Theoretical review on $\sin 2\beta(\phi_1)$ from $b \to s$ penguins

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Recent theoretical results of the standard model expectations on $\sin 2\beta_{\rm eff}$ from penguin-dominated $b \to s$ decays are briefly reviewed.

I. INTRODUCTION

Possible New Physics beyond the Standard Model is being intensively searched via the measurements of time-dependent CP asymmetries in neutral B meson decays into final CP eigenstates defined by

$$\frac{\Gamma(\overline{B}(t) \to f) - \Gamma(B(t) \to f)}{\Gamma(\overline{B}(t) \to f) + \Gamma(B(t) \to f)} = \mathcal{S}_f \sin(\Delta m t) + \mathcal{A}_f \cos(\Delta m t), \qquad (1)$$

where Δm is the mass difference of the two neutral B eigenstates, S_f monitors mixing-induced CP asymmetry and $\mathcal{A}_f$ measures direct CP violation. The CP-violating parameters $\mathcal{A}_f$ and $\mathcal{S}_f$ can be expressed as

$$\mathcal{A}_f = -\frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \qquad \mathcal{S}_f = \frac{2\,{\rm Im}\lambda_f}{1+|\lambda_f|^2}, \qquad (2)$$

where

$$\lambda_f = \frac{q_B}{p_B}\,\frac{A(\overline{B}^0 \to f)}{A(B^0 \to f)}. \qquad (3)$$

In the standard model $\lambda_f \approx \eta_f e^{-2i\beta}$ for $b \to s$ penguin-dominated or pure penguin modes with $\eta_f = 1\ (-1)$ for final CP-even (odd) states and $\beta({\rm or}\ \phi_1) = \arg(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*)$. Therefore, it is expected in the Standard Model that $-\eta_f S_f \approx \sin 2\beta$ and $\mathcal{A}_f \approx 0$.

The mixing-induced CP violation in B decays has already been observed in the golden mode $\overline{B}^0 \to J/\psi K_S$ for several years. The current world average the mixing-induced asymmetry from tree $b \to c\bar{c}s$ transition is [1]

$$\sin 2\beta = 0.681 \pm 0.025\,. \qquad (4)$$

Results of the time-dependent CP-asymmetries in the $b \to sq\bar{q}$ induced two-body decays such as $\overline{B}^0 \to (\phi, \omega, \pi^0, \eta', f_0)K_S$ are shown in Fig. 1 and 2 [1]. In the SM, CP asymmetry in all above-mentioned modes should be equal to $\mathcal{S}_{J/\psi K}$ with a small deviation *at most* $\mathcal{O}(0.1)$ [2]. As discussed in [2], this may originate from the $\mathcal{O}(\lambda^2)$ truncation and from the subdominant (color-suppressed) tree contribution to these processes. Since the penguin loop contributions are sensitive to high virtuality, New Physics beyond the SM may contribute to $\mathcal{S}_f$ through the heavy particles in the loops. In order to detect the signal of New Physics unambiguously in the penguin $b \to s$ modes, it is of great importance to examine how much of the deviation of S_f from $\mathcal{S}_{J/\psi K}$,

$$\Delta\mathcal{S}_f \equiv -\eta_f \mathcal{S}_f - \mathcal{S}_{J/\psi K_S}, \qquad (5)$$

is allowed in the SM [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16].

The decay amplitude for the pure penguin or penguin-dominated charmless B decay in general has the form

$$M(\overline{B}^0 \to f) = V_{ub}V_{us}^* F^u + V_{cb}V_{cs}^* F^c + V_{tb}V_{ts}^* F^t. \qquad (6)$$

Unitarity of the CKM matrix elements leads to

$$\begin{aligned} M(\overline{B}^0 \to f) &= V_{ub}V_{us}^* A_f^u + V_{cb}V_{cs}^* A_f^c \\ &\approx A\lambda^4 R_b e^{-i\gamma} A_f^u + A\lambda^2 A_f^c, \end{aligned} \qquad (7)$$

where we use $A_f^u \equiv F^u - F^t$, $A_f^c \equiv F^c - F^t$ and $R_b \equiv |V_{ud}V_{ub}/(V_{cd}V_{cb})| = \sqrt{\bar\rho^2 + \bar\eta^2}$. The first term

