

$\bar{B} \rightarrow X_s \gamma$: Standard Model and Beyond[†]

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We present a concise review of the recent theoretical progress concerning the standard model calculation of the inclusive radiative $\bar{B} \rightarrow X_s \gamma$ decay. Particular attention is thereby devoted to the calculations of the next-to-next-to-leading order fixed-order $\mathcal{O}(\alpha_s^2)$ contributions, non-local $\mathcal{O}(\alpha_s \Lambda/m_b)$ power corrections, and logarithmic-enhanced $\mathcal{O}(\alpha_s^2)$ cut-effects to the decay rate. The current status of various beyond the standard model calculations of the inclusive $b \rightarrow s \gamma$ mode is also summarized.

1. Introduction

As flavor-changing neutral current (FCNC) processes the inclusive radiative \bar{B} -meson ($\bar{B} = \bar{B}^0, B^-$) decays are both Cabibbo-Kobayashi-Maskawa (CKM) and loop-suppressed within the standard model (SM). They thus allow to probe the structure of non-standard electroweak (EW) physics at the quantum level. To exploit the full potential of $\bar{B} \rightarrow X_s \gamma$ in constraining the parameter space of beyond the SM (BSM) scenarios both the measurements and the SM calculations should be performed as accurately as possible.

The present experimental world average (WA) which includes measurements by CLEO, Belle, and BaBar [1] is performed by the Heavy Flavor Averaging Group [2] and reads for a photon energy cut of $E_\gamma > E_{\text{cut}}$ with $E_{\text{cut}} = 1.6 \text{ GeV}$ in the \bar{B} -meson rest-frame¹

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = (3.55 \pm 0.24_{-0.10}^{+0.09} \pm 0.03) \times 10^{-4}. \quad (1)$$

The total error of the WA is already below 8% and consists of *i*) a combined statistical and systematic error, *ii*) a systematic uncertainty due to the extrapolation from $E_{\text{cut}} = [1.8, 2.0] \text{ GeV}$ to the reference value, and *iii*) a systematic error due to the subtraction of the $\bar{B} \rightarrow X_d \gamma$ event fraction. At the end of the B -factory era the final accuracy of the averaged experimental value is expected to be around 5%.

2. Basic Properties of $\bar{B} \rightarrow X_s \gamma$

The $b \rightarrow s \gamma$ transition is dominated by perturbative QCD effects which replace the power-like Glashow-

Iliopoulos-Maiani (GIM) suppression present in the EW vertex by a logarithmic one. This mild suppression of the QCD corrected amplitude reduces the sensitivity of the process to high-scale physics, but enhances the $\bar{B} \rightarrow X_s \gamma$ branching ratio (BR) with respect to the purely EW prediction by a factor of around three. The logarithmic GIM cancellation originates from operator mixing and the non-conservation of the tensor current which is generated at the EW scale by loop diagrams involving W -boson and top quark exchange. The associated large logarithms $L \equiv \ln M_W/m_b$ have to be resummed at each order in α_s , using techniques of the renormalization group (RG) improved perturbation theory. Factoring out the Fermi constant G_F , the $b \rightarrow s \gamma$ amplitude receives corrections of $\mathcal{O}(\alpha_s^n L^n)$ at leading order (LO), of $\mathcal{O}(\alpha_s^n L^{n-1})$ at next-to-leading order (NLO), and of $\mathcal{O}(\alpha_s^n L^{n-2})$ at next-to-next-to-leading order (NNLO) in QCD.

A suitable framework to achieve the necessary resummation is the construction of an effective theory with five active quarks, photons and gluons by integrating out the EW bosons and the top quark. Including terms of dimension up to six in the local operator product expansion (OPE) the relevant effective Lagrangian at a scale μ reads

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD} \times \text{QED}} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{k=1}^8 C_k(\mu) Q_k. \quad (2)$$

Here the first term is the conventional QCD and QED Lagrangian for the light SM particles. In the second term V_{ij} denotes the elements of the CKM matrix and $C_k(\mu)$ are the Wilson coefficients of the corresponding operators Q_k built out of the light fields.

