Fermions and Little Higgs

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OTTO C. W. KONG
— Nat'l Central U, Taiwan

Standard Model :-

- gauge symmetry
- anomaly free chiral fermion spectrum
- a Higgs multiplet for EW symmetry breaking

Going beyond the Standard Model :-

- extending (EW) gauge symmetry $\Longrightarrow$ new fermions and scalars

Vs

- grand unification (coupling unification)

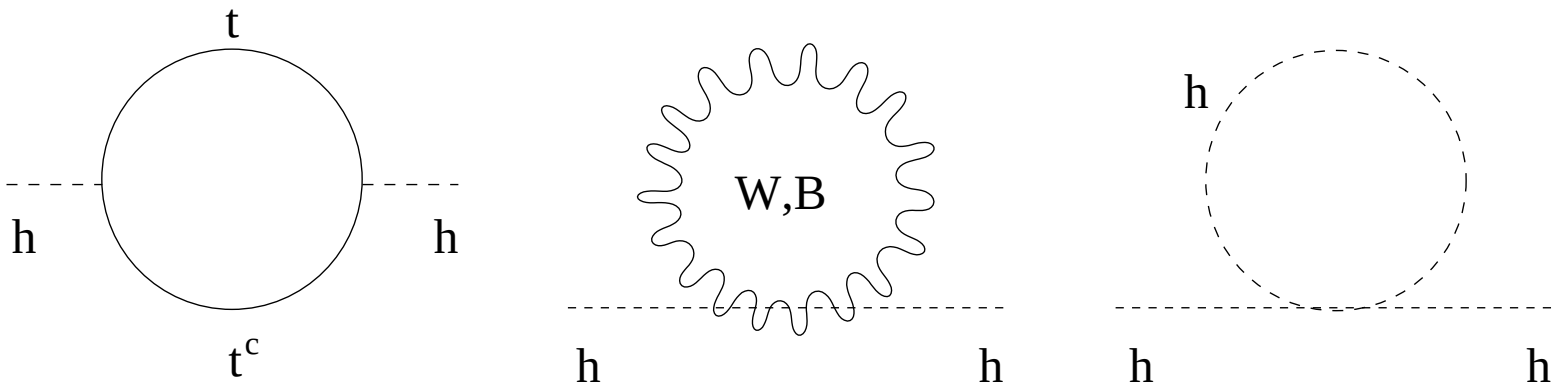
  but desert till 10^{16} GeV

- + supersymmetry (hierarchy problem)

$\star$ Flavor Problem(s) *Why 3 families ? !* .

The Hierarchy Problem :-

— one-loop corrections to the boson mass contain quadratically divergent contributions



(a) (b) (c)

e.g. for $\Lambda = 10$ (TeV)

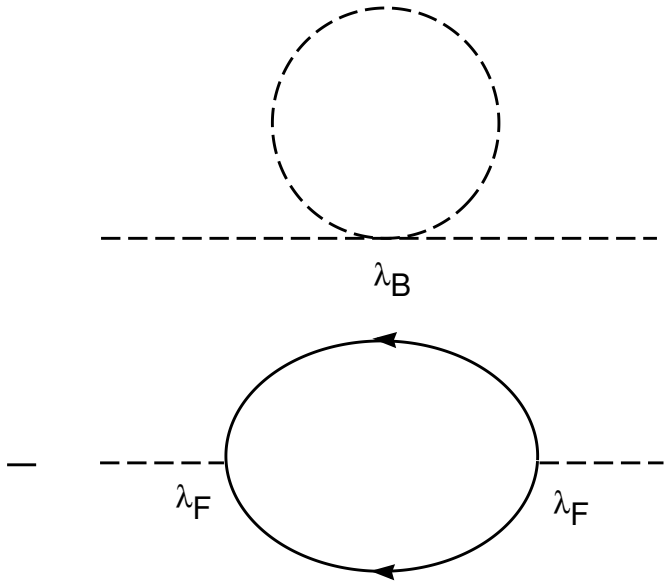
Schmaltz hep-ph/0210415

$$-\frac{3}{8\pi^2}\lambda_t^2\Lambda^2 \sim -(2)^2 \qquad \frac{1}{16\pi^2}g^2\Lambda^2 \sim (0.7)^2 \qquad \frac{1}{16\pi^2}\lambda^2\Lambda^2 \sim (0.5)^2$$

$$m_h^2 = m_{tree}^2 - [100 - 10 - 5]\ (200GeV)^2$$

Solution — cancellation by pairing (protected by symmetry)

e.g. supersymmetry :