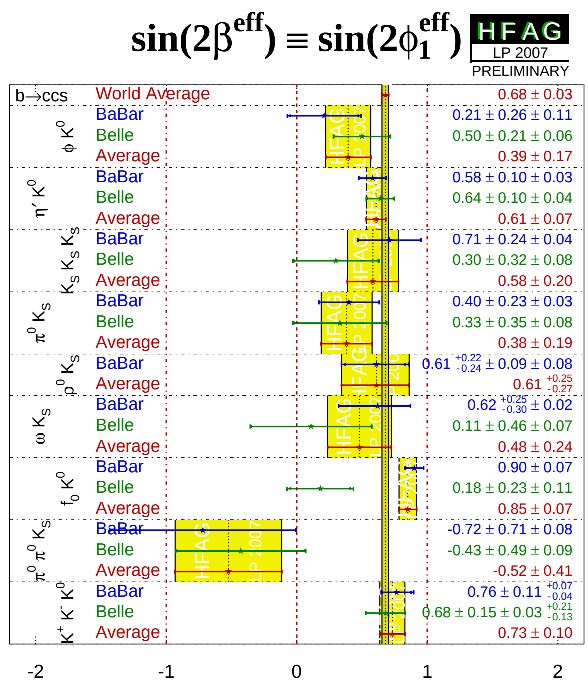

FIG. 1: Experimental results for $\sin 2\beta_{\rm eff}$ from $b \to s$ penguin decays [1].

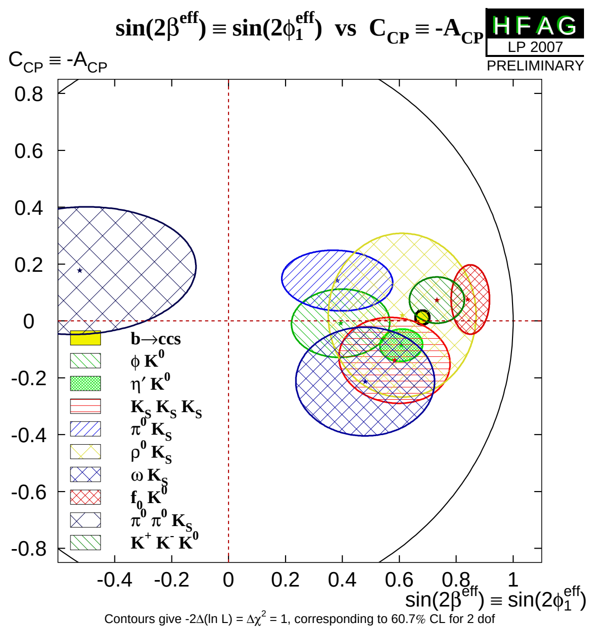

FIG. 2: Experimental results for $\sin 2\beta_{\rm eff}$ and $\mathcal{A}_f$ from $b \to s$ penguin decays [1].

in the above expression is suppressed by a factor of λ^2 relative to the second term. For a pure penguin decay such as $\overline{B}^0 \to \phi K_S$, it is naively expected that A^u_f is in general comparable to A^c_f in magnitude. Therefore, to a good approximation we have $-\eta_f S_f \approx \sin 2\beta \approx S_{J/\psi K}$. For penguin-dominated modes, such as $\omega K_S, \rho^0 K_S, \pi^0 K_S$, A^u_f also receives tree contributions from the $b \to u\bar{u}s$ tree operators. Since the Wilson coefficient for the penguin operator is smaller than the one for the tree operator, it is possible that A^u_f is larger than A^c_f. As the $b \to u$ term carries a weak phase γ, S_f may be subjected to a significant "tree pollution".

