Lattice QCD and Flavour Physics

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I report on recent progress and future prospects for lattice QCD calculations relevant for flavour physics and CP violation. I will focus on lattice studies that incorporate realistic vacuum polarization effects, i.e., with $n_f = 2+1$ sea quarks.

1. Introduction

Theoretical calculations of nonperturbative QCD effects are essential to make full use of the results generated by the experimental flavour program. The combination of theory and experiment can be used to extract the values of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, look for new physics (NP) and constrain beyond the Standard Model (BSM) theories. In order to be useful for achieving these goals, the theoretical calculations must have errors of the same order of the experimental errors. This requires uncertainties at the few percent level.

Lattice QCD is a nonperturbative formulation of QCD based only on first principles. It also provides a quantitative calculation methodology, which has become a precise tool capable of providing some of the accurate determinations needed by phenomenology. The processes that can be analyzed with current lattice QCD methods in a precise way are those with stable (or almost stable) hadrons and no more than one hadron in the initial (final) state. Those processes include leptonic and semileptonic decays and neutral meson mixing.

Accuracy in lattice calculations requires control over all the sources of systematic error. In particular, it is essential to take into account vacuum polarization effects in a realistic way, i.e., including up, down and strange sea quarks on the gauge configurations' generation. The up and down quarks are usually taken to be degenerated, so those simulations are referred to as $n_f = 2+1$. The vacuum polarization effects were almost always neglected in old lattice calculations due to limited computational power. This is known as the quenched approximation and introduces an uncontrolled and irreducible error, which can be as large as 10-30% [1]. Simulations with $n_f = 2$ sea quarks are still missing part of the vacuum polarization effects and the associated systematic error is hard to estimate without repeating the calculation with $n_f = 2+1$ sea quarks.

Another important source of systematic error is associated with the fact that current simulations are unable to simulate up and down quarks as light as the physical ones. The way of connecting lattice results to the physical world in a model independent way is by extrapolating those results using the guide of Chiral Perturbation Theory (ChPT). In order for this extrapolation to have controlled errors and be realistic, simulations must be performed for a range of light sea quark masses smaller than $m_s/2$.

Other systematic errors that must be included in any realistic lattice analysis are discretization, finite volume and renormalization effects. Discretization effects can be estimated by power counting, but this estimate must be explicitly tested by performing the calculation at several values of the lattice spacing. Finite volume effects can be estimated by repeating the calculation at several volumes and/or using ChPT techniques.

It is important to establish the validity of lattice methods by comparing its predictions against well known experimental quantities. Figure 1 shows the ratio between lattice results and experimental measurements for different decay constants, hadron masses and mass splittings. The agreement between lattice QCD and experiment is remarkable once vacuum polarization effects are included in a realistic way. This gives us confidence in lattice techniques.

In the next Sections I will discuss processes relevant for the experimental flavour physics program for which lattice QCD can provide accurate determinations of the nonperturbative inputs. I will thus restrict my discussion to calculations with all sources of systematic error addressed. Among other things, that means that I will focus on simulations with $n_f = 2+1$ sea quarks.

1.1. Lattice fermion formulations

The inclusion of quarks in the lattice QCD action, besides being expensive in computing time, has associated several difficulties. One of them is the so-called doubling problem, that consists of the fact that when one discretizes the QCD quark action in the most straightforward way, the naive discretization, there appear 15 additional unphysical tastes for any continuum flavour. Several methods exist to deal with the doubling problem. The most popular choice is (improved versions of) the Wilson action [3], that solves the doubling problem by adding a dimension five operator with the cost of breaking chiral symmetry. Other fermion formulations keep an exact, Overlap [4], or

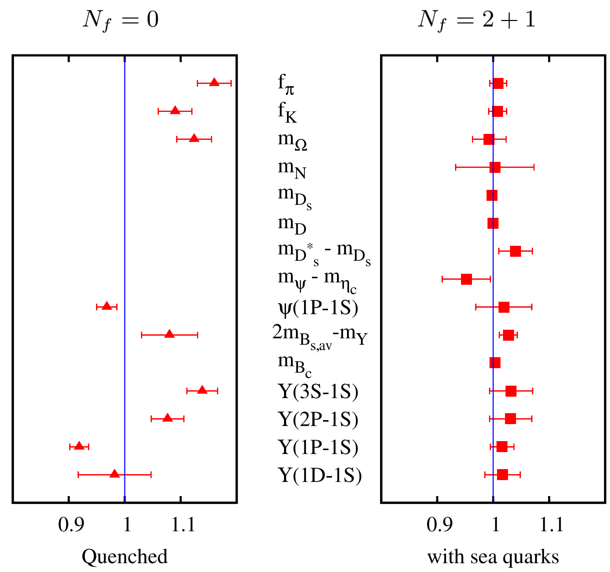

Figure 1: Lattice QCD results divided by experimental results in the quenched approximation (left panel) and with $n_f = 2+1$ sea quarks (right panel). Lattice values are obtained using improved staggered actions to describe sea quarks and u, d, s and c valence quarks, and NRQCD to simulate the b valence quarks. The plot is an updated version of the one in [2].

almost exact, Domain Wall [5], chiral symmetry paying the price of significantly complicating the operator structure and thus increasing the computing cost. The recent progress on the development of algorithms however, is starting to make feasible to perform unquenched calculations with those formulations.

