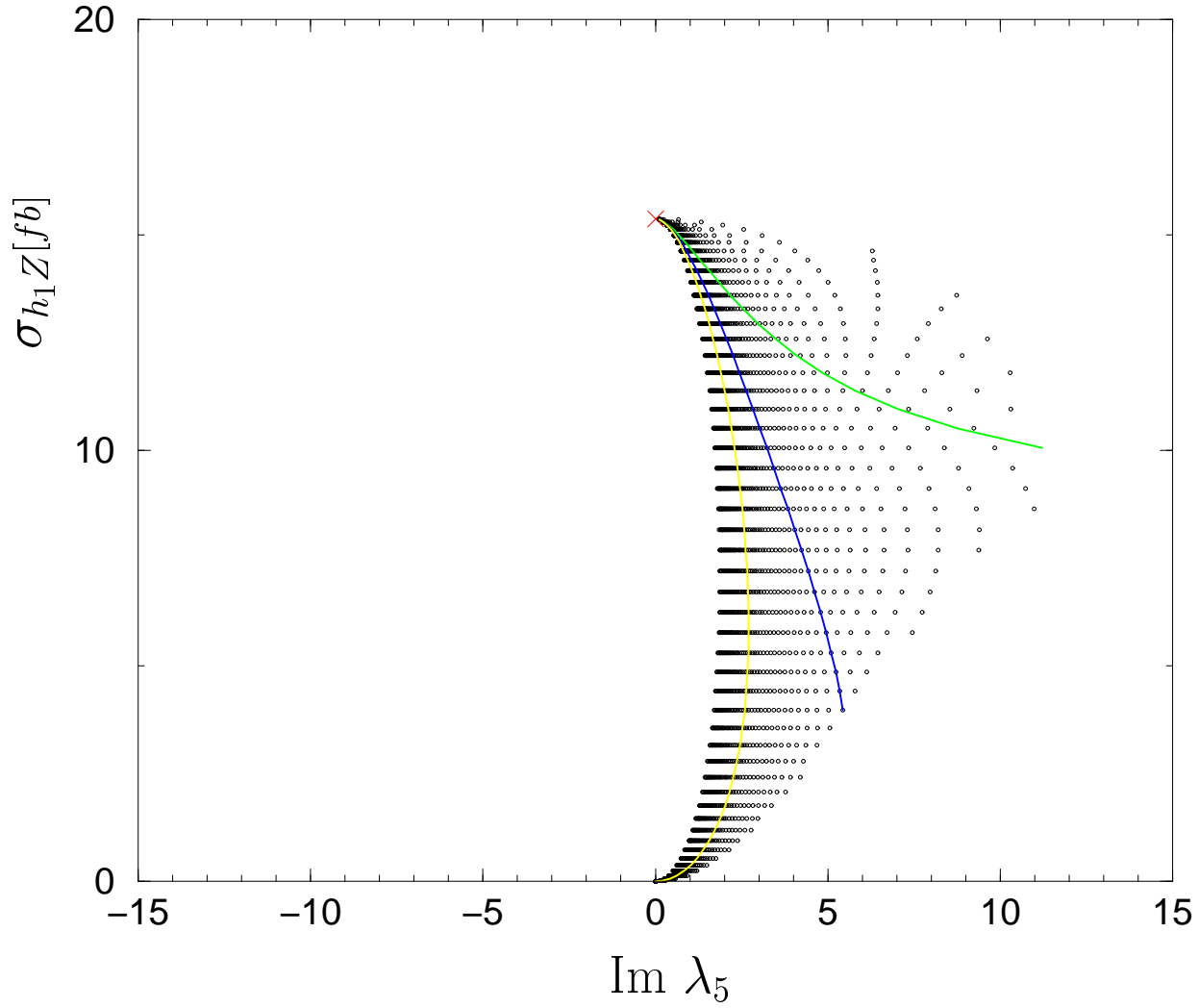


Figure 1: Cross sections for  $e^-e^+ \rightarrow Zh_1$  process.  $G=\theta_c = \pi/6$ ,  $B=\theta_c = \pi/4$ ,  $Y=\theta_c = \pi/3$ .

