# Higgsless models with and without an extra dimension

## — Structure of corrections to electroweak interactions —

November 10, 2004. 7th ACFA Workshop @ National Taiwan Univ.

Masaharu Tanabashi (Tohoku U.)


- R.S. Chivukula, M. Kurachi and M. T., JHEP **0406**, 004 (2004), [arXiv:hep-ph/0403112].
- R.S. Chivukula, E.H. Simmons, H.-J. He, M. Kurachi and M. T., Phys. Rev. **D70**:075008 (2004), [arXiv:hep-ph/0406077].
- R.S. Chivukula, E.H. Simmons, H.-J. He, M. Kurachi and M. T., arXiv:hep-ph/0408262, to appear in Phys. Lett. **B**.
- R.S. Chivukula, E.H. Simmons, H.-J. He, M. Kurachi and M. T., arXiv:hep-ph/0410154.

## §.0. Introduction

**The Higgs boson has not been discoverd.**

- Too heavy for present colliders? $M_H > 114.4\text{GeV}$.

  We just need to wait and see the results of LHC.
- Alternative to Higgs? various theoretical flaws in Higgs scenario.

  Interesting possibility: Higgsless models.

  - Higgsless models in 4D (a.k.a. technicolor) are almost ruled out by precision tests at LEP/SLC
  - Higgsless models in 5D : new comer
    * C.Csaki, C.Grojean, H.Murayama, L.Pilo, and J.Terning, Phys.Rev.D **69**, 055006 (2004).
    * C.Csaki, C.Grojean, L.Pilo, and J.Terning, Phys.Rev.Lett. **92**, 101802 (2004).
    * Y. Nomura, JHEP 11 (2003) 050.
    * R.Barbieri, A.Pomarol, and R.Rattazzi, Phys.Lett. B**591**, 141 (2004).

## Role of Higgs in the $W_L W_L$ scattering amplitude

B.W.Lee, C.Quigg, and H.B.Thacker

- Higgs mass $M_H$ is a free parameter in the standard model (SM).
- Imagine a super-heavy Higgs in the SM.

Equivalence theorem

$$\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) \simeq \mathcal{M}(w^a w^b \to w^c w^d) \qquad \text{for } E \gg M_W.$$

$W_L$ : Longitudinal polarization of $W$ boson

$\simeq w$ : would-be Nambu-Goldstone boson

Low energy theorem (LET) of $ww$ scattering

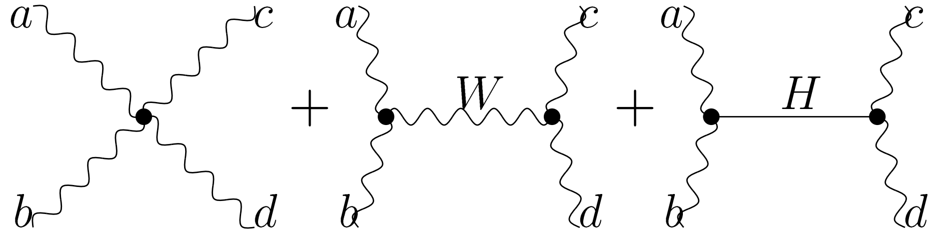
$$\mathcal{M}(w^a w^b \to w^c w^d) = \frac{s}{v^2}\delta^{ab}\delta^{cd} + \frac{t}{v^2}\delta^{ac}\delta^{bd} + \frac{u}{v^2}\delta^{ad}\delta^{bc} + \cdots, \quad v \simeq 246\text{GeV},$$

for energy below $M_H$.

Violation of tree-level unitarity at $\sqrt{s} \simeq 1$TeV. (non-perturbative)

## $W_L W_L$ scattering amplitude in the SM

$$i\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) =$$




$$+\text{crossed.}$$

For $E \gg M_W$

$$\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) = \frac{\lambda s}{M_H^2 - s}\delta^{ab}\delta^{cd} + \frac{\lambda t}{M_H^2 - t}\delta^{ac}\delta^{bd} + \frac{\lambda u}{M_H^2 - u}\delta^{ad}\delta^{bc},$$

with

$$M_H^2 = \lambda v^2.$$

- This result is consistent with the LET for $s, t, u \ll M_H^2$.
- Theory remains perturbative even at high energy $\sqrt{s} \gg 1$ TeV with a light Higgs mass $M_H \ll 1$ TeV.
  A light Higgs "unitarizes" the $W_L W_L$ scattering amplitude.