Staggered fermions have good chiral properties and are computationally more efficient than any other light fermion formulation. The downside is that the doubling problem is not completely eliminated but reduced to four tastes for any continuum flavour. In unquenched simulations the extra tastes are eliminated by taking the root of the fermion determinant in the generation of configurations. This procedure is the focus of intense scrutiny by the lattice community and there has been considerable progress in our understanding of the issues involved during the past years [6]. To date there is no proof of the correctness or incorrectness of the method, but all the tests performed on dynamical simulations taking the root of the determinant have given evidence of controlled effects which disappear in the continuum limit.

The Asqtad [7] is the staggered formulation most widely used. It is improved with respect to the original staggered action so leading discretization effects are removed at tree-level. Remaining errors are therefore $\mathcal{O}(\alpha_s a^2)$ and $\mathcal{O}(a^4)$. That is the fermion action that the MILC collaboration is using in the generation of the $n_f = 2+1$ configurations, which are employed in several of the lattice calculations referred to in the next sections. An even more improved staggered action is the recently developed HISQ (highly improved staggered quark) action [8]. It reduces the remaining $\mathcal{O}(\alpha_s a^2)$ errors coming from taste-changing effects in the Asqtad action by roughly a factor of three.

For heavy quarks, charm and bottom, discretization errors coming in powers of the mass in lattice units, am_Q, are not negligible at typical lattice spacings.

The HISQ action however incorporate corrections that remove the dominant am_Q effects so the leading mass discretization effects are $\mathcal{O}(\alpha_s(am_c)^2)$ and $\mathcal{O}(am_c)^4$. This action can thus be used in a very effective way to describe charm quarks if the lattice spacing is small enough[1].

To describe bottom quarks, however, an effective field theory framework is more adequate. Some implementations of the effective field methods are lattice heavy quark effective theory (HQET) (whose leading term is the static approximation), non-relativistic QCD (NRQCD) and the Fermilab approach. The lattice NRQCD Lagrangian is obtained by discretizing the non-relativistic expansion of the continuum Dirac Lagrangian. The particular lattice NRQCD action used in the HPQCD calculations of B decay constants and B^0 mixing parameters described in next sections is improved through $\mathcal{O}(1/M^2)$, $\mathcal{O}(a^2)$ and leading relativistic $\mathcal{O}(1/M^3)$ [9]. The action parameters are fixed via heavy-heavy simulations, in particular the valence b quark mass is tuned to give the physical value of the Υ mass.

The Fermilab action [10] is obtained following a different approach. It starts with an improved relativistic Wilson action [3], which has the same heavy quark limit as QCD. With the Fermilab interpretation in terms of HQET[10], this action can be used to describe heavy quarks without errors that grow as $(am_Q)^n$. One of the advantages of this approach is that it can be used for both charm and bottom quarks. The errors associated with the use of the Fermilab action mentioned in this paper, used by the FNAL/MILC collaboration, are $\mathcal{O}\left(\alpha_s\Lambda/m_Q\right)$ and $\mathcal{O}\left(\Lambda/m_Q\right)^2$.

2. Leptonic decays

The lattice determination of pseudoscalar decay constants, together with experimental measurements of pseudoscalar leptonic decay widths, can be used to extract the value of the CKM matrix elements in-

[1]The MILC ensembles used by the HPQCD collaboration in its HISQ studies have small enough lattice spacings so the heavy discretization errors for the charm quark are under control.

volved in the process

$$\Gamma(P_{ab} \to l\nu) = (\text{known factors})\, f_P^2 |V_{ab}|^2 \,. \qquad (1)$$

On the other hand, for decay constants well determined experimentally, those for which the CKM matrix elements involved are known with a good precision and experimental measurements are accurate, the comparison with lattice calculations can be used as a test of the theory used to make the theoretical prediction.

The calculation of decay constants on the lattice is done using a simple matrix element,

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 c|P(p)\rangle = i f_P p_\mu \,, \qquad (2)$$

so they can be obtained with a very good accuracy. In the next subsections I will summarize the status of unquenched lattice calculations of charm and bottom decay constants. The current status of the lattice calculations of f_K/f_π, from which one can extract the value of the CKM matrix element $|V_{us}|$, was reviewed by F. Mescia in this conference [11].

2.1. D^+ and D_s decay constants

For the decays constants in the charm sector f_{D^+} and f_{D_s} there are lattice results available from two groups with $n_f = 2+1$ sea quarks, the FNAL/MILC [12] and the HPQCD [13] collaborations, to compare against experiment. Both use configurations generated by the MILC collaboration for three different values of the lattice spacing, $a = 0.15\ fm$, $a = 0.12\ fm$ and $a = 0.9\ fm$. The main difference between the two collaborations is the treatment of the valence quarks. While HPQCD uses the HISQ action for all the valence quarks, FNAL/MILC uses staggered Asqtad for the light quarks (up, down and strange) and the Fermilab action for the charm quark. The HPQCD collaboration has partially conserved currents, so they can extract the value of the decay constant without any renormalization. The FNAL/MILC collaboration needs to renormalize its currents, but they do it in a partially non-perturbative way that generates very small errors, around 1.5%.

