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Theory review of exclusive rare radiative decays

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I briefly review the theory status of exclusive rare radiative decays.

1. Introduction

Experimental measurements of $B \to V\gamma$ decays, with V a light vector meson such as K^*, ρ, ω, ϕ, have continued to improve and will become more precise at the end of the B-factories and at LHC-b. Given their rich CKM phenomenology and potential to constrain new physics models, having reliable theory predictions for these decays is increasingly relevant. In this talk I briefly review the status of this area.

The starting point is the effective weak Hamiltonian, which for $B \to V\gamma$ decays is [1]:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(q)} \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{8} C_i Q_i \right], \tag{1}$$

where $\lambda_p^{(q)} = V_{pq}^* V_{pb}$. The operators with the largest Wilson coefficients are the four-quark operators Q_1^p and Q_2^p, which read

$$\begin{aligned} Q_1^p &= (\bar{q}\, p)_{V-A}\, (\bar{p}\, b)_{V-A} \\ Q_2^p &= (\bar{q}_i p_j)_{V-A}\, (\bar{p}_j b_i)_{V-A}, \end{aligned} \tag{2}$$

and the electromagnetic and chromomagnetic penguin operators Q_7 and Q_8, which are

$$\begin{aligned} Q_7 &= -\frac{e\, \overline{m}_b(\mu)}{8\pi^2}\, \bar{q}\, \sigma^{\mu\nu}\, [1+\gamma_5]\, b F_{\mu\nu}\,, \\ Q_8 &= -\frac{g\, \overline{m}_b(\mu)}{8\pi^2}\, \bar{q}\, \sigma^{\mu\nu}\, [1+\gamma_5]\, T^a\, b G^a_{\mu\nu}. \end{aligned} \tag{3}$$

Here $q = d$ or s, and the convention for the sign of the couplings corresponds to the covariant derivative $iD_\mu = i\partial_\mu + eQ_f A_\mu + gT^a A^a_\mu$, with A_μ and A^a_μ representing the photon and gluon fields respectively, and $Q_e = -1$ etc. The factor $\overline{m}_b(\mu)$ is the $\overline{\rm MS}$ mass of the b quark.

The main theoretical challenge is to evaluate the hadronic matrix elements of the operators in the effective weak Hamiltonian. Common ways of doing this include the QCD factorization [2, 3, 4] and pQCD [5, 6] approaches. Both of these rely on the fact that $\alpha_s(m_b)$ and $\Lambda_{\rm QCD}/m_b$ can be considered as small expansion parameters. In this talk I will focus exclusively on QCD factorization methods, including in this category also strategies which supply additional information on $1/m_b$-suppressed contributions from QCD sum rules [7, 8], or apply a form of renormalization-group (RG) improved perturbation theory based on soft-collinear effective theory (SCET) [9, 10].

The remainder of the talk is organized as follows. Section 2 introduces the QCD factorization formalism, Section 3 gives two sample applications, and Section 4 reviews recent results in higher-order perturbative corrections to the hard-scattering kernels. This is followed by a short discussion of $1/m_b$ power corrections and endpoint divergences in Section 5, using isospin violation in $B \to K^*\gamma$ decays as an example, and a summary in Section 6.

2. QCD factorization

QCD factorization is the statement that in the heavy-quark limit the hadronic matrix element of each operator in the effective weak Hamiltonian can be written in the form

$$\begin{aligned} &\langle V\gamma\, |Q_i|\, \bar{B}\rangle = \zeta_{V_\perp}\, t_i^{\rm I} + \\ &\frac{\sqrt{m_B} F f_{V_\perp}}{4} \phi_+^B \otimes t_i^{\rm II} \otimes \phi_\perp^V \; + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right), \end{aligned} \tag{4}$$