## Two possibilities of Higgsless theories

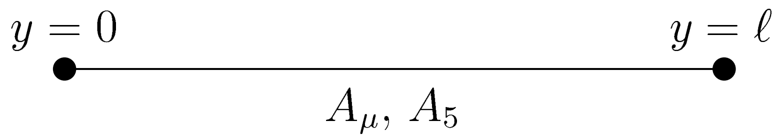
- $W_L W_L$ scattering becomes non-perturbative at $\sqrt{s} \simeq 1\text{TeV}$.
  There is no resonance $\lesssim 1\text{TeV}$ which unitarizes the $W_L W_L$ scattering amplitude.
  Technicolor theories.

- $W_L W_L$ scattering remains perturbative at $\sqrt{s} \simeq 1\text{TeV}$.
  The violation of tree-level unitarity is delayed.
  Resonance $\lesssim 1\text{TeV}$ unitarizes the $W_L W_L$ scattering amplitude.
  Higgsless models in 5D.

# §.1. Higgsless models in 5D

## Gauge symmetry breaking from boundary conditions

Example: 5D gauge theory with an extra dimension compactified on an **interval**.




Dirichlet or Neumann BC?

1. $A_\mu(x,y)|_{y=0} = 0$ (D), $\quad \partial_5 A_5(x,y)|_{y=0} = 0$ (N),
   $A_\mu(x,y)|_{y=\ell} = 0$ (D), $\quad \partial_5 A_5(x,y)|_{y=\ell} = 0$ (N). [DD]
2. $\partial_5 A_\mu(x,y)|_{y=0} = 0$ (N), $\quad A_5(x,y)|_{y=0} = 0$ (D),
   $\partial_5 A_\mu(x,y)|_{y=\ell} = 0$ (N), $\quad A_5(x,y)|_{y=\ell} = 0$ (D). [NN]
3. $A_\mu(x,y)|_{y=0} = 0$ (D), $\quad \partial_5 A_5(x,y)|_{y=0} = 0$ (N),
   $\partial_5 A_\mu(x,y)|_{y=\ell} = 0$ (N), $\quad A_5(x,y)|_{y=\ell} = 0$ (D). [DN]
4. $\partial_5 A_\mu(x,y)|_{y=0} = 0$ (N), $\quad A_5(x,y)|_{y=0} = 0$ (D),
   $A_\mu(x,y)|_{y=\ell} = 0$ (D), $\quad \partial_5 A_5(x,y)|_{y=\ell} = 0$ (N). [ND]

## Spectrum and 4D gauge symmetry

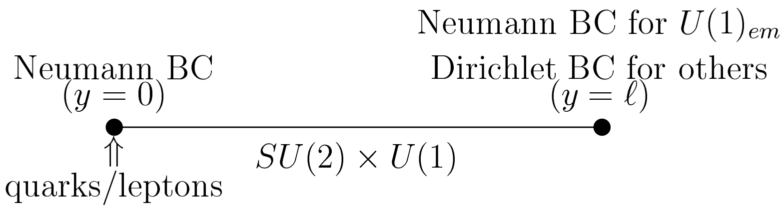
In addition to a tower of massive spin-1 KK-modes, we have

1. [DD]: massless spin-0 particle. 4D gauge sym. is all broken.
2. [NN]: massless spin-1 particle. unbroken 4D gauge sym.
3. [DN]: no massless particle. 4D gauge sym. is all broken.
4. [ND]: no massless particle. 4D gauge sym. is all broken.

Tree level unitarity: Thanks to the exchange of massive spin-1 KK-modes, delay of unitarity violation is achieved with this setup.

- R. Sekhar Chivukula, D. A. Dicus and H. J. He, "Unitarity of compactified five dimensional Yang-Mills theory," Phys. Lett. B **525**, 175 (2002) [arXiv:hep-ph/0111016].
- C. Csaki, C. Grojean, H. Murayama, L. Pilo and J. Terning, "Gauge theories on an interval: Unitarity without a Higgs," Phys. Rev. D **69**, 055006 (2004) [arXiv:hep-ph/0305237].