The operators and the numerical values of their Wilson coefficients at $\mu_b \sim m_b$ are given by

$$\begin{aligned} Q_{1,2} &= (\bar{s} \Gamma_i c)(\bar{c} \Gamma'_i b), & C_{1,2}(m_b) &\sim 1, \\ Q_{3-6} &= (\bar{s} \Gamma_i b) \sum_q (\bar{q} \Gamma'_i q), & |C_{3-6}(m_b)| &< 0.07, \\ Q_7 &= \frac{em_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & C_7(m_b) &\sim -0.3, \\ Q_8 &= \frac{gm_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a, & C_8(m_b) &\sim -0.15, \end{aligned} \quad (3)$$

where Γ and Γ' , entering both the current-current operators $Q_{1,2}$ and the QCD penguin operators Q_{3-6} ,

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¹The recent measurements of BaBar [3] and Belle [4] that give $\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = (3.66 \pm 0.85_{\text{stat}} \pm 0.60_{\text{syst}}) \times 10^{-4}$ for $E_0 = 1.9 \text{ GeV}$ and $\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = (3.31 \pm 0.19_{\text{stat}} \pm 0.37_{\text{syst}} \pm 0.01_{\text{boost}}) \times 10^{-4}$ for $E_0 = 1.7 \text{ GeV}$, respectively, are not taken into account in the average of Eq. (1).

stand for various products of Dirac and color matrices [5]. In the dipole operator Q_7 (Q_8), e (g) is the electromagnetic (strong) coupling constant, $q_{L,R}$ are the chiral quark fields, $F_{\mu\nu}$ ($G_{\mu\nu}^a$) is the electromagnetic (gluonic) field strength tensor, and T^a are the color generators.

After including LO QCD effects the dominant contribution to the partonic decay rate stems from charm quark loops that amount to $\sim +158\%$ of the total $b \rightarrow s\gamma$ decay amplitude. The top contribution is compared to the one from the charm quark with $\sim -60\%$ less than half as big and has the opposite sign. Diagrams involving up quarks are suppressed by small CKM factors and lead to an effect of a mere $\sim +2\%$ at the amplitude level.

All perturbative calculations of $b \rightarrow s\gamma$ involve three steps: *i*) evaluation of the initial conditions $C_k(\mu_w)$ of the Wilson coefficients at the matching scale $\mu_w \sim M_W$ by requiring equality of Green's functions in the full and the effective theory up to leading order in (external momenta)/ M_W , *ii*) calculation of the anomalous dimension matrix (ADM) that determines the mixing and RG evolution of $C_k(\mu)$ from μ_w down to the \bar{B} -meson scale $\mu_b \sim m_b$, and *iii*) determination of the on-shell matrix elements of the various operators at $\mu_b \sim m_b$. Due to the inclusive character of the $\bar{B} \rightarrow X_s\gamma$ mode and the heaviness of the bottom quark, $m_b \gg \Lambda \sim \Lambda_{\text{QCD}}$, non-perturbative effects arise in the last step only as small corrections to the partonic decay rate.

3. Theoretical Progress in $\bar{B} \rightarrow X_s\gamma$

At the NNLO level, the dipole and the four-quark operator matching involves three and two loops, respectively. Renormalization constants up to four loops must be found for $b \rightarrow s\gamma$ and $b \rightarrow sg$ diagrams with four-quark operator insertions, while three-loop mixing is sufficient in the remaining cases. Two-loop matrix elements of the dipole and three-loop matrix elements of the four-quark operators must be evaluated in the last step.

The necessary two- and three-loop matching was performed in [6] and [7]. The mixing at three loops was determined in [8] and at four loops in [9]. The two-loop matrix element of the photonic dipole operator together with the corresponding bremsstrahlung was found in [10] and subsequently confirmed in [11]. These calculations have been extended to include the full charm quark mass dependence [12]. The three-loop matrix elements of the current-current operators were derived in [13] within the so-called large- β_0 approximation. A calculation that goes beyond the large- β_0 approximation employs an interpolation in the charm quark mass [14]. Further progress towards a complete calculation of the three-loop current-current matrix elements has been made recently in [15] by a