where the $\otimes$ stand for convolution integrals. (Sometimes this same formula is written in a different but equivalent form, using the tensor QCD form factor $T_1^{B\to V}$ instead of the SCET soft function $\zeta_{V_\perp}$.) The soft function $\zeta_{V_\perp}$, the meson decay constants $f_{V_\perp}$, F, and the light-cone distribution amplitudes (LCDAs) ϕ_+^B, $\phi_\perp^V$ are non-perturbative but universal objects. At present, numerical values for these objects are taken from light-cone sum rules. The hard-scattering kernels $t_i^{\rm I,II}$ can be calculated as a perturbative series in α_s. The $t_i^{\rm I}$ are referred to as "vertex corrections", and the $t_i^{\rm II}$ are referred to as "spectator corrections". Only the hard-scattering $t_7^{\rm I}$ is non-vanishing at zeroth order in α_s. Corrections from this and the other operators appear at order α_s, and have been known completely for some time [2, 3, 4]. I shall refer to these α_s corrections as next-to-leading order (NLO), even though the hard-spectator kernels $t_i^{\rm II}$ first start at this order. In Section 4 I describe some recent results for the NNLO kernels.

An all orders proof of the QCD factorization formula (4) was performed in [9], using the technology of SCET. In the effective-theory approach the hard-scattering kernels are short distance Wilson coefficients of operators whose hadronic matrix elements define the $\zeta_{V_\perp}$ and the LCDAs. For the vertex term

this makes little practical difference, but for the hard-spectator term one can show that the hard-scattering kernels $t_i^{\rm II}$ can be further factorized into the form

$$t_i^{\rm II}(u,\omega,\mu_i) = [C_i^{B1}(\mu_h)\otimes U(\mu_h,\mu_i)]\otimes j_\perp(\mu_i)\,. \quad (5)$$

The hard coefficients C_i^{B1} contain physics at the scale m_b, while the jet function $j_\perp$ is independent of the operator and contains physics at the intermediate scale $\sqrt{m_b\Lambda_{\rm QCD}}\sim 1.5\,{\rm GeV}$. The evolution factor U is derived by solving the RG-equations in the effective theory. Because these equations are non-local, the evolution factor appears in a convolution with the Wilson coefficient C_i^{B1}. In the limit where $m_b^2 \gg m_b\Lambda_{\rm QCD}$, this evolution matrix resums perturbative logs in the ratio $\Lambda_{\rm QCD}/m_b$, and allows one to evaluate the hard coefficients and the jet function at their natural scales $\mu_h\sim m_b$ and $\mu_i\sim\sqrt{m_b\Lambda_{\rm QCD}}$. If one does not wish to do this, then making the choice $\mu_h=\mu_i$ sets the evolution factor U to unity, and one recovers the original QCD factorization formula (4). So for these decays, the "SCET approach" is just QCD factorization, plus the added opportunity to perform a further scale separation in the hard scattering kernels $t_i^{\rm II}$.

It is necessary to keep in mind that the factorization formula (4) is valid only up to $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ power corrections. Since SCET is an effective theory which sets up a systematic expansion in α_s and $\Lambda_{\rm QCD}/m_b$, it has the potential to provide a complete classification of the subleading terms in $1/m_b$. However, there has been no serious attempt to do this, because the generalization of the leading-order factorization formula to corrections in $1/m_b$ presumably contains many more non-perturbative objects. Even more troublesome is the fact that, in the cases where power corrections been calculated, the convolution integrals over momentum fractions do not always converge. These "endpoint divergences" are at present a principle limitation on the entire formalism.

In the absence of a comprehensive theoretical framework to deal with the $1/m_b$ suppressed corrections, it has become common practice to focus on the contributions which are believed to be large, or which play an especially important role for phenomenology. In this talk I will discuss the power corrections which are important for the calculation of isospin violation in $B\to K^*\gamma$ decays as performed in [11]. This is sufficient to describe the problem of endpoint divergences. Other power corrections, due to long-distance photon emission and soft-gluon emission from quark loops, were calculated within the context of light-cone sum rules in [8].