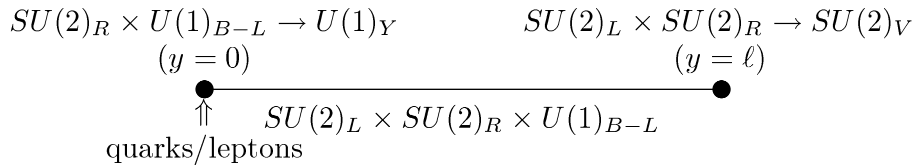
## A toy model of 5D Higgsless




- Massless neutral boson: $\gamma$
- The lightest massive neutral KK mode: $Z$
- The lightest massive charged KK mode: $W$

$\times$ custodial symmetry $\Rightarrow$ $\Delta\rho \neq 0$.

## More realistic(?) model




custodial symmetry $\Rightarrow$ $\Delta\rho = 0$

Realistic configuration satisfying all of electroweak constraints has not yet been found even with this setup.

- C. Csaki, C. Grojean, L. Pilo, and J. Terning, hep-ph/0308038.
- Y. Nomura, hep-ph/0309189.
- R. Barbieri, A. Pomarol, and R. Rattazzi, hep-ph/0310285.

$\vdots$

Exchanges of massive spin-1 KK-modes $\Rightarrow$ Tree level unitarity at 1TeV

## §.2. Deconstruction of 5D Higgsless models

c.f., R.Foadi, S.Gopalakrishna, and C.Schmidt, JHEP 03 (2004) 042.

### Deconstruction (latticization) of extra dimension

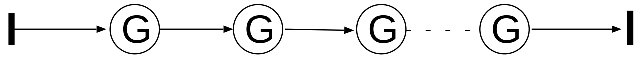
- N. Arkani-Hamed, A. G. Cohen and H. Georgi, "(De)constructing dimensions," Phys. Rev. Lett. **86**, 4757 (2001) [arXiv:hep-th/0104005].
- C. T. Hill, S. Pokorski and J. Wang, "Gauge invariant effective Lagrangian for Kaluza-Klein modes," Phys. Rev. D **64**, 105005 (2001) [arXiv:hep-th/0104035].

Lattice spacing $a$.

- $A_\mu^j(x) = A_\mu(x, y = ja)$ : Gauge field at the site $j$.
- $U_j(x) = \exp(i \int_{(n-1)j}^{ja} dy A_5(x, y))$ : Link field between sites $j-1$ and $j$. Nonlinear $\sigma$ model field.

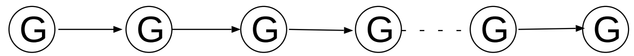
Deconstruction of an interval in "moose" notation:

[DD] case



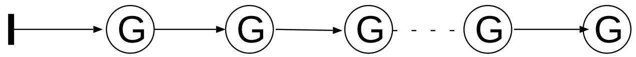

$$\#(U_j) = \#(A^j_\mu) + 1.$$

[NN] case



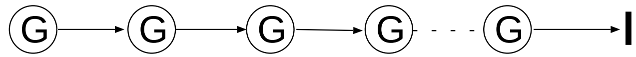

$$\#(U_j) = \#(A^j_\mu) - 1.$$

[DN] case




$$\#(U_j) = \#(A^j_\mu).$$

[ND] case

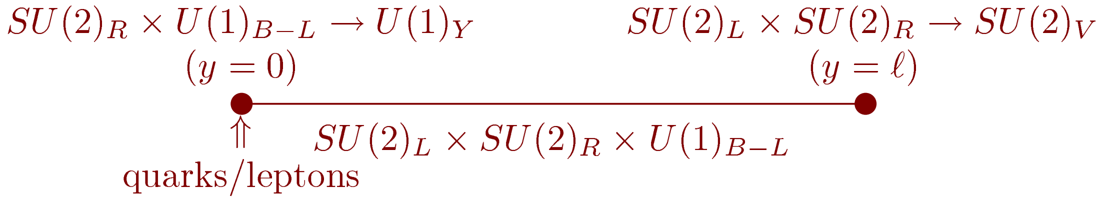



$$\#(U_j) = \#(A^j_\mu).$$

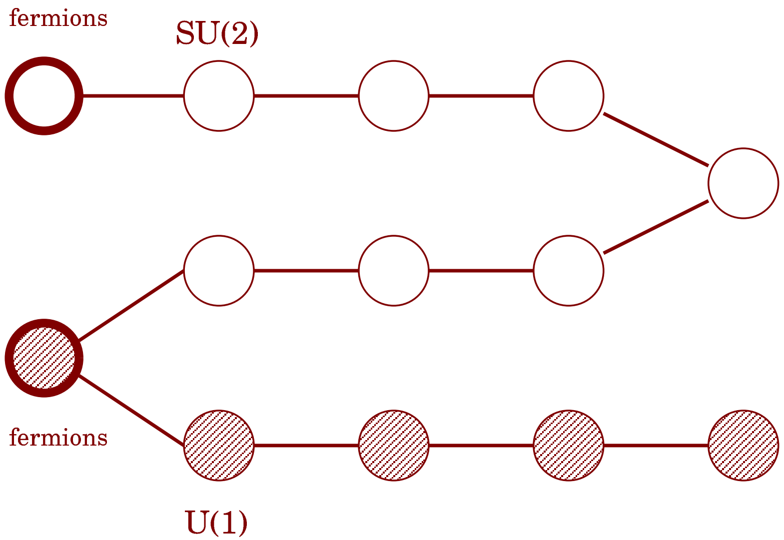
Advantage of deconstruction in 5D Higgsless models

- Familiar language of spontaneous gauge symmetry breaking (gauged nonlinear $\sigma$ model).
- Easier to understand the physics behind the delay of unitarity violation.
- Easier to calculate corrections to electroweak interactions.
- Allows for arbitrary background 5D geometry, spatially dependent gauge couplings, and brane kinetic terms.

## More realistic(?) model




## and its deconstruction




$$\frac{1}{v_\pm^2} = \sum_{j=1}^{N+1} \frac{1}{f_j^2}, \qquad \frac{1}{v_0^2} = \sum_{j=1}^{N+1} \frac{1}{f_j^2}$$

We thus find

$$\rho = 1, \quad \Delta\rho = \rho - 1 = 0.$$

**5D Higgsless models are described by linear moose.**

## §.3. Technicolor & BESS model

We note a similarity between technicolor and 5D Higgsless models.

Existence of massive spin-1 resonance:

- Techni-$\rho$ in technicolor ($\bar{Q}Q$ bound state)
- KK-mode of $W$ and $Z$

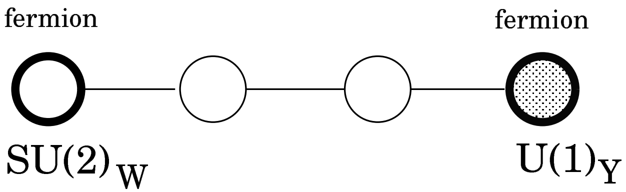
The QCD $\rho$ meson is described as a dynamical gauge boson in the Hidden Local Symmetry (HLS) formalism.

- M.Bando, T.Kugo, S.Uehara, K.Yamawaki, and T.Yanagida, Phys.Rev.Lett. **54** (1985) 1215.
- M.Bando, T.Kugo, K.Yamawaki, Phys.Rept. **164** (1988) 217.

Application of HLS formalism in the electroweak symmetry breaking (a low energy effective theory of technicolor): BESS model

- R.Casalbuoni, S.De Curtis, D.Dominici, and R. Gatto, Phys.Lett.**B155** (1985) 95.

The effective theory of technicolor (BESS model) is described in terms of a linear moose




$SU(2)_W$ and $U(1)_Y$ are weakly gauged.

**Techni-$\rho$ and techni-$a_1$ are described also by a linear moose!**

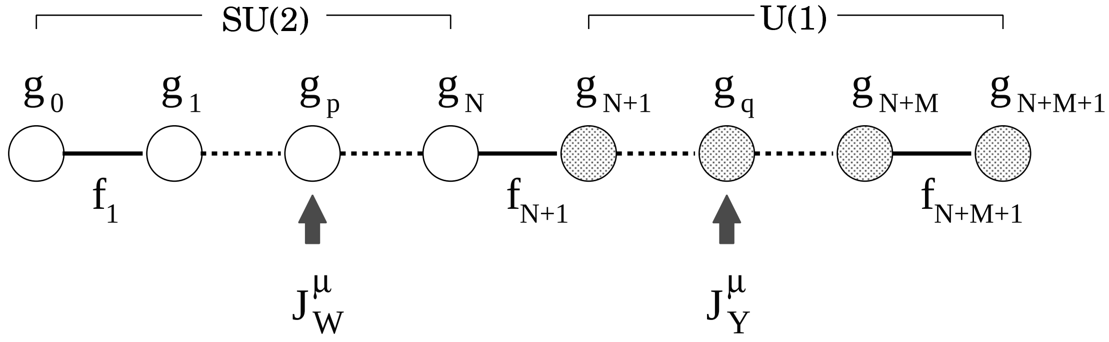
**Linear moose provides a general framework to describe Higgsless models with and without an extra dimesion.**

The precision tests of the electroweak interactions severely restrict technicolor models.