To quantify the deviation, it is known that [5, 17]

$$\Delta\mathcal{S}_f = 2|r_f|\cos 2\beta \sin\gamma \cos\delta_f,\ \ \mathcal{A}_f = 2|r_f|\sin\gamma\sin\delta_f,$$

with $r_f \equiv (V_{ub}V^*_{us}A^u_f)/(V_{cb}V^*_{cs}A^c_f)$ and $\delta_f \equiv \arg(A^u_f/A^c_f)$ and only terms up to the first order in r_f are shown. Hence, the magnitude of the CP asymmetry difference $\Delta\mathcal{S}_f$ and direct CP violation are both governed by the size of A^u_f/A^c_f. For the aforementioned penguin-dominated modes, the tree contribution is color suppressed and, hence, in practice, the deviation of $\mathcal{S}_f$ is expected to be small [2]. It is useful to note that $\Delta\mathcal{S}_f$ is proportional to the real part of A^u_f/A^c_f as shown in the above equation.

Below I will briefly review the results of the SM expectations on $\Delta\mathcal{S}_f$ from the SU(3)$_F$ approach, short-distance and long-distance calculations.

II. $\Delta\mathcal{S}_f$ FROM THE SU(3)$_F$ APPROACH

I briefly review the underlying reasoning of the SU(3)$_F$ approach (using [5] as an example) and summarize the present results. Recent reviews of results obtained from the SU(3)$_F$ approach can be found in [18, 19].

For a $\Delta S = 0$ decay, such as $\overline{B}^0 \to f'$ decay, the decay amplitude is given by

$$A(\overline{B}^0 \to f') = V_{ub}V^*_{ud}B^u_{f'} + V_{cb}V^*_{cd}B^c_{f'}. \qquad (8)$$

Note that comparing with the $\Delta S = 1$ amplitude, we have s replaced by d in the CKM matrices, resulting an opposite hierarchy of tree and penguin amplitudes. Hence, ratio of (tree dominated) $\Delta S = 0$ and (penguin dominated) $\Delta S = 1$ amplitudes may provide information on r_f.

Through SU(3)$_F$ symmetry, one can obtain

$$A^{u(c)}_f = \sum_{f'} C^{f'}_f B^{u(c)}_{f'}, \qquad (9)$$

where $C^{f'}_f$ are some SU(3) Clebsch-Gordan coefficients. Consequently, a suitable sum of $\overline{B}^0 \to f'$ amplitudes gives

$$\begin{aligned} A'(\overline{B}^0 \to f) &\equiv \sum_{f'} C^{f'}_f A(\overline{B}^0 \to f') \\ &= V_{ub}V^*_{ud}A^u_f + V_{cb}V^*_{cd}A^c_f, \qquad (10) \end{aligned}$$

which is identical to $A(\overline{B}^0 \to f)$, except with $V_{us,cs}$ replaced by $V_{ud,cd}$. Note that $A'(\overline{B}^0 \to f)$ is not a $\Delta S = 1$ decay amplitude, but a sum of several $\Delta S = 0$ amplitudes. The absolute value of the ratio of $A'(\overline{B}^0 \to f)$ and $A(\overline{B}^0 \to f)$ with a suitable CKM factor, gives

$$\xi_f \equiv \left|\frac{V_{us}A'(\overline{B}^0 \to f)}{V_{ud}A(\overline{B}^0 \to f)}\right| = \left|\frac{r_f + V_{us}V_{cd}/V_{ud}V_{cs}}{1 + r_f}\right|, \qquad (11)$$

which can be used to constrain r_f. There are two comments: (i) From the above expression, we see that the bound on r_f cannot be better than $|V_{us}V_{cd}/V_{ud}V_{cs}| = \mathcal{O}(\lambda^2)$. (ii) If no assumption on phases of $\overline{B}^0 \to f'$ amplitudes is made, the above ratio is bounded by

$$\xi_f \le \sum_{f'} |V_{us}/V_{ud}||C^{f'}_f|\sqrt{\frac{\mathcal{B}(\overline{B}^0 \to f')}{\mathcal{B}(\overline{B}^0 \to f)}}, \qquad (12)$$

which is, however, a rather conservative bound. The bounds work better for modes with less ($\Delta S = 0$) $\overline{B}^0 \to f'$ modes involved in the sum.

Present results on the bounds are briefly summarized, while more detail discussions can be found in recent reviews [18, 19]. Bounds on various ξ_f are found to be: $\xi_{\eta' K_s} < 0.116$ [18, 20], $\xi_{K^+K^-K^0} < 1.02$

TABLE I: $\Delta\mathcal{S}_f$ from various short-distance calculations.