The values for f_{D^+}, f_{D_s} and the ratio of both quantities, together with the new CLEO-c experimental results presented in this conference [14], are collected in Table I. Both lattice collaborations agree very well in the central values obtained for the decay constants. The errors of the HPQCD calculation are smaller than those for the FNAL/MILC due to the fact that the HISQ action is more improved[2] than the Fermilab action and thus discretization errors are sensibly reduced. The FNAL/MILC results agree with experiment for f_{D^+} and f_{D_s} within errors. The HPQCD result for f_{D^+} agrees also very well with the experimental number. However there is a discrepancy of over 3σ between the HPQCD and experimental values for f_{D_s}. This discrepancy has been recently suggested to be a possible hint of beyond the Standard Model effects [15]. More work is needed to resolve this issue. From the experimental side, a reduction of the errors that will constrain the possible statistical fluctuations is expected. It is also desirable that certain issues like the assumption of three-generation CKM unitarity to set $V_{cs} = V_{ud}$ or the inclusion of radiative corrections from experimental data or Monte Carlo simulations are addressed. From the theory side, the error on the FNAL/MILC result for f_{D_s} is currently larger by roughly a factor of four than the error on the HPQCD result. The FNAL/MILC collaboration plans to reduce the uncertainties in their calculation in the near future by increasing statistics, using a smaller lattice spacing, and improving the determination of the inputs needed. This will constitute an important check of the HPQCD numbers.

Group	Reference	f_{D^+}	f_{D_s}	f_{D_s}/f_{D^+}
experiment	[14]	(*CLEO-c*) 205.8(8.9)	(*average*) 269.6(8.3)	1.31(7)
HPQCD	[13]	207(4)	241(3)	1.164(11)
FNAL/MILC	[12]	215(14)	254(14)	1.188(26)

Table I Comparison of f_{D^+} and f_{D_s} as obtained from experiment and from unquenched lattice QCD calculations.

Calculations of f_{D^+} using Wilson fermions are also making progress, although they are still restricted to $N_f = 2$ simulations. A value of $f_{D^+} = 201 \pm 22^{+4}_{-9}$ based on a single value of the lattice spacing was reported in [16]. The number is compatible with the $N_f = 2+1$ calculations but with larger errors. The authors in that reference determine the ratio f_{D^+}/f_π and use the experimental value of f_π instead of directly determine f_{D^+} since the chiral logarithms are suppressed in the ratio and the associated systematic error is thus reduced. Preliminary results using twisted mass fermions with $N_f = 2$ for f_{D^+} and f_{D_s} were also reported by the ETM collaboration in [17] for two different values of the lattice spacing.

2.2. B and B_s decay constants

Lattice results for the decay constants in the B sector are more needed than in the D sector since the corresponding CKM matrix elements to extract the information from experiment are worse known. The

[2] Improvement in this context refers to the addition of higher-dimensional operators to the action.

value of the B decay constants are used in the SM predictions for processes very sensitive to beyond SM effects, such as $B_s \to \nu^+\nu^-$. The purely leptonic decays themselves are also a sensitive probe of effects from charged Higgs bosons.

The way of calculating these decay constants on the lattice is the same as for the charm decay constants. In fact, the FNAL/MILC collaboration has also calculated f_B, f_{B_s} and the ratio of both with the same choice of actions, same ensembles and same procedure as for the D decay constants in [12]. The errors in both the charm and bottom mesons are thus very similar. The results are listed in Table II. In that table, the results from the other lattice $n_f = 2+1$ calculation, by the HPQCD collaboration, are also included. In this case the HPQCD collaboration used the NRQCD action mentioned in section 1.1 to describe the b valence quark. Errors are then significantly larger than in their analysis of D^+ and D_s decay constants where they use the HISQ action for the c quark. A dominant source of uncertainty in their NRQCD calculation is the error associated with the one-loop renormalization applied.

Group	Reference	f_B	f_{B_s}	f_{B_s}/f_B
FNAL/MILC	[12]	197(13)	240(12)	1.22(3)
HPQCD	[18]	216(22)	260(26)	1.20(3)

Table II Values of f_B and f_{D_s} as obtained from the two lattice QCD calculations with $n_f = 2+1$.

A suggested way of reducing the uncertainty in the calculation of f_{B_s}/f_B is by extracting it from the double ratio $[f_{B_s}/f_B]/[f_K/f_\pi]$ which can be calculated very accurately since it is very close to one in ChPT.

3. Semileptonic decays

Semileptonic decays can be used to extract CKM matrix elements like $|V_{cb}|$, $|V_{ub}|$, $|V_{cd}|$, $|V_{cs}|$ and $|V_{us}|$. The theory input needed to get those parameters from experimentally measured semileptonic widths are the form factors in terms of which the hadronic matrix elements involved on those decays are parametrized. For example, for the decay $D \to Kl\nu$, the differential decay rate is given by

$$\frac{d\Gamma}{dq^2} = (\text{known factors})|V_{cs}|^2 f_+^2(q^2)\,, \tag{3}$$

where $f_+(q^2)$ is the vector form factor, which can be extracted from the matrix element of the vector current

$$\langle K|V^\mu|D\rangle = f_+(q^2)(p_D + p_K - \Delta)^\mu + f_0(q^2)\Delta^\mu\,, \tag{4}$$

with $\Delta^\mu = (m_D^2 - m_K^2)q^\mu/q^2$.