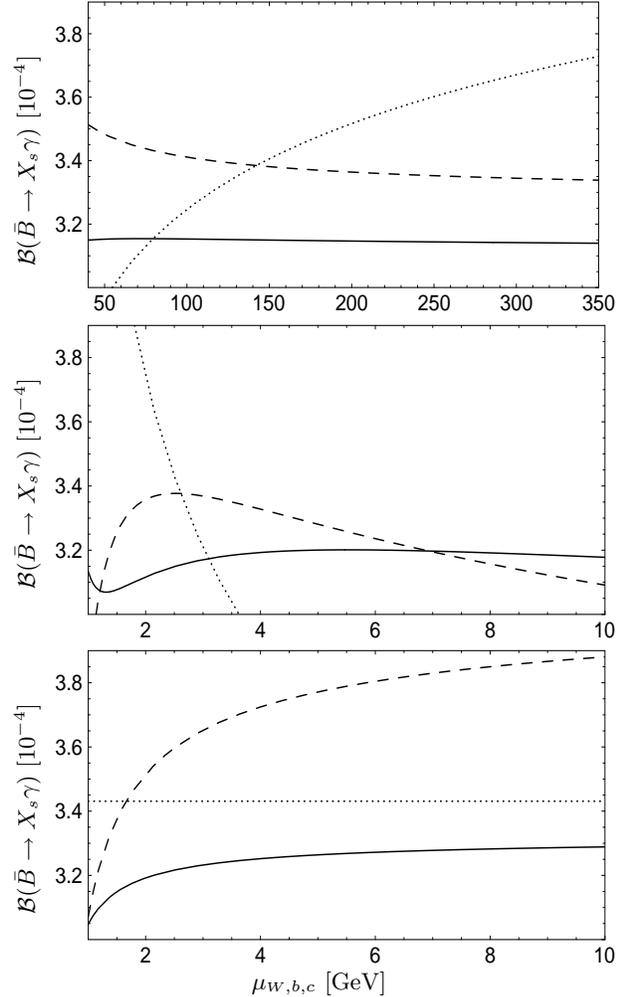


Figure 1: Renormalization scale dependences of $\mathcal{B}(\bar{B} \rightarrow X_s\gamma)$ at LO (dotted lines), NLO (dashed lines), and NNLO (solid lines) in QCD. The plots show from top to bottom the dependence on the matching scale μ_w , the \bar{B} -meson scale μ_b , and the charm quark mass renormalization scale μ_c .

calculation of fermionic contributions involving a massive charm and bottom quark loop insertion into the gluon propagator. The effect of still unknown NNLO contributions is believed to be smaller than the uncertainty that has been estimated after incorporating the above corrections into the SM calculation [14, 16]. To dispel possible doubts about the correctness of this assumption, calculations of the missing pieces are being pursued [17–19].

A crucial part of the NNLO calculation is the interpolation in the charm quark mass performed in [14]. The three-loop $\mathcal{O}(\alpha_s^2)$ matrix elements of the current-current operators contain the charm quark, and the NNLO calculation of these matrix elements is essential to reduce the overall theoretical uncertainty of the SM calculation. In fact, the largest part of the theoretical

uncertainty in the NLO analysis of the BR is related to the definition of the mass of the charm quark [24] that enters the $\mathcal{O}(\alpha_s)$ matrix elements $\langle s\gamma|Q_{1,2}|b\rangle$. The latter matrix elements are non-vanishing at two loops only and the scale at which m_c should be normalized is therefore undetermined at NLO. Since varying m_c between $m_c(m_c) \sim 1.25$ GeV and $m_c(m_b) \sim 0.85$ GeV leads to a shift in the NLO BR of more than 10% this issue is not an academic one.

Finding the complete NNLO correction to $\langle s\gamma|Q_{1,2}|b\rangle$ is a formidable task, since it involves the evaluation of hundreds of three-loop on-shell vertex diagrams that are presently not even known in the case $m_c = 0$. The approximation made in [14] is based on the observation that at the physical point $m_c \sim 0.25 m_b$ the large $m_c \gg m_b$ asymptotic form of the exact $\mathcal{O}(\alpha_s)$ [5] and large- β_0 $\mathcal{O}(\alpha_s^2\beta_0)$ [13] result matches the small $m_c \ll m_b$ expansion rather well. This feature prompted the analytic calculation of the leading term in the $m_c \gg m_b$ expansion of the three-loop diagrams, and to use the obtained information to perform an interpolation to smaller values of m_c assuming the $\mathcal{O}(\alpha_s^2\beta_0)$ part to be a good approximation of the full $\mathcal{O}(\alpha_s^2)$ result for vanishing charm quark mass. The uncertainty related to this procedure has been assessed in [14] by employing three ansätze with different boundary conditions at $m_c = 0$. A complete computation of the $\mathcal{O}(\alpha_s^2)$ corrections to $\langle s\gamma|Q_{1,2}|b\rangle$ in the latter limit or, if possible, a calculation of the virtual corrections for $m_c \sim 0.25 m_b$, would allow to resolve this ambiguity and are therefore highly desirable. Such cutting-edge calculations are in progress [19].