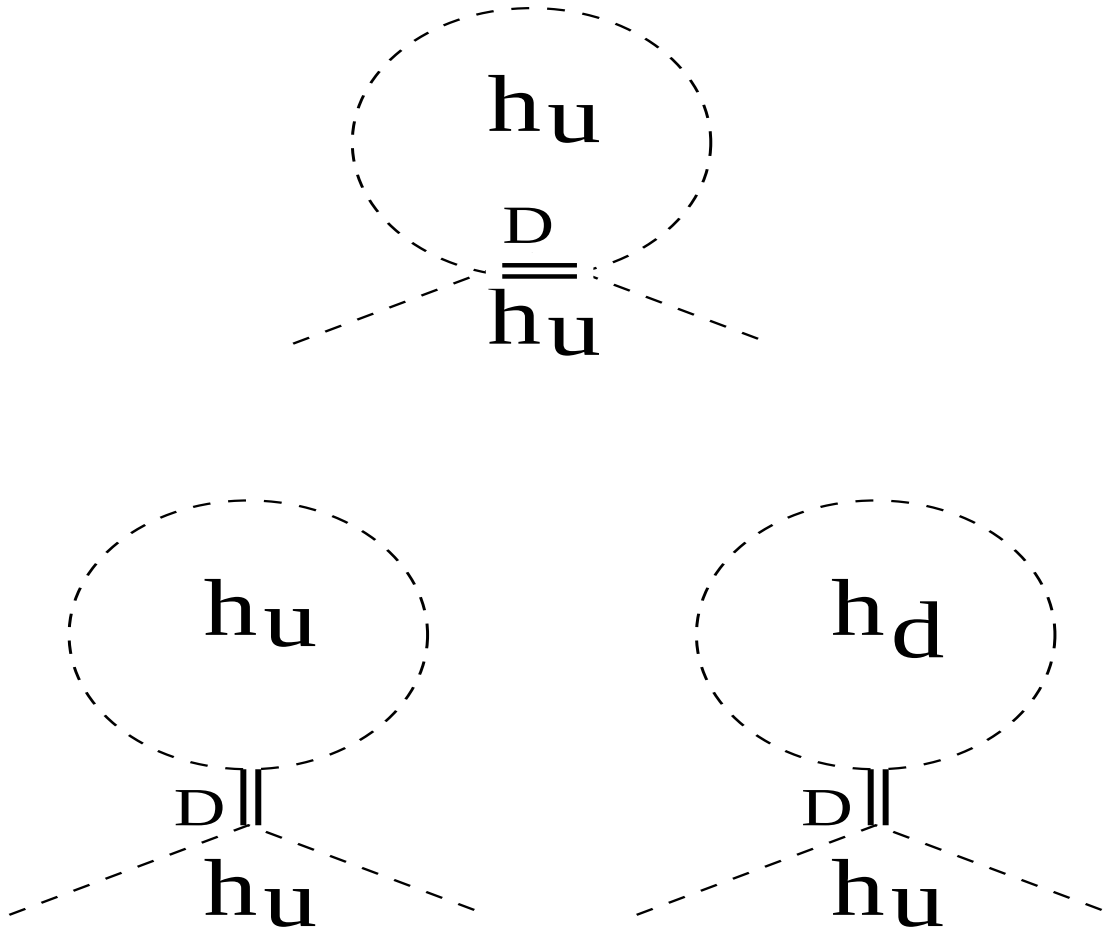


$$\delta M_B^2 \;=\; \frac{\lambda_B}{16\pi^2}\,\Lambda^2 \;-\; \frac{\lambda_F^2}{16\pi^2}\,\Lambda^2$$