3. Sample applications

The formalism above can be used to calculate many different observables, see for instance [12, 13, 14]. In this section I briefly touch on two examples, taking results from the recent studies [7, 8] for simplicity. The first is the determination of the ratio of CKM matrix elements $|V_{td}/V_{ts}|$ from the ratio of branching fractions in $B\to\rho\gamma$ and $B\to K^*\gamma$ decays. This is an independent check on the measurements from B_s oscillations [15]. The most recent theory results are [7]

$$\begin{aligned} R \;\equiv\; & \frac{\overline{\mathcal{B}}(B\to(\rho,\omega)\gamma)}{\overline{\mathcal{B}}(B\to K^*\gamma)} = \frac{|V_{td}|^2}{|V_{ts}|^2} \\ & \times\;(0.75\pm 0.11(\xi)\pm 0.02({\rm UT\ param.},\,\mathcal{O}(1/m_b)))\;, \end{aligned} \quad (6)$$

which combined with February 2007 HFAG data leads to $|V_{td}|/|V_{ts}| = 0.192\pm 0.014({\rm th})\pm 0.016({\rm exp})$. This central value is compatible with that from B_s oscillations, although the experimental errors are significantly larger and the theory errors about twice as large. The dominant theory error is in the form factor ratio $\xi = T_1^{B\to K^*}/T_1^{B\to\rho}$, and according to the recent review [16] this should go down by about a factor of two with improved lattice results for the tensor decay constants. It should be mentioned that the experimental measurements of the ratio in (6) combine data from ρ and ω decays under the assumption of exact isospin asymmetry, while the theory calculations can actually predict the magnitude of isospin breaking. Improved experimental measurements such as those in [17, 18] will make this isospin averaging unnecessary.

As a second application I quote results for the isospin asymmetry in $B\to K^*\gamma$ decays. This asymmetry was calculated in the context of QCD factorization in [11] and the numbers updated in [8], which gives the result

$$\begin{aligned} A_I(K^*) \;&=\; \frac{\Gamma(\bar B^0\to\bar K^{*0}\gamma)-\Gamma(B^-\to\bar K^{*-}\gamma)}{\Gamma(\bar B^0\to\bar K^{*0}\gamma)+\Gamma(B^-\to\bar K^{*-}\gamma)} \\ &=\; (5.4\pm 1.4)\%. \end{aligned} \quad (7)$$

The current result by HFAG in [19] is (3±4)%. Results from [11] showed that this asymmetry is particularly sensitive to the penguin operator Q_6. Improved experimental measurements of this asymmetry can thus provide constraints on new physics. The theory prediction above, however, does not include any α_s corrections to the perturbative hard-scattering kernels, even though some of these were calculated in [11]. The reason for this is the presence of endpoint divergences, a topic discussed in more detail in Section 5.

4. NNLO perturbative corrections

Recently, a set of the NNLO corrections to the vertex and spectator kernels were obtained in [10]. In this section I summarize the new results from that

work and also what remains to be done to complete the NNLO calculation.

Consider first the vertex corrections and the hard-scattering kernel $t^{\rm I}$. This hard-scattering kernel is most easily obtained as a matching coefficient in SCET; details were given in [9, 10]. The only technical point I mention here is that the matching coefficients are independent of the external states used in the calculation, and it is possible to use the partonic matrix element $\langle q\gamma|Q_i|b\rangle$ in the matching. The SCET calculation is trivial for on-shell quarks, because the loop corrections are given by scaleless integrals which vanish in dimensional regularization, so the main challenge is to calculate the partonic matrix elements in full QCD. However, these are just virtual corrections to the $b \to q\gamma$ process needed also for inclusive $B \to X_s\gamma$ decay, and are currently known very accurately due to the efforts of many people (see [10, 20] for a complete list of references). Using those multi-loop calculations, $t^{\rm II}_{7,8}$ can be obtained completely to NNLO. For the four-quark operators $Q_{1,2}$, on the other hand, results to NNLO are known only in the large-β_0 limit, obtained by calculating the $\alpha_s^2 C_F n_f$ terms and replacing $n_f \to -3/2\beta_0$, where $\beta_0 = 11 - 2/3 n_f$ in QCD. The remaining terms require to calculate a large number of three-loop graphs depending on the ratio m_c^2/m_b^2 and containing imaginary parts. Given that this is also a missing piece in the $B \to X_s\gamma$ calculation, it is likely that this will be done in the near future.