Lattice QCD can be used to calculate the value of those form factors as a function of the virtual W momentum transfer, q^2, or, equivalently, the recoil momentum of the daughter meson. On the lattice, the smallest discretization errors correspond to the form factor at the largest momentum transfer, where the experimental data are less precise. In addition, the finite volume provides an infrared cutoff and there is a finite minimum value for the momenta that can be simulated. A way of circumventing this limitation is by using twisted boundary conditions that allow for arbitrary small values of the momenta [19]. Another set of important techniques that can be applied to semileptonic decay analysis are double ratio methods [20]. These methods can yield a reduction of both statistical and systematic uncertainties by a partial or total cancellation of those uncertainties between numerator and denominator in the ratios.

In the case of $|V_{us}|$, experimental data for $K \to \pi l\nu$ and lattice results for the corresponding form factors at zero momentum transfer yield a determination of this parameter with an error claimed to be 0.5 % -see F. Mescia's [11] and P. Gambino's [21] talks at this conference.

The FNAL/MILC calculation of the shape of the form factor f_+^K for the decay $D \to Kl\nu$ [22] in 2004 constituted a prediction since the result appeared before the experimental measurements by the FOCUS [23] and Belle [24] collaborations. The comparison of this lattice calculation with Belle experimental data is shown in Figure 2 as a function of q^2. The determination of the vector form factor for $D \to Kl\nu$ and $D \to \pi l\nu$ are being currently updated by the FNAL/MILC collaboration. The most straightforward improvement is the extension of the calculation to smaller values of the lattice spacing. This will reduce discretization errors which are the main source of uncertainty in the previous calculation.

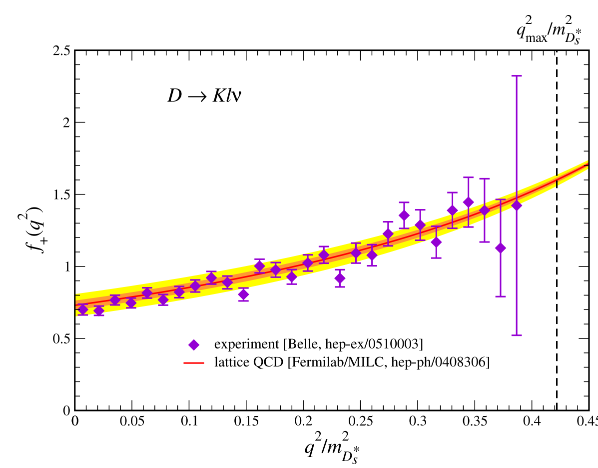

Figure 2: Comparison of the shape of the vector form factor $f_+^K(q^2)$ as measured experimentally and obtained on the lattice [25].

The matrix element $|V_{cb}|$ can be extracted from the decay $B \to D^* l\nu$. The experimental results for this process at zero recoil have smaller errors than those for $B \to Dl\nu$. A new lattice calculation of the form factor describing the decay, the axial vector form factor at zero recoil $\mathcal{F}_{\mathcal{B}\to\mathcal{D}^*}(1)$, was presented in the lattice conference last year [26]. This calculation eliminates the quenching errors from previous calculations since it includes $n_f = 2+1$ sea quarks. It also introduces a new double ratio method which gives the form factor at zero recoil directly and with a reduction of the computational cost. The relation between the double ratio and the form factor,

$$|\mathcal{F}_{\mathcal{B}\to\mathcal{D}^*}(1)|^2 = \frac{\langle D^*|\bar{c}\gamma_j\gamma_5 b|\bar{B}\rangle\langle\bar{B}|\bar{b}\gamma_j\gamma_5 c|D^*\rangle}{\langle D^*|\bar{c}\gamma_4 c|D^*\rangle\langle\bar{B}|\bar{b}\gamma_4 b|\bar{B}\rangle}\,, \quad (5)$$

is exact to all orders in the heavy-quark expansion in the continuum. Statistical errors in the numerator and denominator are highly correlated and largely cancel. And also, most of the renormalization cancels, yielding a small uncertainty for the perturbative matching.

The final result obtained for the form factor after chiral and continuum extrapolation is $\mathcal{F}_{B\to D^*}(1) = 0.921 \pm 0.013 \pm 0.021$ [27], where the first error is statistical and the second one includes all sources of systematic errors and is dominated by heavy-quark discretization errors. The CKM matrix element $|V_{cb}|$ extracted from this value of the form factor and the experimental averages in [28] is

$$|V_{cb}| = (39.2 \pm 0.6 \pm 1.0) \times 10^{-3}\,. \quad (6)$$

This value differs by 2σ from the one extracted from inclusive decays.