$$\lambda_B = \lambda_F^2 \longrightarrow \delta M_B^2 = 0$$

- *note* (Schmaltz) there is also bosonic pairing

— even in SUSY (SSM) : squark, gaugino, & *higgsino*



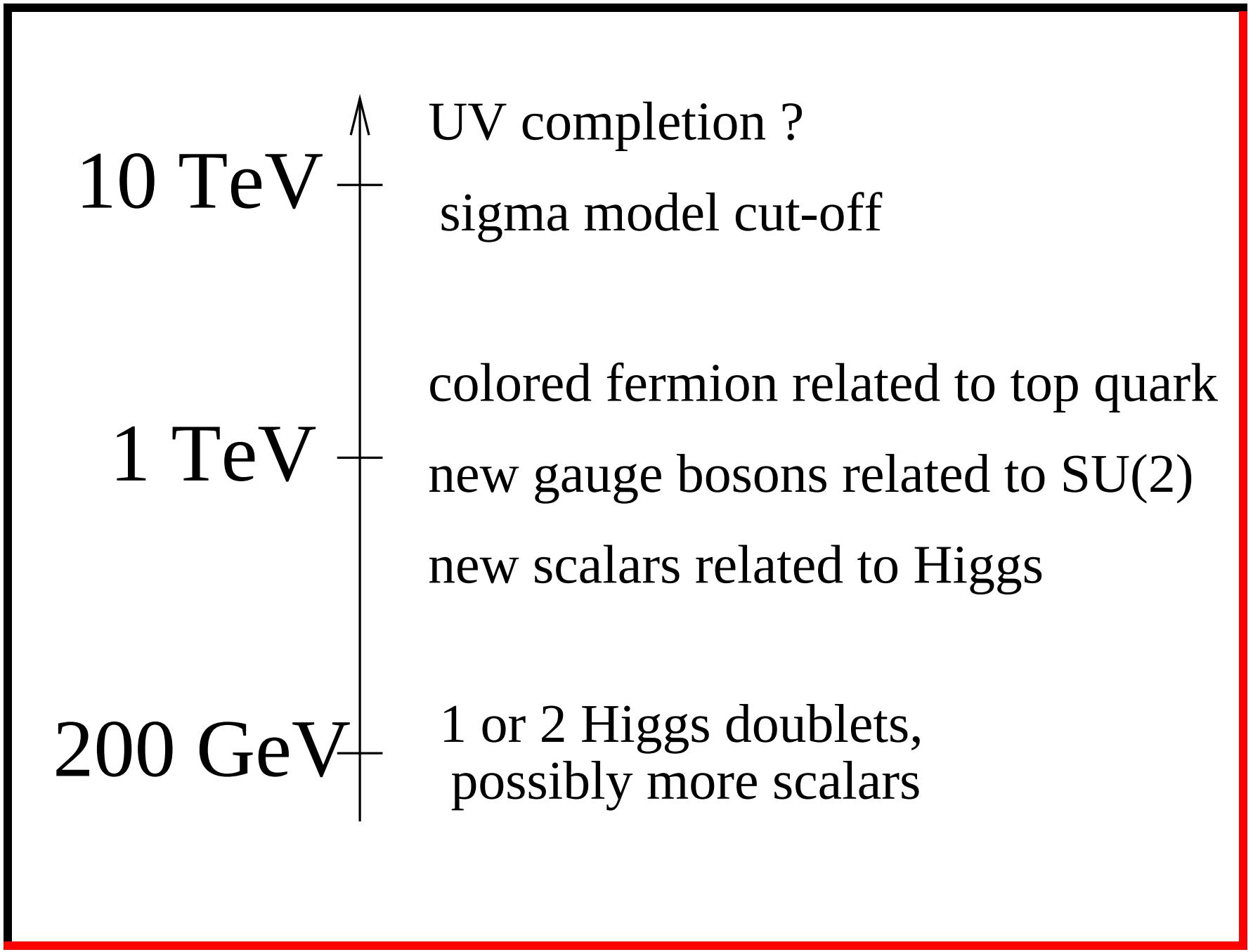
The Little Higgs Idea :-

- cancel *only* one-loop quadratic divergence
- mainly for the t contribution
- use only bosonic symmetry

— Higgs as *pseudo-Nambu-Goldstone boson*
described by nonlinear sigma model

$\Longrightarrow$ produce *safe* little hierarchy

— to be completed by background strong dynamics at 10-50 TeV
cf. (TeV scale) technicolor


10 TeV
UV completion ?
sigma model cut-off
1 TeV
colored fermion related to top quark
new gauge bosons related to SU(2)
new scalars related to Higgs
200 GeV
1 or 2 Higgs doublets,
possibly more scalars