$$m_1 = 100 \text{ GeV} \quad m_2 = 300 \text{ GeV} \quad m_{H^+} = 350 \text{ GeV}$$

$$\tan \beta = 0.7 \quad \sqrt{s} = 500 \text{ GeV}$$

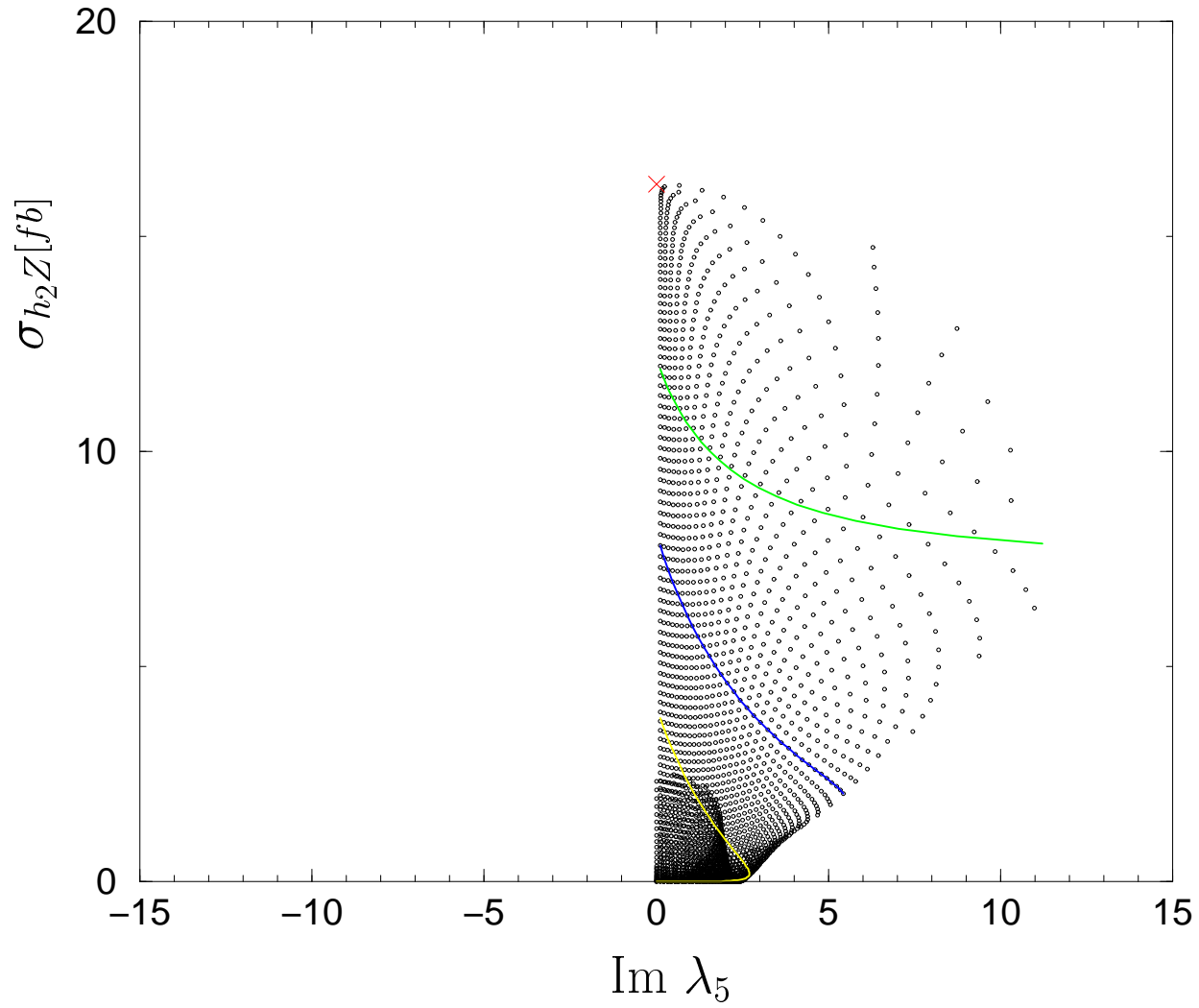


Figure 2: Cross sections for  $e^-e^+ \rightarrow Zh_2$  process.