Combining the aforementioned results it was possible to obtain the first theoretical estimate of the total BR of $\bar{B} \rightarrow X_s\gamma$ at NNLO. For the reference value $E_{\text{cut}} = 1.6$ GeV the result of the improved SM evaluation is given by [14, 16]²

$$\mathcal{B}(\bar{B} \rightarrow X_s\gamma) = (3.15 \pm 0.23) \times 10^{-4}, \quad (4)$$

where the uncertainties from hadronic power corrections (5%), parametric dependences (3%), higher-order perturbative effects (3%), and the interpolation in the charm quark mass (3%) have been added in quadrature to obtain the total error.

The reduction of the renormalization scale dependences at NNLO is clearly seen in Fig. 1. The most pronounced effect occurs in the case of the charm quark mass renormalization scale μ_c that was the

main source of uncertainty at NLO. The current uncertainty of 3% due to higher-order effects is estimated from the variation of the NNLO curves. The central value in Eq. (4) corresponds to the choice $\mu_{w,b,c} = (160, 2.5, 1.5)$ GeV. More details on the phenomenological analysis including the list of input parameters can be found in [14].

It is well-known that the OPE for $\bar{B} \rightarrow X_s\gamma$ has certain limitations which stem from the fact that the photon has a hadronic substructure. In particular, the local expansion does not apply to contributions from operators other than Q_7 , in which the photon couples to light quarks [25, 26]. While the presence of non-local corrections was thus foreseen such terms have been studied until recently only in the case of the (Q_8, Q_8) interference [25]. In [27] the analysis of non-perturbative effects that go beyond the local OPE have been extended to the enhanced non-local terms emerging from (Q_7, Q_8) insertions. The found correction scales like $\mathcal{O}(\alpha_s\Lambda/m_b)$ and its effect on the BR was estimated using the vacuum insertion approximation to be $-[0.3, 3.0]\%$. A measurement of the flavor asymmetry between $\bar{B}^0 \rightarrow X_s\gamma$ and $B^- \rightarrow X_s\gamma$ could help to sustain this numerical estimate [27]. Potentially as important as the latter correction are those arising from the $(Q_{1,2}, Q_7)$ interference. Naive dimensional analysis suggests that some non-perturbative corrections to them also scale like $\mathcal{O}(\alpha_s\Lambda/m_b)$. Since at the moment there is not even an estimate of those corrections, a non-perturbative uncertainty of 5% has been assigned to the result in Eq. (4). This error is the dominant theoretical uncertainty at present and thought to include all known [27] and unknown $\mathcal{O}(\alpha_s\Lambda/m_b)$ terms. A dedicated study of non-perturbative effects in $b \rightarrow s\gamma$ that goes beyond the local OPE is near completion [28]. Calculating the precise impact of the enhanced non-local power corrections may, however, remain notoriously difficult given the limited control over non-perturbative effects on the light cone.

A further complication in the calculation of $\bar{B} \rightarrow X_s\gamma$ arises from the fact that all measurements impose stringent cuts on the photon energy to suppress the background from other \bar{B} -meson decay processes. Restricting E_γ to be close to the physical endpoint $E_{\text{max}} = m_B/2$, leads to a breakdown of the local OPE, which can be cured by resummation of an infinite set of leading-twist terms into a non-perturbative shape function [29]. A detailed knowledge of the shape function and other subleading effects is required to extrapolate the measurements to a region where the conventional OPE can be trusted.