I now turn to the spectator corrections and the hard scattering kernels $t_i^{\rm II}$. Again in this case their calculation is most conveniently formulated as a matching calculation in SCET. In contrast to the vertex corrections, however, the Feynman diagrams needed in the matching calculation have no analog in inclusive decay, where interactions with the spectator quarks do not contribute at leading power in $1/m_b$. Therefore, results for the $t_i^{\rm II}$ cannot simply be extracted from existing calculations and must be obtained from scratch. The one exception is the α_s^2 contribution to $t_7^{\rm II}$, which can be taken from studies of heavy-to-light form factors carried out in [21, 22]. The α_s^2 contribution to $t_8^{\rm II}$ was calculated in [10]. The most important missing pieces are from $Q_{1,2}$, but these are also the hardest to calculate, since they involve two-loop graphs depending on m_c^2/m_b^2 in addition to the momentum fraction u of the quark in the V-meson.

A natural place to explore the numerical impact of the NNLO corrections is on branching fractions in $B \to K^*\gamma$ decays, since for these decays the annihilation topology is CKM suppressed. The branching fraction is

$$\mathcal{B}(B \to K^*\gamma) = \frac{\tau_B m_B}{4\pi}\left(1 - \frac{m_{K^*}^2}{m_B^2}\right)|\mathcal{A}_{\rm v} + \mathcal{A}_{\rm hs}|^2 \ , \tag{8}$$

where the $\mathcal{A}_{\rm v}$ ($\mathcal{A}_{\rm hs}$) are the amplitudes for the vertex (hard spectator) corrections. These are separately RG-invariant and so can be studied individually. For the vertex corrections, the ratio of the NNLO amplitude to LO amplitude for the default set of parameters in [10] is

$$\begin{aligned}\frac{\mathcal{A}_{\rm v}^{\rm NNLO}}{\mathcal{A}_{\rm v}^{\rm LO}} = 1 \ &+ \ (0.096 + 0.057i)\,[\alpha_s] \\ &+ \ (-0.007 + 0.030i)\left[\alpha_s^2\right],\end{aligned}$$

where the first term in parentheses is the NLO (α_s) correction and the second term the NNLO (α_s^2) correction. One sees that the real part of the NNLO correction is extremely small. It is instructive to further split the above amplitude into the pieces originating from the various operators Q_i. In that case the result reads

$$\begin{aligned}&\frac{\mathcal{A}_{\rm v}^{\rm NNLO}}{\mathcal{A}_{\rm v}^{\rm LO}} - 1 = \\ &\Big((.26 + .03i)\,[Q_1] - .18\,[Q_7] + (.02 + .02i)\,[Q_8]\Big)\,[\alpha_s] \\ &+\Big((.07 + .02i)\,[Q_1] - .08\,[Q_7] + (.002 + .01i)\,[Q_8]\Big)\,[\alpha_s^2]\,.\end{aligned} \tag{9}$$

From this one sees that the NNLO contributions are so small because of a large cancellation between the Q_1 and Q_7 contributions. It is an open question whether this cancellation will persist when results for Q_1 are obtained beyond the large-β_0 approximation used above. This will be discussed in more detail below, where results for branching fractions are given.

For the hard spectator amplitude, one finds

$$\frac{\mathcal{A}_{\rm hs}^{\rm NNLO}}{\mathcal{A}_{\rm v}^{\rm LO}} = \big(0.11 + 0.05i\big)\,[\alpha_s] + \big(0.03 + 0.01i\big)\,[\alpha_s^2]. \tag{10}$$

Unlike the case of the vertex corrections, the individual contributions from the different operators are rather small at NLO and especially NNLO. The exact numbers, including a split into components at the jet and hard scales in (5), can be found in [10].

Putting the amplitudes together, one can find the branching fractions and their uncertainties. Including isospin and SU(3) breaking from meson masses, lifetimes, and V-meson distribution amplitudes, [10] estimated

$$\begin{aligned}\mathcal{B}(B^+ \to K^{*+}\gamma) &= (4.6 \pm 1.4) \times 10^{-5}, \\ \mathcal{B}(B^0 \to K^{*0}\gamma) &= (4.3 \pm 1.4) \times 10^{-5}, \\ \mathcal{B}(B_s \to \phi\gamma) &= (4.3 \pm 1.4) \times 10^{-5}.\end{aligned} \tag{11}$$