$\Delta\mathcal{S}_f$	QCDF	pQCD	SCET	Expt
ϕK_S	0.02 ± 0.01	0.03 ± 0.03	0.01	-0.29 ± 0.17
ωK_S	0.13 ± 0.08	$0.16^{+0.04}_{-0.07}$	$-0.18^{+0.06}_{-0.07}$ 0.12 ± 0.03	-0.20 ± 0.24
$\rho^0 K_S$	$-0.08^{+0.08}_{-0.12}$	$-0.18^{+0.10}_{-0.07}$	$0.17^{+0.05}_{-0.06}$ $-0.12^{+0.03}_{-0.04}$	$-0.07^{+0.25}_{-0.27}$
$\eta' K_S$	0.01 ± 0.01		-0.02 ± 0.01 -0.01 ± 0.01	-0.07 ± 0.08
ηK_S	$0.10^{+0.11}_{-0.07}$		-0.03 ± 0.17 $+0.07 \pm 0.14$	
$\pi^0 K_S$	$0.07^{+0.05}_{-0.04}$	$0.06^{+0.02}_{-0.03}$	0.08 ± 0.03	-0.30 ± 0.19
$f_0 K_S$	0.02 ± 0.00			$+0.17 \pm 0.07$
$a_0 K_S$	0.02 ± 0.01			
$\bar{K}_0^{*0}\pi^0$	$0.00^{+0.03}_{-0.05}$ $0.02^{+0.00}_{-0.02}$			

and $\xi_{K_SK_SK_S} < 0.31$ [16]. Other results on $\eta' K_S$ and $\pi^0 K_S$ modes can be found in [20]. These bounds can be improved by measuring relevant $\Delta S = 0$ modes as much as possible. For example, measurements of $\pi^0\eta^{(\prime)}$ and $\eta^{(\prime)}\eta^{(\prime)}$ rates can improve the $\xi_{\eta' K_S}$ bound (see [21] for recent update on the data).

III. $\Delta\mathcal{S}_f$ FROM SHORT-DISTANCE CALCULATIONS

A. $\Delta\mathcal{S}_f$ in two-body modes

There are several QCD-based approaches in calculating hadronic B decays [22, 23, 24]. $\Delta\mathcal{S}_f$ from calculations of QCDF [9, 10], pQCD [11], SCET [12, 13] are summarized in Table 1. The QCDF calculations on PP, VP modes are from [9] [32], while those in SP modes are from [10]. The SCET calculations on PP modes are from [12], while those on VP modes are from [13]. It is interesting to note that (i) $\Delta\mathcal{S}_f$ are predicted to be small and positive in most cases, while experimental central values for $\Delta\mathcal{S}_f$ are all negative, except the one from f_0K_S; (ii) In most cases, QCDF and pQCD results agree with each other, since the main difference of these two approach is the (penguin) annihilation contribution, which hardly affects S_f; (iii) The SCET results involve some non-perturbative contributions fitted from data. These contributions affect ΔS_f. In some modes results different from other short distance calculations are obtained.

It is instructive to understand the size and sign of $\Delta\mathcal{S}_f$ in the QCDF approach [9], for example. Recall that ΔS_f is proportional to the real part of A^u_f/A^c_f, which we shall pay attention to. We follow [9] to denote a complex number x by $[x]$ if $\mathrm{Re}(x) > 0$. In QCDF the dominant contributions to A^u_f/A^c_f are basically given by [9, 25]

$$\begin{aligned}
\frac{A^u_{\phi K_S}}{A^c_{\phi K_S}} &\sim \frac{[-(a^u_4 + r_\chi a^u_6)]}{[-(a^c_4 + r_\chi a^c_6)]} \sim \frac{[-P^u]}{[-P^c]},\\
\frac{A^u_{\omega K_S}}{A^c_{\omega K_S}} &\sim \frac{+[a^u_4 - r_\chi a^u_6] + [a^u_2 R]}{+[a^c_4 - r_\chi a^c_6]} \sim \frac{+[P^u] + [C]}{+[P^c]},\\
\frac{A^u_{\rho K_S}}{A^c_{\rho K_S}} &\sim \frac{-[a^u_4 - r_\chi a^u_6] + [a^u_2 R]}{-[a^c_4 - r_\chi a^c_6]} \sim \frac{-[P^u] + [C]}{-[P^c]},\\
\frac{A^u_{\pi^0 K_S}}{A^c_{\pi^0 K_S}} &\sim \frac{[-(a^u_4 + r_\chi a^u_6)] + [a^u_2 R']}{[-(a^c_4 + r_\chi a^c_6)]} \sim \frac{[-P^u] + [C]}{[-P^c]},\\
\frac{A^u_{\eta' K_S}}{A^c_{\eta' K_S}} &\sim \frac{-[-(a^u_4 + r_\chi a^u_6)] + [a^u_2 R'']}{-[-(a^c_4 + r_\chi a^c_6)]} \sim \frac{[-P^u] - [C]}{[-P^c]},
\end{aligned} \tag{13}$$