A Toy Model (Kaplan & Schmaltz)

hep-ph/0302049

- extended EW gauge symmetry

— $SU(3)_L \times U(1)_X$ with $Q^a = (T^a, t^a, b^a)^T$

- Higgs sector global symmetry protecting hierarchy

— $[SU(3)/SU(2)]^2$

— *violated* by gauge and Yukawa couplings

— but one-loop diagrams are protected

e.g. Φ_1 & Φ_2 with *aligned* VEVs

$$\mathcal{L}_{top} = y_1\, \bar{t}'_a\, \Phi_1\, Q^a + y_2\, \bar{T}'_a\, \Phi_2\, Q^a$$

$$= f\,(y_1\, \bar{t}' + y_2\, \bar{T}')\, T + \frac{i}{\sqrt{2}}\,(y_1\, \bar{t}' - y_2\, \bar{T}')\, h \begin{pmatrix} t \\ b \end{pmatrix} + \cdots$$

- either $\Phi \longrightarrow$ 5 NGB states from gauged $SU(3) \to SU(2)$

- either $\Phi \longrightarrow$ 5 NGB states from (axial) global $SU(3) \to SU(2)$

— protected from having mass term

- nonlinear sigma model —

$$\Phi_1 = e^{i\theta/f} \begin{pmatrix} 0 & 0 & f \end{pmatrix}^T \qquad \Phi_2 = e^{-i\theta/f} \begin{pmatrix} 0 & 0 & f \end{pmatrix}^T$$

$$\theta = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h^\dagger & & 0 \end{pmatrix} + \frac{\eta}{4} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Towards Realistic Little Higgs Models :-

— Is it a really viable alternative to SUSY (SSM) ?

- nice toy models *that work*
- need to have realistic models

— has to be complete

— does incorporate everything in SM

— need consistent fermion spectrum

- the Major Step :

— anomaly free fermionic spectra for the extended EW symmetries

(3 families SM fermion + vectorlike states)

SM spectrum :-

— one-family spectrum *very unique*

- taking $SU(3)_C \times SU(2)_L \times U(1)_Y$
- assuming a $(3, 2, 1)$ multiplet
- *obtained* as the minimal chiral set free from all gauge anomalies

O.K. Mod. Phys. Lett. A11, 2547
O.K. Phys. Rev. D55, 383

— $SU(3)$ requires $(\bar{3}, 1, a)$ and $(\bar{3}, 1, b)$

— $SU(2)$ requires an extra $(1, 2, c)$

— $U(1)$ anomalies have no solution

$\longrightarrow$ adding a $(1, 1, k)$ give *the* unique solution

$\star$ idea extended to derive the 3-family spectrum

331 little Higgs Model :-

— there is a solution (existence not *a priori* clear)

	Gauge anomalies					$U(1)_Y$ states	
	tX	LLL	LLX	CCX	X^3		
$(\mathbf{3_C}, \mathbf{3_L}, \frac{\mathbf{1}}{\mathbf{3}})$	3	3	1	1	$\frac{1}{3}$	$\frac{\mathbf{1}}{\mathbf{6}}[Q]$	$\frac{\mathbf{2}}{\mathbf{3}}(T)$
2 $(\mathbf{3_C}, \mathbf{\bar{3}_L}, \mathbf{0})$	0	-6	0	0	0	2 $\frac{\mathbf{1}}{\mathbf{6}}[Q]$	2 $\frac{\mathbf{-1}}{\mathbf{3}}(D, S)$
3 $(\mathbf{l_C}, \mathbf{3_L}, \frac{\mathbf{-1}}{\mathbf{3}})$	-3	3	-1		$\frac{-1}{3}$	3 $\frac{\mathbf{-1}}{\mathbf{2}}[L]$	3 $\mathbf{0}(N)$
4 $(\mathbf{\bar{3}_C}, \mathbf{1_L}, \frac{\mathbf{-2}}{\mathbf{3}})$	-8			$\frac{-8}{3}$	$\frac{-32}{9}$	4 $\frac{\mathbf{-2}}{\mathbf{3}}$ $(\bar{u}, \bar{c}, \bar{t}, \bar{T})$	
5 $(\mathbf{\bar{3}_C}, \mathbf{1_L}, \frac{\mathbf{1}}{\mathbf{3}})$	5			$\frac{5}{3}$	$\frac{5}{9}$	5 $\frac{\mathbf{1}}{\mathbf{3}}$ $(\bar{d}, \bar{s}, \bar{b}, \bar{D}, \bar{S})$	
3 $(\mathbf{1_C}, \mathbf{1_L}, \mathbf{1})$	3				3	3 $\mathbf{1}$ (e^+, μ^+, τ^+)	
Total	0	0	0	0	0		