The transition from the shape function to the OPE region can be described by a multi-scale OPE (MSOPE) [30]. In addition to the hard scale $\mu_h \sim m_b \sim 5$ GeV, this expansion involves a hard-collinear scale $\mu_{hc} \sim \sqrt{m_b\Delta} \sim 2.5$ GeV corresponding to the typical hadronic invariant mass of the final state

²Including several perturbative and non-perturbative effects [9, 12, 15, 18, 20–22] leads to a total correction to $\mathcal{B}(\bar{B} \rightarrow X_s\gamma)$ of +1.6% with respect to Eq. (4). Using the recent determination [23] of the semileptonic normalization factor entering Eq. (4) causes a further enhancement of $\mathcal{B}(\bar{B} \rightarrow X_s\gamma)$ by +4.8% and a slight increase of the quoted overall uncertainty.

Model	Accuracy	Effect	Bound
THDM type II	NLO [39, 40]	↑	$M_H^\pm > 295 \text{ GeV}$ (95% CL) [16]
MFV MSSM	NLO [40–45]	↕	—
MFV SUSY GUTs	NLO [46]	↓	—
LR	NLO [40]	↕	—
general MSSM	LO [47]	↕	$ \delta_{23}^d _{LL} \lesssim 4 \times 10^{-1}$, $ \delta_{23}^d _{RR} \lesssim 8 \times 10^{-1}$, $ \delta_{23}^d _{LR} \lesssim 6 \times 10^{-2}$, $ \delta_{23}^d _{RL} \lesssim 2 \times 10^{-2}$ [47]
UED5	LO [49]	↓	$1/R > 600 \text{ GeV}$ (95% CL) [50]
UED6	LO [51]	↓	$1/R > 650 \text{ GeV}$ (95% CL) [51]
RS	LO [52]	↑	$M_{\text{KK}} \gtrsim 2.4 \text{ TeV}$
LH	LO [53]	↑	—
LHT	LO [54]	↑	—

Table I Theoretical accuracy, effect on $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$ relative to the SM prediction, and if applicable, constraint on the parameter space following from $\bar{B} \rightarrow X_s \gamma$ in popular BSM scenarios. Arrows pointing upward (downward) indicate that the BSM effects interfere constructively (destructively) with the SM $b \rightarrow s \gamma$ amplitude. Single (double) arrows specify whether the maximal possible shift is smaller (larger) than the theoretical uncertainty of the SM expectation. See text for details.

X_s , and a soft scale $\mu_s \sim \Delta \sim 1.5 \text{ GeV}$ related to the width $\Delta/2 = m_b/2 - E_{\text{cut}}$ of the energy window in which the photon spectrum is measured. In the MSOPE framework, the perturbative tail of the spectrum receives calculable corrections at all three scales, and may be subject to large perturbative corrections due to the presence of terms proportional to $\alpha_s(\sqrt{m_b \Delta}) \sim 0.27$ and $\alpha_s(\Delta) \sim 0.36$.

A systematic MSOPE analysis of the (Q_7, Q_7) interference at NNLO has been performed in [31]. Besides the hard matching corrections, it involves the two-loop logarithmic and constant terms of the jet [30, 33] and soft function [34]. The three-loop ADM of the shape function remains unknown and is not included. The MSOPE result can be combined with the fixed-order prediction by computing the fraction of events $1 - T$ that lies in the range $E_{\text{cut}} = [1.0, 1.6] \text{ GeV}$. The analysis [31] yields

$$1 - T = 0.07_{-0.03}^{+0.05}_{\text{pert}} \pm 0.02_{\text{hadr}} \pm 0.02_{\text{pars}}, \quad (5)$$

where the individual errors are perturbative, hadronic, and parametric. The quoted value is almost twice as large as the NNLO estimate $1 - T = 0.04 \pm 0.01_{\text{pert}}$ obtained in fixed-order perturbation theory [14, 16, 32] and plagued by a significant additional theoretical

error related to low-scale perturbative corrections. These large residual scale uncertainties indicate a slow convergence of the MSOPE series expansion in the tail region of the photon energy spectrum. Given that Δ is always larger than 1.4 GeV and thus fully in the perturbative regime this feature is unexpected [35].