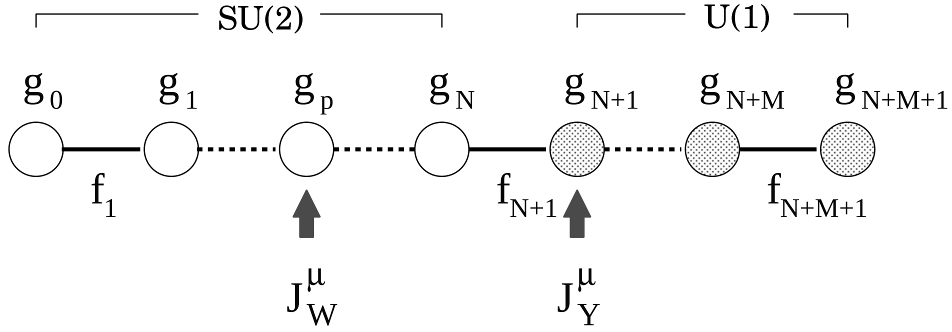
General structure of corrections to electroweak interactions in linear moose models

## §.4. Linear moose models

General linear moose with localized $J_W^\mu$ and $J_Y^\mu$:

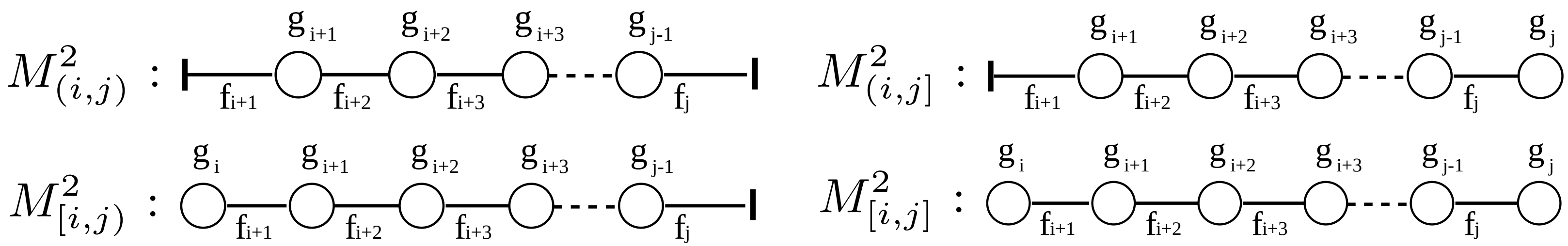



In this talk, we concentrate our attention on "Case I" models ($q = N + 1$):




in which $\Delta\rho = 0$ is satisfied automatically.

“open” and “closed” interval notation:




Gauge boson mass matrices and propagators:

- Neutral current (NC):

$$M^2_{\rm NC} = M^2_{[0,N+M+1]}, \quad [G_{\rm NC}(Q^2)]_{i,j} = g_i g_j([Q^2 + M^2_{\rm NC}]^{-1})_{i,j}$$

- Charged current (CC):

$$M^2_{\rm CC} = M^2_{[0,N+1)}, \quad [G_{\rm CC}(Q^2)]_{i,j} = g_i g_j([Q^2 + M^2_{\rm CC}]^{-1})_{i,j}$$

We also define

$$[G_{\rm CC}(Q^2)]_{WW} = [G_{\rm CC}(Q^2)]_{p,p}, \quad [G_{\rm NC}(Q^2)]_{WW} = [G_{\rm NC}(Q^2)]_{p,p},$$
$$[G_{\rm NC}(Q^2)]_{WY} = [G_{\rm NC}(Q^2)]_{p,q}, \quad [G_{\rm NC}(Q^2)]_{YY} = [G_{\rm NC}(Q^2)]_{q,q}.$$