$\star$ realistic little Higgs models $\Longrightarrow$ *new perspective* on flavor physics
(also neutrino physics)

• may need to go to $SU(4)_L \times U(1)_X$ model (*also fine*)

Flavor Structure of the 331-little Higgs Model(s) :-

- top Yukawa — $y_1\,\bar{t}'_a\,\Phi_1\,Q^a + y_2\,\bar{T}'_a\,\Phi_2\,Q^a \longrightarrow y_t\,\bar{t}\,h\begin{pmatrix} t \\ b \end{pmatrix}$
- bottom Yukawa — no $\bar{b}\,\Phi_i^\dagger\,Q^a$ but $\bar{b}\,\Phi_i^\dagger\,\Phi_j^\dagger\,Q^a$
- extra S and D may be relevant to B (b) physics

$\star$ others — from the gauge symmetry only

— u and c Yukawa $\to 1_L\,\Phi_i\,\Phi_j\,\bar{3}_L$

— d and s Yukawa $\to 1_L\,\Phi_i^\dagger\,\bar{3}_L$

— *family universal* leptonic Yukawa $\to \ell^+\,\Phi_i^\dagger\Phi_j^\dagger\,L$

- also extra singlet neutrinos $\cdots\cdots$

quark masses — admit generic mass matrices

note : L-handed quark mixings

$$\mathcal{L}_{top} = \lambda_1^t\, \bar{t'}_a\, \Phi_1\, Q^a + \lambda_2^t\, \bar{T'}_a\, \Phi_2\, Q^a$$

$$= f\,(\lambda_1^t\, \bar{t'} + \lambda_2^t\, \bar{T'})\, T + \frac{i}{\sqrt{2}}\,(\lambda_1^t\, \bar{t'} - \lambda_2^t\, \bar{T'})\, h \begin{pmatrix} t \\ b \end{pmatrix} + \cdots$$

$$\mathcal{L}_{Q'} = \frac{1}{M}\lambda^u_{\alpha j}\, \bar{u'}_\alpha\, \Phi_1\, \Phi_2\, Q'_j = \frac{-i\sqrt{2}\, f}{M}\lambda^u_{\alpha j}\, \bar{u'}_\alpha\, h \begin{pmatrix} u_j \\ d_j \end{pmatrix} + \cdots$$

$$\mathcal{L}_{down} = \lambda^{d1}_{\beta j}\, \bar{d'}_\beta\, \Phi_1^\dagger\, Q'_j + \lambda^{d2}_{\beta j}\, \bar{d'}_\beta\, \Phi_2^\dagger\, Q'_j + \frac{1}{M}\lambda^b_\beta\, \bar{d'}_\beta\, \Phi_1^\dagger\, \Phi_2^\dagger\, Q$$

$$= f\,(\lambda^{d1}_{\beta j}\, \bar{d'}_\beta + \lambda^{d2}_{\beta j}\, \bar{d'}_\beta)\, D_j - \frac{i}{\sqrt{2}}\,(\lambda^{d1}_{\beta j}\, \bar{d'}_\beta - \lambda^{d2}_{\beta j}\, \bar{d'}_\beta)\, h^\dagger \begin{pmatrix} u_j \\ d_j \end{pmatrix}$$

$$+ \frac{i\sqrt{2}\, f}{M}\lambda^b_\beta\, \bar{d'}_\beta\, h^\dagger \begin{pmatrix} t \\ b \end{pmatrix} + \cdots$$

$$\mathcal{M}^q = \begin{pmatrix} m^q & 0 \\ m^{Qq} & m^Q \end{pmatrix} \quad — \begin{cases} 3+1 & \mathcal{M}^u \\ 3+2 & \mathcal{M}^d \end{cases}$$

SMALL PART OF THE STORY :-

- Z^0 boson couplings — $T_f^3 - \mathcal{Q}_f \sin^2\theta_W$

— mixings of states with different T_f^3 values

— CKM unitarity violation, FCNC, ...