Additional theoretical information on the shape of the photon energy spectrum can be obtained from the universality of soft and collinear gluon radiation. Such an approach can be used to predict large logarithms of the form $\ln(E_{\text{max}} - E_{\text{cut}})$. These computations have also achieved NNLO accuracy [36] and incorporate Sudakov and renormalon resummation via dressed gluon exponentiation (DGE) [36, 37]. The present NNLO estimate of $1 - T = 0.016 \pm 0.003_{\text{pert}}$ [36, 38] indicates a much thinner tail of the photon energy spectrum and a considerable smaller perturbative uncertainty than reported in [31]. The DGE analysis thus supports the view that the integrated photon energy spectrum below $E_{\text{cut}} = 1.6 \text{ GeV}$ is well approximated by a fixed-order perturbative calculation, complemented by local OPE power corrections. To understand how precisely the tail of the photon energy spectrum can be calculated requires nevertheless further theoretical investigations.

4. Beyond the Standard Model Physics in $\bar{B} \rightarrow X_s \gamma$

Compared with the experimental WA of Eq. (1), the new SM prediction of Eq. (4) is lower by more than 1σ . Potential beyond SM contributions should now

be preferably constructive, while models that lead to a suppression of the $b \rightarrow s \gamma$ amplitude are more severely constrained than in the past, where the theoretical determination [5] has been above the experimental one.

BSM physics can affect the initial conditions of the Wilson coefficients of the operators in the low-energy

effective theory and might also induce new operators besides those already present in the SM. Complete NLO matching calculations are available only in the case of the two-Higgs-doublet models (THDMs) [39, 40], the minimal supersymmetric SM (MSSM) with minimal flavor-violation (MFV) for small and large $\tan\beta$ [40–45], and left-right (LR) symmetric models [40]. In the general MSSM [47], universal extra dimensional models with one (UED5) [49, 50] and two (UED6) [51] additional flat dimensions, Randall-Sundrum (RS) scenarios [52], littlest Higgs (LH) models without [53] and with T -parity (LHT) [54], and the minimal 3-3-1 model featuring a leptophobic neutral Z' gauge boson [55], the accuracy is in general strictly LO and hence far from the one achieved in the SM. The main features and results of recent analyses of beyond SM physics in $\bar{B} \rightarrow X_s \gamma$ are listed in Tab. I. In the following we will briefly review the most important findings.

Even though the effect of charged Higgs boson contributions in the THDM type II model is necessarily constructive [39, 40, 56], the lower bound on M_{H^\pm} following from $\bar{B} \rightarrow X_s \gamma$ remains in general stronger than all other direct and indirect constraints. In particular, $\bar{B} \rightarrow X_s \gamma$ still prevails over $B \rightarrow \tau\nu$ [57–59], $B \rightarrow D\tau\nu$ [58, 59], and $K \rightarrow \mu\nu$ [60] for values of $\tan\beta \lesssim 40$. This is illustrated in the upper panel of Fig. 2. The derived 95% confidence level (CL) limit amounts to $M_{H^\pm} > 295$ GeV independently of $\tan\beta$ [16]. In the THDM type I model, the strongest constraint on M_{H^\pm} stems from the ratio of the widths of the Z -boson decay into bottom quarks and hadrons, R_b , and not from $\bar{B} \rightarrow X_s \gamma$.

In the MFV MSSM the complete NLO corrections to $\bar{B} \rightarrow X_s \gamma$ are also known. The needed two-loop diagrams containing gluons and gluinos were evaluated in [40, 41] and [42, 43], respectively. Since EW interactions affect the quark and squark mass matrices in a different way, their alignment is not RG invariant and MFV can only be imposed at a certain scale μ_{MFV} that is related to the mechanism of supersymmetry (SUSY) breaking [43]. For μ_{MFV} much larger than the SUSY masses M_{SUSY} , the ensuing large logarithms can lead to sizable effects in $\bar{B} \rightarrow X_s \gamma$, and need to be resummed by solving the RG equation of the flavor-changing gluino-quark-squark couplings.