An alternative method for the extraction of $|V_{cb}|$ is the analysis of the decay $B \to Dl\nu$. A study of the form factors needed for such determination with quenched Wilson fermions was presented in [29]. An interesting aspect of that analysis is that the authors calculate the scalar form factor as well as the momentum transfer dependence to avoid needing to extrapolate to zero recoil, where the experimental data suffer from phase space suppression compared to the $B \to D^* l\nu$ case. The scalar form factor, which only contributes for $l = \tau$ can be used to constrain BSM physics [30].

The decay $B \to \pi l\nu$ provides a way of extracting $|V_{ub}|$ that is competitive with $b \to u$ inclusive decays. The only unquenched ($N_f = 2+1$) lattice determination of the form factor needed for this extraction to date is the HPQCD calculation in [31], which uses the Asqtad staggered formulation for the light quarks and NRQCD for the b quarks. Together with experimental results, from this calculation one obtains [3] $|V_{ub}| = (3.55 \pm 0.25 \pm 0.50) \times 10^{-3}$. The error is larger than it could be due to the fact that lattice and experimental results for the corresponding decay rate have a poor overlap in q^2. There are several methods that can be adopted to try to address this issue. The HPQCD collaboration is reducing the recoil momentum of the pion required for small q^2 by using a lattice frame in which the B meson is moving in the opposite direction to the pion. A modified version of NRQCD (moving-NRQCD) provides the description of the corresponding b quark with a large velocity [32]. It is then possible to calculate the form factors at small q^2 without needing large recoil momenta, hence keeping discretization and statistical errors under control. The FNAL/MILC collaboration is implementing another method to overcome the poor overlap [33]. They are using the so called $z-$expansion to parameterize the shape of the form factors. This parameterization is model independent and based only on unitarity and analyticity [34]. When the experimental data are analyzed using the same model-independent parameterization, the results for the shape parameters from theory and experiment can be compared directly even if the respective q^2 ranges have poor overlap. Hence, Lattice QCD must provide only the normalization of the form factor, which can be calculated at the q^2 values where theoretical errors are smallest. The FNAL/MILC collaboration uses Fermilab action for the b quark and improved staggered formalism to describe the light quark in the numerical simulations for this work. The preliminary results obtained by this collaboration are shown in Figure 3. They look quite promising.

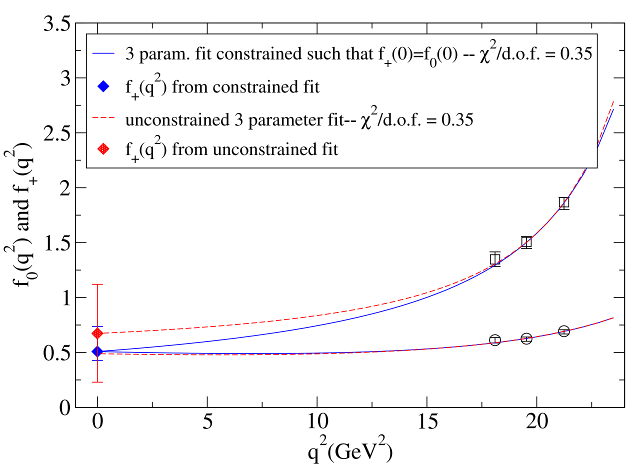

Figure 3: Preliminary results in [33] for the form factor describing the decay $B \to \pi l\nu$ as a function of q^2.

The calculation of semileptonic and leptonic decays on the lattice can be used to construct ratios indepen-

[3]First error is experimental, the second one is the theoretical error from the lattice calculation of the vector form factor.

dent of CKM matrix elements, such as $\frac{\Gamma(D\to l\nu)}{\Gamma(D\to\pi l\nu)}$ or $\frac{\Gamma(D_s\to l\nu)}{\Gamma(D\to K l\nu)}$, with which one could test the consistency of lattice calculations against experiment or constrain BSM physics.

4. Neutral meson mixing

The theoretical determination of the parameters that describe the mixing in the neutral Kaon and B systems are needed in order to test for the consistency of the SM description of CP violation when compared against experiment.

In the SM, the neutral K and B meson mixing is due to box diagrams with exchange of two W-bosons. These box diagrams can be rewritten in terms of an effective Hamiltonian with four-fermion operators describing processes with $\Delta F = 2$ (F=S or B). The matrix elements of the operators between the neutral meson and antimeson encode the non-perturbative information on the mixing and can be calculated using lattice QCD techniques.

4.1. Indirect CP violation in neutral kaon decays: B_K.

The non-perturbative input to study CP violating effects in $K^0 - \bar{K}^0$ mixing is parametrized by B_K, defined as

$$B_K(\mu) \equiv \frac{\langle \bar{K}^0 | Q_{\Delta S=2}(\mu) | K^0 \rangle}{\frac{8}{3}\langle \bar{K}^0 | \bar{s}\gamma_\mu\gamma_5 d|0\rangle\langle 0|\bar{s}\gamma_\mu\gamma_5 d|K^0\rangle}\,. \quad (7)$$

The theoretical calculation of this parameter, together with the experimental measurement of $\varepsilon_K \equiv \left|\frac{A(K_L\to(\pi\pi)_{I=0})}{A(K_S\to(\pi\pi)_{I=0})}\right|$, gives a hyperbole in the $\rho - \eta$ plane, where ρ and η are the usual unitarity triangle parameters. That corresponds to the light blue band in Figure 8.