$$\Longrightarrow \quad U_L^f = \begin{pmatrix} K^f & R^f \\ S^f & T^f \end{pmatrix} \qquad e.g. \quad S^c \simeq \frac{-1}{m_T}\, m^{Tc}$$

$g_L(c) = \frac{1}{2}\left[1 - |S_c|^2\right] - \frac{2}{3}\sin^2\theta_W \qquad g_L(\bar{u}c) = \frac{1}{2}\left[-S_u^* S_c\right]$

— hadronic Z-width gives < 0.014

$SU(4)_L \times U(1)_X$ **little Higgs Model** — 2nd example

	$U(1)_Y$-states		
$(\mathbf{3_C}, \mathbf{4_L}, \frac{\mathbf{1}}{\mathbf{6}})$	$\frac{\mathbf{1}}{\mathbf{6}}[Q]$	$\frac{\mathbf{2}}{\mathbf{3}}(T)$	$\frac{\mathbf{-1}}{\mathbf{3}}(B)$
$2\ (\mathbf{3_C}, \bar{\mathbf{4}}_\mathbf{L}, \frac{\mathbf{1}}{\mathbf{6}})$	$2\ \frac{\mathbf{1}}{\mathbf{6}}[2\ Q]$	$2\ \frac{\mathbf{-1}}{\mathbf{3}}(D, S)$	$2\ \frac{\mathbf{2}}{\mathbf{3}}(U, C)$
$3\ (\mathbf{l_C}, \mathbf{4_L}, \frac{\mathbf{-1}}{\mathbf{2}})$	$3\ \frac{\mathbf{-1}}{\mathbf{2}}[3\ L]$	$3\ \mathbf{0}(3\ N)$	$3\ \mathbf{-1}(3\ E^-)$
$6\ (\bar{\mathbf{3}}_\mathbf{C}, \mathbf{1_L}, \frac{\mathbf{-2}}{\mathbf{3}})$	$4\ \frac{\mathbf{-2}}{\mathbf{3}}\ (\bar{u}, \bar{c}, \bar{t}, \bar{T})$		$2\ \frac{\mathbf{-2}}{\mathbf{3}}\ (\bar{U}, \bar{C})$
$6\ (\bar{\mathbf{3}}_\mathbf{C}, \mathbf{1_L}, \frac{\mathbf{1}}{\mathbf{3}})$	$5\ \frac{\mathbf{1}}{\mathbf{3}}\ (\bar{d}, \bar{s}, \bar{b}, \bar{D}, \bar{S})$		$\frac{\mathbf{1}}{\mathbf{3}}\ (\bar{B})$
$6\ (\mathbf{1_C}, \mathbf{1_L}, \mathbf{1})$	$3\ \mathbf{1}\ (e^+, \mu^+, \tau^+)$		$3\ \mathbf{1}(3\ E^+)$

Flavor Structure of new 341-little Higgs Model :-

- top Yukawa — standard
- bottom Yukawa — same as t
- Higgs sector — 2 doublets (+ singlets) with NFC (?), large tanβ
— 2 set of 2 aligned Φ's with opposite X-charges
- b-loop quadratic divergence canceled by B (as t-T)

$\star$ duplicated fermion spectrum (*e.g.* U, D, C, S, also T, B)

- need the global $SU(4)$'s to suppress other Yukawa
- also extra singlet neutrinos $\cdots\cdots$

Why gauge anomaly cancellation among chiral fermions ?

— for an effective field theory

bottom line :

- it works perfectly for the SM

  an accident Vs a guiding principle

- taking the principle $\Longrightarrow$ definite spectrum predicted

Vs fermion spectrum is arbitrary

+ (Georgi) survival hypothesis : gauge invariant mass $\sim M_{\rm cutoff}$

Final Remarks :-

- fermion spectra is the key
  for taking any (scalar sector) little Higgs model realistic
- $SU(N) \times U(1)$ extended EW symmetry models exist
  with little Higgs compatibility options
- fermion spectra likely to be (SM) family non-universal

— strong implications on flavor structure issues

- anomaly cancellation conditions link the 3 families

— may be a way to answer why 3 $(= N_c)$

- spectra *not* as beautiful as one-family SM (SUSY ?)
  but viable alternatives
- strong flavor constraints $\cdots\cdots$ **other little Higgs models ?**

THANK YOU !