where a^p_i are effective Wilson coefficients [33], $r_\chi = O(1)$ are the chiral factors and $R^{(\prime,\prime\prime)}$ are (real and positive) ratios of form factors and decay constants.

From Eq.(8), it is clear that $\Delta\mathcal{S}_f > 0$ for ϕK_S, ωK_S, $\pi^0 K_S$, since their $\mathrm{Re}(A^u_f/A^c_f)$ can only be positive. Furthermore, due to the cancellation between a_4 and $r_\chi a_6$ in the ωK_S amplitude, the corresponding penguin contribution is suppressed. This leads to a large and positive $\Delta\mathcal{S}_{\omega K_S}$ as shown in Table I. For the cases of $\rho^0 K_S$ and $\eta' K_S$, there are chances for $\Delta\mathcal{S}_f$ to be positive or negative. The different signs in front of $[P]$ in $\rho^0 K_S$ and ωK_S are originated from the second term of the wave functions $(u\bar{u} \pm d\bar{d})/\sqrt{2}$ of ω and ρ^0 in the $\overline{B}^0 \to \omega$ and $\overline{B}^0 \to \rho^0$ transitions, respectively. The $[P]$ in $\rho^0 K_S$ is also suppressed as the one in ωK_S, resulting a negative $\Delta\mathcal{S}_{\rho^0 K_S}$. On the other hand, $[-P]$ in $\eta' K_S$ is not only unsuppressed (no cancellation in the a_4 and a_6 terms), but, in fact, is further enhanced due to the constructive interference of various penguin amplitudes [26]. This enhancement is responsible for the large $\eta' K_S$ rate [26] and also for the small $\Delta\mathcal{S}_{\eta' K_S}$ [9, 14].

B. $\Delta\mathcal{S}_f$ in KKK modes

$\overline{B}^0 \to K^+K^-K_S$ and $\overline{B}^0 \to K_SK_SK_S$ are penguin-dominated and pure penguin decays, respectively. They are also used to extracted $\sin 2\beta_{\mathrm{eff}}$ with results shown in Fig. 1 and 2.

Three-body modes are in general more complicated than two-body modes. A factorization approach is used to study these KKK modes [15]. For a review on charmless three body modes, one is referred to [31]. Results of CP asymmetries for these modes are summarized in Table II.

To study $\Delta\mathcal{S}_f$ and $\mathcal{A}_f$, it is crucial to know the size of the $b \to u$ transition term (A^u_f). For the pure-penguin $K_SK_SK_S$ mode, the smallness of $\Delta\mathcal{S}_{K_SK_SK_S}$ and $\mathcal{A}_{K_SK_SK_S}$ can be easily understood. For the

TABLE II: Mixing-induced and direct CP asymmetries for various charmless 3-body B decays [15, 31]. Experimental results are taken from [1].