The most recent unquenched lattice determinations of $B_K(\mu)$ are shown in Figure 4. The current most accurate value is the one by the RBC/UKQCD collaboration using domain wall fermions with $n_f = 2+1$ sea quarks [37]

$$B_K^{\overline{MS}}(2\text{ GeV}) = 0.524 \pm 0.010 \pm 0.013 \pm 0.025\,, \quad (8)$$

where the first error is statistical, the second one is associated with the non-perturbative renormalization and the third one corresponds to the other systematic errors. The main source of uncertainty in this calculation is discretization errors. The result in (8) is obtained from simulations with a single value of the lattice spacing, $a^{-1} = 1.729(28)$GeV. The discretization error corresponding to the use of this single value of a is estimated in [37] by using the scaling behaviour of a previous quenched calculation with the same light

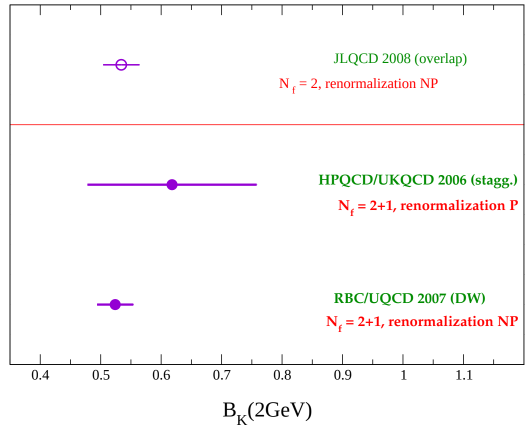

Figure 4: Recent unquenched lattice values of B_K. The parameter is given in the $\overline{MS} - NDR$ scheme and at an scale equal to 2GeV. The results are taken from [35] (JLQCD), [36] (HPQCD/UKQCD) and [37] (RBC/UKQCD). The matching is done nonperturbatively in [35] and [37], and perturbatively at one-loop in [36]. The perturbative matching is the origin of the rather large error.

quark action. The authors found that error to be of around a 4%, which dominates the systematic uncertainty. The RBC/UKQCD collaboration is planning to explicitly check discretization effects by performing the unquenched calculation with the same quark and gluon actions but with a smaller lattice spacing.

The JLQCD result in Figure 4 with $N_f = 2$ sea quarks is affected by the fact that part of the vacuum polarization effects are still missing and are difficult to estimate *a priori*. This collaboration is planning on extending its methodology to a $N_f = 2+1$ calculation of B_K. The needed configurations are being currently generated. The ensemble will include also configurations for two different volumes. This will allow them to explicitly study and reduce the finite volume effects that are the main source of uncertainty in their $N_f = 2$ calculation.

There are two other unquenched determinations of B_K in progress which use mixed actions[4]. A $N_f = 2$ calculation outlined in [38] uses overlap fermions for the valence quarks and twisted mass fermions for the sea quarks. A $N_f = 2+1$ study whose preliminary results can be found in [39] uses domain wall valence fermions and improved staggered sea fermions. The errors due to the matching to the continuum affecting previous staggered calculations can be highly re-

[4]In a mixed action calculation the valence and sea quarks are described with different fermion formulations.

duced in this analysis by performing the renormalization non-perturbatively. The use of domain wall valence quarks makes the chiral extrapolation more continuum-like than in the purely staggered case. Another advantage of this calculation is that there are staggered configurations generated by the MILC collaboration for a large range of lattice spacings, volumes and small sea quark masses. This allows for good control over the systematic error from chiral and continuum extrapolations. The expected final error from this calculation is around 5%.

In the near future, there will be thus several lattice calculations of B_K using different discretizations and with errors at the 5% level.

4.2. B^0 mixing: $\Delta M_{d,s}$, $\Delta\Gamma_{d,s}$ and ξ

The mixing in the $B^0_q - \bar{B}^0_q$ system is an interesting place to look for NP effects. The BSM effects can appear as new tree level contributions, or through the presence of new particles in the box diagrams. In fact, it has been recently claimed that there is a disagreement between direct experimental measurement of the phase of B^0_s mixing amplitude and the SM prediction [40]. Possible NP effects have also been reported to show up in the comparison between direct experimental measurements of $\sin(2\beta)$ and SM predictions using B^0 mixing parameters [41]. Studies of neutral B meson mixing parameters can also impose important constraints on different NP scenarios [42].

The quantities that describe the mixing in the B^0 system are the mass differences, $\Delta M_{s,d}$, and decay width differences, $\Delta\Gamma_{s,d}$, between the heavy and light B^0_s and B^0_d mass eigenstates. The non-perturbative physics of those processes is contained in hadronic matrix elements of the four-fermion operators in the effective Hamiltonian with $\Delta B = 2$. Those matrix elements are parametrized by products of B decay constants and bag parameters. For example, for the mass difference

$$\Delta M_{s(d)}|_{theor.} \propto |V^*_{ts(d)} V_{tb}|^2 f^2_{B_{s(d)}} \hat{B}_{B_{s(d)}} \,, \qquad (9)$$

with $\langle \bar{B}^0_s | Q^{s(d)}_L | B^0_s \rangle = \frac{8}{3} M^2_{B_{s(d)}} f^2_{B_{s(d)}} B_{B_{s(d)}}(\mu)$ and $O^{s(d)}_L = \left[\bar{b}^i \gamma_\mu (1-\gamma_5) s^i (d^i)\right] \left[\bar{b}^j \gamma^\mu (1-\gamma_5) s^j (d^j)\right]$.