$$m_1 = 100 \text{ GeV} \quad m_2 = 300 \text{ GeV} \quad m_{H^+} = 350 \text{ GeV}$$

$$\tan \beta = 0.7 \quad \sqrt{s} = 500 \text{ GeV}$$

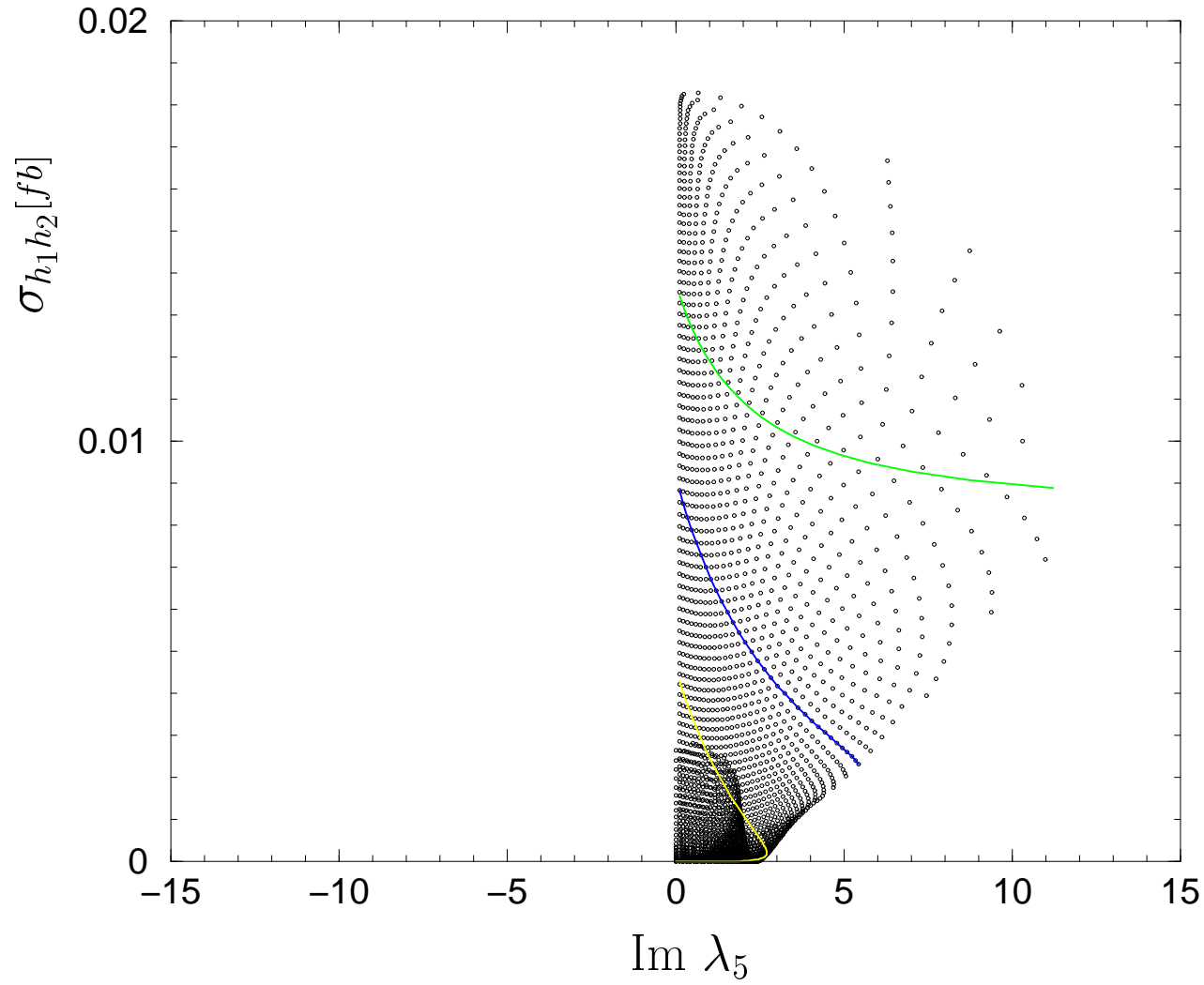


Figure 3: Cross sections for  $e^-e^+ \rightarrow h_1 h_2$  process.

# Summary

- The 2 Higgs doublet model with CP violation may enhance the  $e^-e^+ \rightarrow Zh$  and  $e^-e^+ \rightarrow h_i h_j$  cross sections compared with those of CP conserving case.
- In the limit of  $\sin \theta_c \rightarrow 0$  and  $\sin \theta_b \rightarrow 1$ , the ratio of  $hZZ$  and  $HZZ$  couplings are reversed to that of the CP conserving case and the mixing angle  $\alpha(= \theta_a)$  is close to  $-\beta$ .
- The neutral Higgs boson production has very sensitive behavior near the CP conserving case.
- The 2 Higgs doublet model with CP violation will be able to be tested at the LC through neutral Higgs boson production.