In the limit of $M_{\text{SUSY}} \gg M_W$, SUSY effects can be absorbed into the coupling constants of local operators in an effective theory [44, 45]. The Higgs sector of the MSSM is modified by these non-decoupling corrections and can differ notably from the native THDM type II model. Some of the corrections to $\bar{B} \rightarrow X_s \gamma$ in the effective theory are enhanced by $\tan\beta$. As a result, they can be sizable, of order $\alpha_s \tan\beta \sim 1$ for values of $\tan\beta \gg 1$, and need to be resummed if applicable. In the large $\tan\beta$ regime the relative sign of the chargino contribution is given by $-\text{sign}(A_t \mu)$. For $\text{sign}(A_t \mu) > 0$, the chargino and charged Higgs

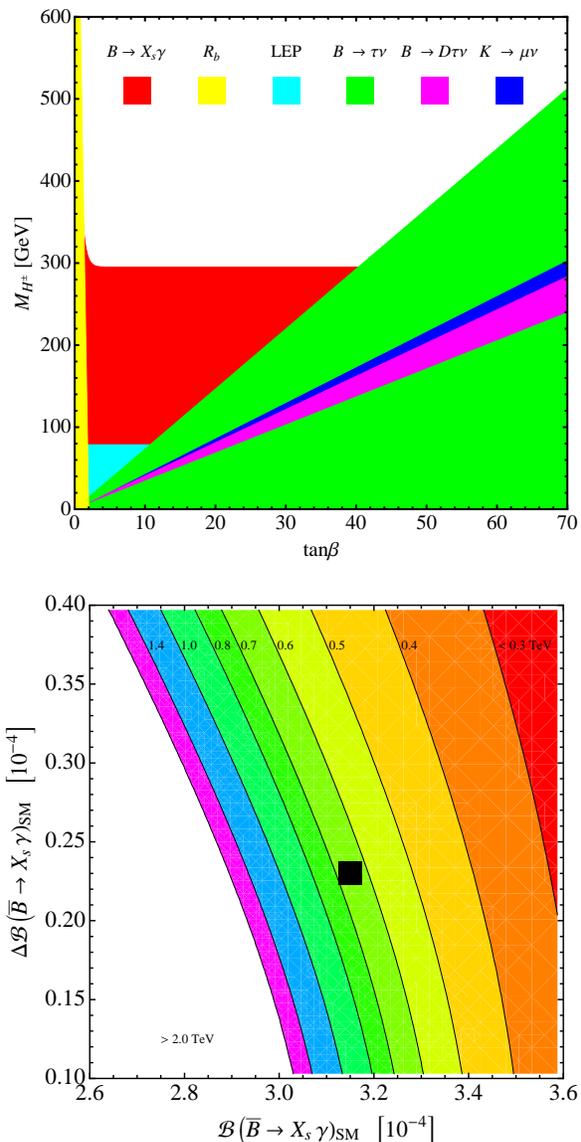


Figure 2: Top: Direct and indirect bounds on M_{H^\pm} in the THDM type II model as a function of $\tan\beta$. The colored areas are excluded by the constraints at 95% CL. Bottom: 95% CL limits on the compactification scale $1/R$ in the UED6 model as a function of the SM central value and total error. The present SM result is indicated by the black square. See text for details.

contributions interfere hence constructively with the SM result and this tends to rule out large positive values of the product of the trilinear soft SUSY breaking coupling A_t and the Higgsino parameter μ .

In the MSSM with generic sources of flavor violation a complete NLO analysis is still missing up to date. Experimental constraints on generic $b \rightarrow s$ flavor violation have been studied extensively [47], and radiative inclusive \bar{B} -meson decays play a central role in these analyses. In particular, for small and moderate values of $\tan\beta$ all four mass insertions $(\delta_{23}^d)_{AB}$

with $A, B = L, R$ except for $(\delta_{23}^d)_{RR}$ are determined entirely by $\bar{B} \rightarrow X_s \gamma$. The bounds on the mass insertions $(\delta_{23}^d)_{AB}$ corresponding to $\tan \beta = 10$ are given in Tab. I. For large values of $\tan \beta$ neutral Higgs penguin contributions become important and the constraints from both $B_s \rightarrow \mu^+ \mu^-$ and $B_s - \bar{B}_s$ mixing surpass the one from $\bar{B} \rightarrow X_s \gamma$. The effect of the precision measurement of the mass difference ΔM_s [48] is especially strong in the case of $(\delta_{23}^d)_{RL, RR}$. At large $\tan \beta$ the limits on both mass insertions are now imposed by the $B_s - \bar{B}_s$ mixing constraint alone.