Many of the uncertainties that affect the theoretical calculation of the decay constants and bag parameters cancel totally or partially if one takes the ratio $\xi^2 = f^2_{B_s} B_{B_s} / f^2_{B_d} B_{B_d}$. Hence, this ratio and therefore the combination of CKM matrix elements related to it, $|\frac{V_{td}}{V_{ts}}|$, can be determined with a significantly smaller error than the individual matrix elements. The ratio ξ is also an important ingredient in the unitarity triangle analyses.

The first lattice calculation of the B^0 mixing parameters with $n_f = 2+1$ sea quarks, which only studied the B^0_s sector, was performed by the HPQCD collaboration in [43]. The authors obtained

$$\Delta M_s = 20.3(3.0)(0.8) ps^{-1} \quad \text{and} \quad \Delta\Gamma_s = 0.10(3) ps^{-1} \,, \qquad (10)$$

which is compatible with experiment.

The FNAL/MILC and HPQCD collaborations are currently working on a more complete study of $B^0 - \bar{B}^0$ mixing, including B^0_s and B^0_d parameters. The main goal of both projects is obtaining the ratio ξ fully incorporating vacuum polarization effects. The choice of actions and the setup is the same as for their f_B, f_{B_s} calculations described in Section 2 -more details can be found in [44].

Results from the two collaborations are still preliminary. Figures 5 and 6 show some examples of the values of $f_B\sqrt{M_B B_B}$ obtained as function of the light sea quark masses. The dependency on the light sea quark mass is in both studies very mild, so only the chiral extrapolation in the d quark mass for B^0_d parameters is expected to be a significant source of error. In Figure 5, it can also be appreciated that the results for the two different lattice spacings are very similar, which indicates small discretization errors.

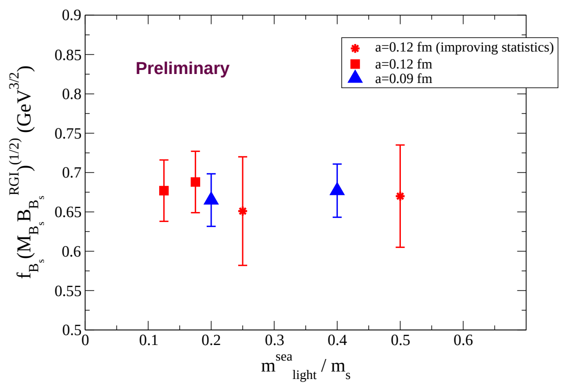

Figure 5: Values of $f_{B_s}\sqrt{M_{B_s}\hat{B}_{B_s}}$ in GeV$^{3/2}$ as a function of the light sea quark mass normalized to the physical strange quark mass from the HPQCD collaboration. The data include statistical, perturbative and scale errors. The bottom valence quark is fixed to its physical value and the strange valence quark is very close to its physical value. The strange sea quark mass is also very close to its physical value.

A comparison of the preliminary results from the two collaborations for the ratio ξ is shown in Figure 7. Only the full QCD points[5] are included. The results of the two collaborations agree within statistical errors. This is very encouraging since both analyses use

[5] Where valence and sea quark masses are the same.

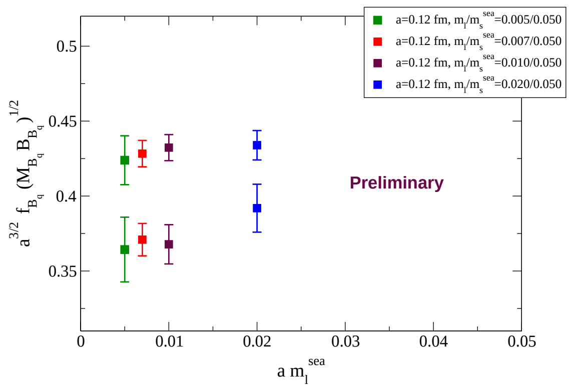

Figure 6: Bare values of $f_{B_q}\sqrt{M_{B_q}B_{B_q}}$ in lattice units as a function of the light sea quark mass also in lattice units from the FNAL/MILC collaboration. Results are shown for both B_s^0 and B_d^0 including only statistical errors. The results correspond to one of the three lattice spacings at which the FNAL/MILC's study is performed. The bottom valence quark is fixed to its physical value and the strange valence quark is very close to its physical value. The strange sea quark mass is also very close to its physical value.