Modes	$\mathcal{S}_f$	$\Delta\mathcal{S}_f$	Expt	$\mathcal{A}_f(\%)$	Expt
$K^+K^-K_S$	$0.728^{+0.001+0.002+0.009}_{-0.002-0.001-0.020}$	$0.041^{+0.028}_{-0.033}$	0.05 ± 0.11	$-4.63^{+1.35+0.53+0.40}_{-1.01-0.54-0.34}$	-7 ± 8
$K_SK_SK_S$	$0.719^{+0.000+0.000+0.008}_{-0.000-0.000-0.019}$	$0.039^{+0.027}_{-0.032}$	-0.10 ± 0.20	$0.69^{+0.01+0.01+0.05}_{-0.01-0.03-0.07}$	14 ± 15
$K_S\pi^0\pi^0$	$0.729^{+0.000+0.001+0.009}_{-0.000-0.001-0.020}$	$0.049^{+0.027}_{-0.032}$	-1.20 ± 0.41	$0.28^{+0.09+0.07+0.02}_{-0.06-0.06-0.02}$	-18 ± 22
$K_S\pi^+\pi^-$	$0.718^{+0.001+0.017+0.008}_{-0.001-0.007-0.018}$	$0.038^{+0.031}_{-0.032}$		$4.94^{+0.03+0.03+0.32}_{-0.02-0.05-0.40}$	

$K^+K^-K_S$ mode, there is a $b \to u$ transition in the $\langle \overline{B}^0 \to K^+K_S\rangle \otimes \langle 0 \to K^-\rangle$ term. It has the potential of giving large tree pollution to $\Delta\mathcal{S}_{K^+K^-K_S}$.

It is useful to note that the $K^+K^-K_S$ final state in the $b \to u$ transition is not CP self-conjugated. This can be easily seen by noting that the K^- meson from the $\langle \overline{B}^0 \to K^+K_S\rangle \times \langle 0 \to K^-\rangle$ term is produced from the virtual W^- meson. Therefore, the CP conjugated term, $\langle \overline{B}^0 \to K^-K_S\rangle \times \langle 0 \to K^+\rangle$ is missing in the weak decay amplitude. Hence, the $b \to u$ transition term should contribute to both CP-even and CP-odd configurations with similar strength. Consequently, information in the CP-odd part can be used to constrain its size and impact on $\Delta\mathcal{S}_f$ and $\mathcal{A}_f$. Indeed, it is found that the CP-odd part is highly dominated by ϕK_S, where other contributions (at $m_{K^+K^-} \neq m_\phi$) are highly suppressed [1]. Since the $\langle \overline{B}^0 \to K^+K_S\rangle \times \langle 0 \to K^-\rangle$ term favors a large $m_{K^+K^-}$ region, which is clearly separated from the ϕ-resonance region, the result of the CP-odd configuration strongly constrains the contribution from this $b \to u$ transition term. Therefore, the tree pollution is constrained and the $\Delta\mathcal{S}_{K^+K^-K_S}$ should not be large.

IV. FSI CONTRIBUTIONS TO $\Delta\mathcal{S}_f$

It was realized recently that long distance FSI may play indispensable role in B decays [27]. The possibility of final-state interactions in bringing in possible tree pollution sources to $\mathcal{S}_f$ are considered in [14]. Both A^u_f and A^c_f will receive long-distance tree and penguin contributions from rescattering of some intermediate states. In particular, there may be some dynamical enhancement on light u-quark loop. If tree contributions to A^u_f are sizable, then final-state rescattering will have the potential of pushing S_f away from the naive expectation. Take the penguin-dominated decay $\overline{B}^0 \to \omega\overline{K}^0$ as an illustration. It can proceed through the weak decay $\overline{B}^0 \to K^{*-}\pi^+$ followed by the rescattering $K^{*-}\pi^+ \to \omega\overline{K}^0$. The tree contribution to $\overline{B}^0 \to K^{*-}\pi^+$, which is color allowed, turns out to be comparable to the penguin one because of the absence of the chiral enhancement characterized by the a_6 penguin term. Consequently, even within the framework of the SM, final-state rescattering may provide a mechanism of tree pollution to $\mathcal{S}_f$. By the same token, we note that although $\overline{B}^0 \to \phi\overline{K}^0$ is a pure penguin process at short distances, it does receive tree contributions via long-distance rescattering. Note that in addition to these charmless final states contributions, there are also contributions from charmful $D_s^{(*)}D^{(*)}$ final states, see Fig. 3. These final-state rescatterings provide the long-distance u- and c-penguin contributions.