The inclusive $b \rightarrow s \gamma$ transition also plays a central role in the dedicated analyses of FCNC processes [46, 61] in the framework of SUSY grand unified theories (GUTs). In particular, the specific minimal $SO(10)$ model with D_3 family symmetry [62] is unable to accommodate simultaneously the value of the bottom quark mass and $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$ once the stringent CDF upper bound on the decay $B_s \rightarrow \mu^+ \mu^-$ [63] is imposed, unless the squark mass spectrum is pushed into the 10 TeV range [61]. This little hierarchy problem seems to be a generic feature of all SUSY GUT models with exact third generation Yukawa unification and universal squark and gaugino masses at the GUT scale [46]. A possible cure to this illness consists in relaxing Yukawa unification to $b - \tau$ unification, but still then SUSY GUTs tend to give values for $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$ at the lower end of the range favored by the experimental measurements [46].

Since Kaluza-Klein (KK) modes in the UED5 model interfere destructively with the SM $b \rightarrow s \gamma$ amplitude [49], $\bar{B} \rightarrow X_s \gamma$ leads to a very powerful bound on the inverse compactification radius of $1/R > 600$ GeV at 95% CL [50]. This exclusion is independent from the Higgs mass and therefore stronger than any limit that can be derived from EW precision measurements. In the UED6 model the corresponding limit reads $1/R > 650$ GeV at 95% CL [51]. This bound exceeds by far the limits that can be derived from any other direct measurement, and is at variance with the parameter region $1/R \lesssim 600$ GeV preferred by the dark matter abundance [64]. The 95% CL bound on the compactification scale $1/R$ in the UED6 scenario and its dependence on the SM central value and error is shown in the lower panel of Fig. 2. In RS models, KK modes tend to enhance the BR relative to the SM [52], and the bound on the KK masses is in consequence with $M_{KK} \gtrsim 2.4$ TeV significantly weaker than the constraint that derive from other FCNC processes or EW precision data.

The contributions to $\bar{B} \rightarrow X_s \gamma$ from new heavy vector bosons, scalars, and quarks appearing in LH models, have been studied in [53] for the original model, and in [54] for an extension in which an additional Z_2 symmetry called T -parity is introduced to preserve custodial $SU(2)$ symmetry. While in the former case the new contributions always lead to an enhancement of $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$ [53], in the latter case also a suppres-

sion with respect to the SM expectation is possible [54]. As the found LH effects in $\bar{B} \rightarrow X_s \gamma$ are generically smaller than the theoretical uncertainties in the SM, they essentially do not lead to any restriction on the parameter space.

An alternative avenue to BSM analyses of $\bar{B} \rightarrow X_s \gamma$ consists in constraining the Wilson coefficients of the operators in the low-energy effective theory. This model-independent approach has been applied combining various B - and K -meson decay modes both neglecting [65, 66] and including [45, 67] operators that do not contribute in the SM. In particular, in the former case, merging the information on $\bar{B} \rightarrow X_s \gamma$ with the one on $\bar{B} \rightarrow X_s l^+ l^-$ [68], one can infer that the sign of the $b \rightarrow s \gamma$ amplitude is in all probability SM-like [69]. In the case of the Z -penguin amplitude the same conclusion can be drawn on the basis of the precision measurements of R_b and the other $Z \rightarrow b\bar{b}$ pseudo observables [66]. The inclusive $\bar{B} \rightarrow X_s \gamma$ decay also provides stringent bounds on the potential size of anomalous Wtb couplings. Due to the chiral m_t/m_b enhancement the bounds on specific effective couplings turn out to be significantly stronger than the limits expected from future measurements of top quark production and decay at the LHC [70].

5. Conclusions

The inclusion of NNLO QCD corrections has lead to a significant suppression of the renormalization scale dependences of the $\bar{B} \rightarrow X_s \gamma$ branching ratio that have been the main source of theoretical uncertainty at NLO. The central value of the SM prediction is shifted downward relative to all previously published NLO results. It is now more than 1σ below the experimental average. This revives the possibility for explorations of beyond the standard model contributions to rare flavor-changing B -decay processes. The dominant theoretical uncertainty in the SM is currently due to unknown non-perturbative effects. A reduction of this error, together with a calculation of the three-loop matrix elements of the current-current operators and a better understanding of the tail of the photon energy spectrum is essential to further increase the power of $\bar{B} \rightarrow X_s \gamma$ in the search for beyond the standard model physics.

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