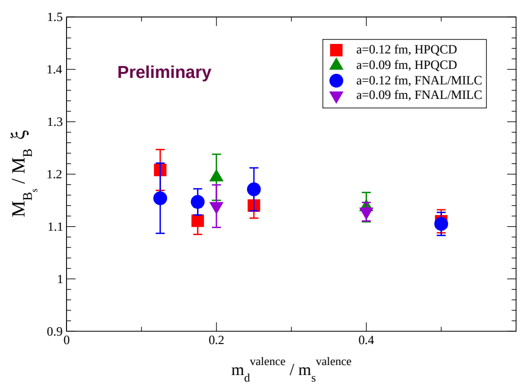

Figure 7: Product of the ratios ξ and M_{B_s}/M_{B_d} as a function of the down quark mass normalized to the strange quark mass. Results for both FNAL/MILC and HPQCD collaborations including only statistical errors are shown for two different values of the lattice spacing a.

completely different descriptions for the heavy quarks.

The final step in those calculations is extrapolating the results to the physical values of the light quark masses and, in the case of the FNAL/MILC collaboration, performing simultaneously the extrapolation to the continuum. Systematic errors on those extrapolations are currently being studied. These two analyses are expected to have final results very soon with total errors ranging $5-7\%$ for $f_{B_q}\sqrt{B_{B_q}}$ and $2-3\%$ for ξ.

The effects of heavy new particles in the box diagrams that describe the B^0 mixing can be seen in the form of effective operators built with SM degrees of freedom. The NP could modify the Wilson coefficients of the four-fermion operators that already contribute to B^0 mixing in the SM and gives rise to new four-fermion operators in the $\Delta B = 2$ effective Hamiltonian -see [45, 46] for a list of the possible operators in the SUSY basis. The calculation of those Wilson coefficients for a particular BSM theory, together with the lattice calculation of the matrix elements of all the possible four-fermion operators in the SM and beyond and experimental measurements of B^0 mixing parameters, can constraint BSM parameters and help to understand new physics. To date, there does not exist an unquenched determination of the complete set of matrix elements of four-fermion operators in that general $\Delta B = 2$ effective Hamiltonian. However, the two collaborations currently working on B^0 mixing in the SM are planning to extend their analysis to BSM operators in the near future. Actually, the HPQCD collaboration has already calculated the one-loop matching coefficients needed for such an analysis [47].

5. Conclusions

Hints of discrepancies between SM predictions and experimental measurements have started to show up in some CP violating observables [48]. As claimed in [48], the precise determination of parameters like $\hat{B}_K$, f_K and ξ, and CKM matrix elements like $|V_{cb}|$ is crucial in order to fully exploit the potential of CP violating observables on constraining NP. Lattice QCD has a fundamental role in that program. There has recently been important progress in order to achieve realistic lattice calculations, with $n_f = 2{+}1$ sea quarks and a serious study of systematic errors. New results relevant for phenomenology have appeared in the last year in the Kaon and D meson sectors with errors at the few percent level. In the near future, results for B^0 mixing parameters and B decay constants will be also available with errors at the few percent level.

Figure 8 summarizes the impact of the lattice calculations with $n_f = 2+1$ on the unitarity triangle analysis. In generate the plot, the value of B_K is the one by the RBC/UKQCD collaboration [37], V_{us} is taken from leptonic decays with f_K/f_π given by the HPQCD collaboration [13], $|V_{cb}|$ is from semileptonic $B \to D^* l\nu$ with the form factor by the FNAL/MILC collaboration [27], $|V_{ub}|$ is from Flynn and Nieves [49] using, among other information, the form factor for $B \to \pi l\nu$ by the HPQCD [31] and FNAL/MILC [33, 50] collaborations. A final $n_f = 2+1$ result for ξ is not yet available. In order to illustrate its effect on the $\rho - \eta$ plane in Figure 8, we assumed a value for ξ with a 3% error. As explained in Section 4.2, this is

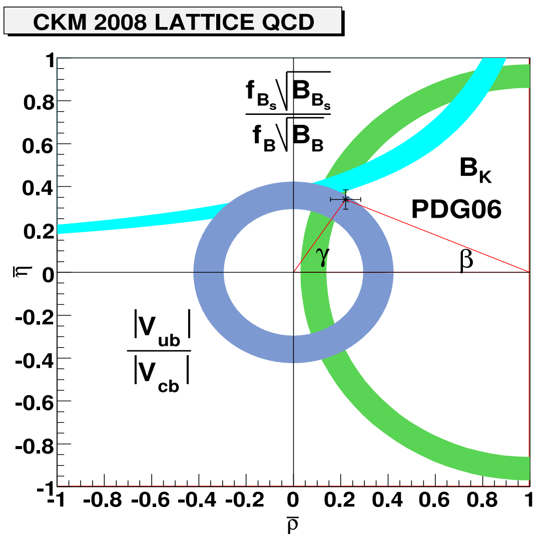

Figure 8: Constraints on the $\bar{\rho}-\bar{\eta}$ plane imposed by recent unquenched lattice QCD calculations. The black point and error bars correspond to the values of $\bar{\rho}$ and $\bar{\eta}$ from the PDG 2006 review of particle physics [51]. See the text for more explanations. The plot is an updated version of the one in [2].

the expected error for ξ from both the HPQCD and the FNAL/MILC calculations.

Several lattice collaborations are currently producing accurate $n_f = 2+1$ results as discussed in this paper, and other collaborations are starting to generate $n_f = 2+1$ ensembles using different sea quark actions. This will allow an important consistency check of lattice methods.

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