Four-fermion amplitudes ($f\bar{f} \to f'\bar{f}'$) in the linear moose model

- Neutral current process:

$$
\begin{aligned}
-\mathcal{A}_{\rm NC} &= [G_{\rm NC}(Q^2)]_{WW} I_3 I_3' + [G_{\rm NC}(Q^2)]_{WY} I_3(\mathcal{Q}' - I_3') \\
&\quad + [G_{\rm NC}(Q^2)]_{WY}(\mathcal{Q} - I_3)I_3' + [G_{\rm NC}(Q^2)]_{YY}(\mathcal{Q} - I_3)(\mathcal{Q}' - I_3') \\
&= A(Q^2) I_3 I_3' + B(Q^2)(I_3\mathcal{Q}' + \mathcal{Q}I_3') + C(Q^2)\mathcal{Q}\mathcal{Q}',
\end{aligned}
$$

with

$$
\begin{aligned}
A(Q^2) &\equiv [G_{\rm NC}(Q^2)]_{WW} - 2[G_{\rm NC}(Q^2)]_{WY} + [G_{\rm NC}(Q^2)]_{YY}, \\
B(Q^2) &\equiv [G_{\rm NC}(Q^2)]_{WY} - [G_{\rm NC}(Q^2)]_{YY}, \\
C(Q^2) &\equiv [G_{\rm NC}(Q^2)]_{YY}.
\end{aligned}
$$

- Charged current process:

$$
-\mathcal{A}_{\rm CC} = [G_{\rm CC}(Q^2)]_{WW}(I_+ I_-' + I_- I_+')/2
$$

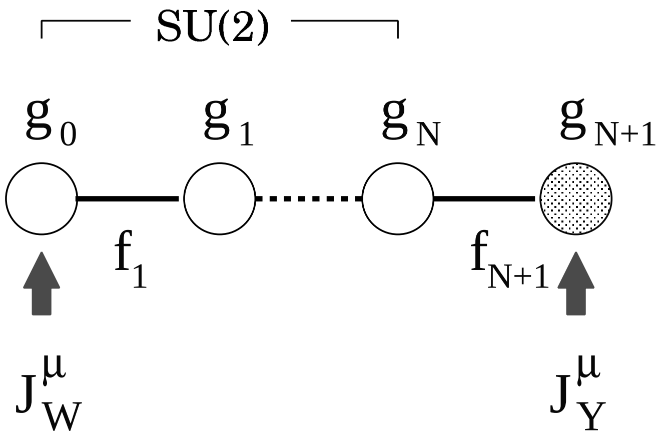
Deviations of $[G_{\rm NC}(Q^2)]$ and $[G_{\rm CC}(Q^2)]$ from their SM values

$\Downarrow$

Corrections to the electroweak interactions in linear moose models

## §.5. Corrections to electroweak interactions

We first consider a special class of models $p = 0$, $M = 0$ in "Case I":




The BESS model (effective theory of technicolor) is included in this class of models.

Number of free parameters: $2N + 3 = (N + 2) + (N + 1)$.
The model is specified completely once $e^2, G_F, M_Z, M_{Z\hat{k}}, M_{W\hat{n}}$ $(\hat{k}, \hat{n} = 1, 2, \cdots, N)$ are fixed.

Properties of $[G_{\rm NC}(Q^2)]_{WY}$:

- Poles at $Q^2 = 0,\ -M_Z^2,\ -M_{Z_1}^2, \cdots, -M_{Z_N}^2$.
- Charge universality of QED determines the pole residue of photon:

$$\lim_{Q^2 \to 0} Q^2 [G_{\rm NC}(Q^2)]_{WY} = e^2.$$

- Generalized Weinberg sum rules:

$$[G_{\rm NC}(Q^2)]_{WY} \propto \frac{1}{Q^{2(N+2)}}, \qquad \text{for } Q^2 \gg M_{Z_N}^2.$$

These conditions are enough to determine the form of $[G_{\rm NC}(Q^2)]_{WY}$ completely:

$$[G_{\rm NC}(Q^2)]_{WY} = \frac{e^2}{Q^2} \frac{M_Z^2}{Q^2 + M_Z^2} \left[ \prod_{\hat{k}=1}^{N} \frac{M_{Z_{\hat{k}}}^2}{Q^2 + M_{Z_{\hat{k}}}^2} \right].$$

We rewrite

$$[G_{\rm NC}(Q^2)]_{WY} = \frac{[\xi_\gamma]_{WY}}{Q^2} + \frac{[\xi_Z]_{WY}}{Q^2 + M_Z^2} + \sum_{\hat{k}=1}^{N} \frac{[\xi_{Z_{\hat{k}}}]_{WY}}{Q^2 + M_{Z_{\hat{k}}}^2},$$

with

$$[\xi_\gamma]_{WY} = e^2, \qquad [\xi_Z]_{WY} = -e^2 \left[ \prod_{\hat{k}=1}^{N} \frac{M_{Z_{\hat{k}}}^2}{M_{Z_{\hat{k}}}^2 - M_Z^2} \right], \qquad \cdots.$$