The magnitude of various sources of uncertainty can be found in [10]. Most significant is about a 25% uncertainty due to the soft functions $\zeta_{V_\perp}$, which is expected. Somewhat surprising is that the α_s^2 corrections from Q_1 can make a large impact, and since they are taken only in the large-β_0 limit the true result is still rather uncertain. For instance, assigning a

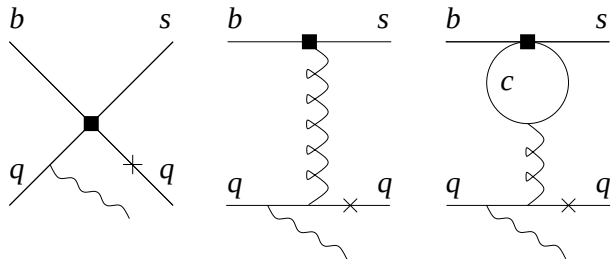

Figure 1: Feynman diagrams contributing to isospin violation in $B \to K^*\gamma$ decays.

100% uncertainty to the α_s^2 piece from Q_1 in (9), one finds an uncertainty of about ± 0.5 in the branching fractions. Also, because the $C_F n_f$ terms do not fix the perturbative definition of the charm-quark mass in the α_s contribution in (9), one can formally use values ranging from a low $\overline{\text{MS}}$ mass to a high pole mass, which can make a large numerical difference, about ± 0.4 in the range used in [10]. The message to be gained from this is that the full NNLO corrections from the set $Q_{1,2}$ also need to be calculated, in order to get the reduced perturbative uncertainty expected from a higher-order calculation.

5. Isospin violation and endpoint divergences

Calculations of interesting observables such as isospin violation in $B \to K^*\gamma$ decays require to include $1/m_b$ suppressed contributions to the factorization formula. Particularly important corrections come from topologies such as those in Figure 1, which I have taken from [11]. The square vertex indicates an insertion of $Q_1 \ldots Q_6$ for the graph on the left, an insertion of Q_8 for the graph in the center, and an insertion of $Q_{1,2}$ for the graph on the right. The crosses denote alternate attachments of the photon line. Note that the center and right graphs are α_s corrections compared to the graphs on the left. The contributions from the graphs in the figure, as well as some α_s terms linked to the graph on the left by RG-invariance, were calculated in [11]. To finish the calculation to this order would require to calculate the α_s corrections to the graphs on the left. This calculation is in progress [23]. However, once completed, the calculation will still face the problem that a straightforward application of the QCD factorization formalism leads to endpoint divergences in the convolution integrals. For example, the center graph in Figure 1 was shown in [11] to contain the integral

$$X_\perp = \int_0^1 du\, \phi_\perp^{K^*}(u) \frac{2-u}{3(1-u)^2}, \qquad (12)$$

which is equal to infinity under the conventional assumption that the LCDA vanishes as $1-u$ in the endpoint. The quantity $X_\perp$ is multiplied by the Wilson coefficient C_8 and was estimated by introducing an IR cutoff in [11] and found to be small, but clearly this is a conceptual problem. Moreover, there is no guarantee that similar endpoint divergences will not appear for the other operators in the weak Hamiltonian, once the full set of one-loop corrections to the graphs on the left in Figure 1 are included. Therefore, it is fair to say that a better understanding of power corrections is needed, in order to achieve a consistent framework for the calculation of quantities such as isospin violation.

6. Summary

Rare radiative $B \to V\gamma$ decays are of increasing interest as experimental measurements become more precise. QCD factorization and SCET have provided a theoretical framework which can be used to calculate observables for these decays to leading order in $1/m_b$. The perturbative hard-scattering kernels have been known at NLO in α_s for some time, and recently a set of NNLO results have been obtained for the operators Q_1, Q_7, and Q_8, although results for the four-quark operators Q_1 and Q_2 are not yet complete. The interesting observables such as isospin asymmetries and branching fractions in the $b \to d\gamma$ modes are sensitive to power corrections in $1/m_b$. Some of these have been estimated in the framework of light-cone sum rules, but a systematic treatment in SCET or QCD factorization is still missing due to the presence of endpoint divergences; a solution to this problem would be a much desired advance in this field.

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