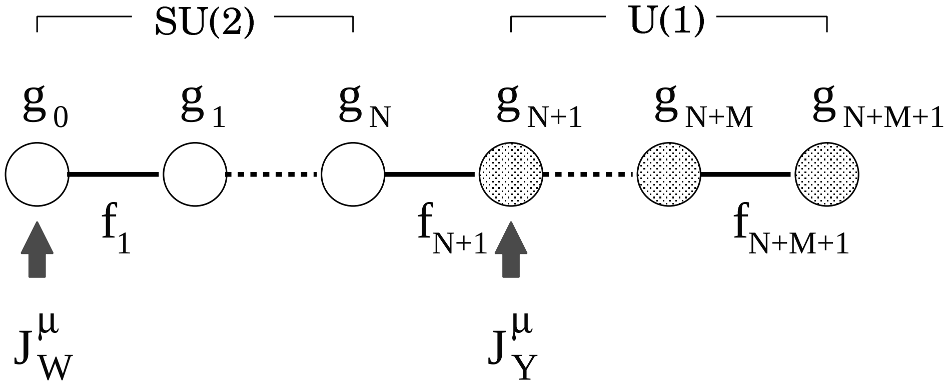
The $S$ parameter can be defined as $[\xi_Z]_{WY} = -e^2 \left[ 1 + \frac{\alpha}{4 s_Z^2 c_Z^2} S \right]$.

We thus obtain manifestly positive expression of $S$

$$\alpha S = 4 s_Z^2 c_Z^2 \sum_{\hat{k}=1}^{N} \frac{M_Z^2}{M_{Z_{\hat{k}}}^2} > 0. \qquad (M_{Z_{\hat{k}}}^2 \gg M_Z^2 \text{ is assumed.})$$

We need $Z'$ lighter than a TeV in order to achieve tree-level unitarity at 1TeV. Such a light $Z'$ gives significant effect to the $S$ parameter: $S \gtrsim 1$, and ruled out by the existing precision tests.

We next consider a little bit general class of models $p = 0$, $M \neq 0$:




Deconstructed version of "more realistic(?)" 5D Higgsless model is included in this class of models.

In this class of models, we find

$$\alpha S = 4 s_Z^2 c_Z^2 M_Z^2 (\Sigma_Z - \Sigma_{(N+1,K+1]}),$$

with

$$\Sigma_Z \equiv \sum_{\hat{k}=1}^{K} \frac{1}{M_{Z\hat{k}}^2}, \qquad \Sigma_{(N+1,K+1]} \equiv \mathrm{tr}\left[\frac{1}{M_{(N+1,K+1]}^2}\right].$$

with $K \equiv N + M$. Negative contribution to $S$!

Electroweak correction parameters $S$, $T$, $\alpha\delta$:

$$
\begin{aligned}
-\mathcal{A}_{\rm NC} &= e^2 \frac{\mathcal{Q}\mathcal{Q}'}{Q^2} + \frac{(I_3 - s^2\mathcal{Q})(I_3' - s^2\mathcal{Q}')}{\left(\frac{s^2c^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2} - \alpha T\right)} \\
&\quad + 4\sqrt{2}G_F \frac{\alpha\delta}{4s^2c^2} I_3 I_3' - 4\sqrt{2}G_F \alpha T (\mathcal{Q} - I_3)(\mathcal{Q}' - I_3'), \\
-\mathcal{A}_{\rm CC} &= \frac{(I_+ I_-' + I_- I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} \\
&\quad + 4\sqrt{2}G_F \frac{\alpha\delta}{4s^2c^2} \frac{(I_+ I_-' + I_- I_+')}{2}.
\end{aligned}
$$

Here $\Delta\rho = 0$ is assumed.

We find

$$\alpha S = 4 s_Z^2 c_Z^2 M_Z^2 \left(\Sigma_Z - \Sigma_{(N+1,K+1]}\right),$$
$$\alpha T = s_Z^2 M_Z^2 \left(\Sigma_Z - \Sigma_W - \Sigma_{(N+1,K+1]}\right),$$

where

$$\Sigma_W \equiv \sum_{\hat{n}=1}^{N} \frac{1}{M_{W\hat{n}}^2}.$$

Thus

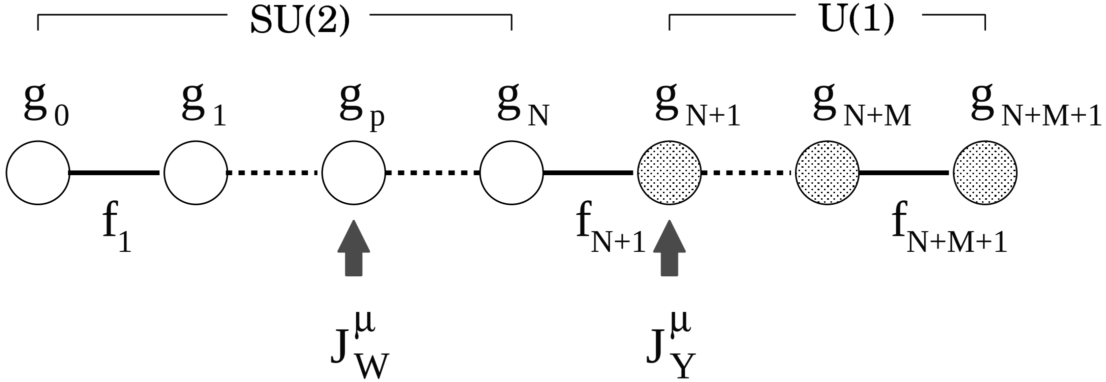
$$\alpha S - 4c_Z^2 \alpha T = 4 s_Z^2 M_W^2 \Sigma_W > 0$$

In any unitary theory, we thus conclude

$$S - 4c_Z^2 T \gtrsim 1,$$

which is clearly inconsistent with precision tests of electroweak interactions.

We finally consider general "Case I" models:




We find

$$\alpha S - 4c_Z^2\alpha T + \frac{\alpha\delta}{c_Z^2} = 4s_Z^2 M_W^2 \Sigma_{(p,N+1)} > 0.$$

Tree level unitarity says $\Sigma_{(p,N+1)} > \dfrac{1}{8\pi v^2}$. We thus obtain

$$S - 4c_Z^2 T + \frac{\delta}{c_Z^2} \gtrsim 1,$$

in any unitary theory.

## §.6. Which model is viable?

**No linear moose model with localized $J^\mu_{W,Y}$ can satisfy the precision electroweak tests and the tree level unitarity at 1 TeV simultaneously.**

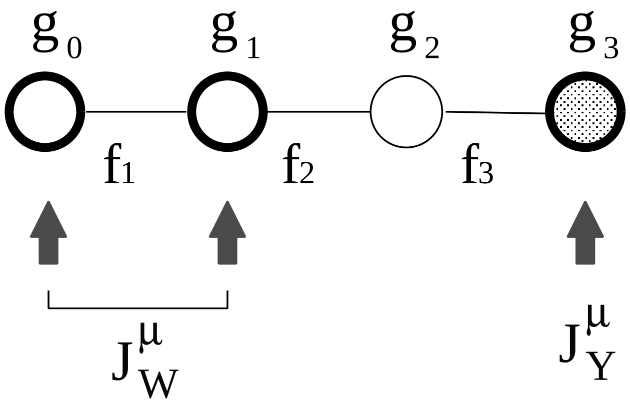
**This conclusion is independent of**

- **background 5D geometry**
- **spatially dependent gauge couplings**
- **brane kinetic terms**
- **etc.**

**We thus need "delocalization" of $J_{W,Y}$ to construct a viable model.**

- G.Cacciapaglia, C.Csaki, C.Grojean, and J.Terning, hep-ph/0409126.
- R.Foadi, S.Gopalakrishna, and C.Schmidt, hep-ph/0409266.

Example: a linear moose with delocalized ferimion




This model can satisfy precision electroweak tests and unitality delay simultaneously by choosing the delocalization parameter.

**Collider signature of this viable Higgsless model**

Existence of KK-mode of $W$ and $Z$ ($W'$,$Z'$ or techni-$\rho$) below a TeV.

## §.7. Summary and Outlooks

- **Higgsless theory is an interesting alternative to the standard model Higgs, achieving tree level unitarity at 1TeV.**

- **Both 4D and 5D Higgsless models are described in terms of linear moose.**

- **We need "delocalization" of fermions in order to construct a viable Higgsless model satisfying various requirements of the precision electroweak tests.**

- **KK-modes of $W$ and $Z$ ($W'$, $Z'$ or techni-$\rho$) exist below a TeV in these models. Collider phenomenology of these particles should be investigated.**