An updated version [28] of results in [14] are shown in Table III. Several comments are in order. (i) ϕK_S and $\eta' K_S$ are the theoretical and experimental cleanest modes for measuring $\sin 2\beta_{\rm eff}$ in these penguin modes. The constructive interference behavior of penguins in the $\eta' K_S$ mode is still hold in the LD case, resulting a tiny $\Delta\mathcal{S}_{\eta' K_S}$. (ii) Tree pollutions in ωK_S and $\rho^0 K_S$ are diluted due to the LD c-penguin contributions. (iii) In general, in this approach, the main contributions to decay amplitudes are charming-penguin like and do not sizably affect $\mathcal{S}_f$.

Recent measurements on $K\pi$ direct CP violations show a more than 5 σ deviation (known as the $K\pi$ puzzle) between $\mathcal{A}(B^- \to K^-\pi^0)$ and $\mathcal{A}(\overline{B}^0 \to K^-\pi^+)$ [1] . The data indicates the needs of other

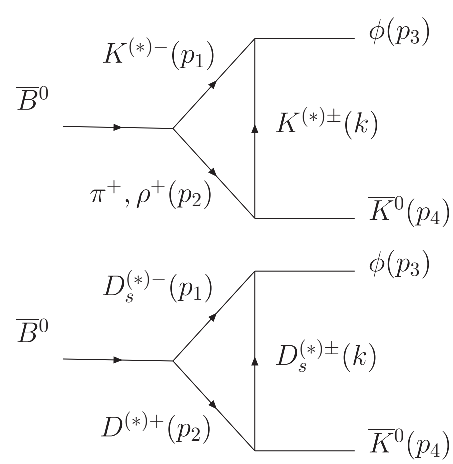

FIG. 3: Final-state rescattering contributions to the $\overline{B}^0 \to \phi\overline{K}^0$ decay.

TABLE III: Direct CP asymmetry parameter $\mathcal{A}_f$ and the mixing-induced CP parameter $\Delta\mathcal{S}_f^{SD+LD}$ for various modes. The first and second theoretical errors correspond to the SD and LD ones, respectively [14].

Final State	$\Delta\mathcal{S}_f$			$\mathcal{A}_f(\%)$		
	SD	SD+LD	Expt	SD	SD+LD	Expt
ϕK_S	$0.02^{+0.01}_{-0.02}$	$0.04^{+0.01+0.01}_{-0.02-0.02}$	-0.29 ± 0.17	$0.8^{+0.5}_{-0.2}$	$-2.3^{+0.9+2.2}_{-1.0-5.1}$	1 ± 12
ωK_S	$0.12^{+0.06}_{-0.05}$	$0.02^{+0.03+0.03}_{-0.04-0.02}$	-0.20 ± 0.24	$-6.8^{+2.4}_{-4.0}$	$-13.5^{+3.5+2.4}_{-5.7-1.5}$	20 ± 19
$\rho^0 K_S$	$-0.08^{+0.03}_{-0.10}$	$-0.04^{+0.07+0.10}_{-0.10-0.12}$	$-0.07^{+0.25}_{-0.27}$	$7.8^{+4.5}_{-2.0}$	$48.9^{+15.8+5.8}_{-13.7-12.5}$	-2 ± 29
$\eta' K_S$	$0.01^{+0.01}_{-0.02}$	$0.00^{+0.01+0.00}_{-0.02-0.00}$	-0.07 ± 0.08	$1.7^{+0.4}_{-0.3}$	$2.1^{+0.2+0.1}_{-0.5-0.4}$	9 ± 6
ηK_S	$0.07^{+0.03}_{-0.03}$	$0.07^{+0.03+0.00}_{-0.03-0.01}$	–	$-5.7^{+2.0}_{-5.5}$	$-3.9^{+1.8+2.5}_{-5.0-1.6}$	–
$\pi^0 K_S$	$0.06^{+0.03}_{-0.03}$	$0.04^{+0.01+0.02}_{-0.02-0.02}$	-0.30 ± 0.19	$-3.2^{+1.1}_{-2.3}$	$3.7^{+1.9+1.7}_{-1.6-1.7}$	-14 ± 11

sub-leading contributions, such as long distance FSI and charming penguins and so on (see, for example [29, 30]). It is found that in cases where the $K\pi$ direct CP data are reproduced, these sub-leading contributions do not sizably affect the magnitudes of $\Delta\mathcal{S}_f$ [29], but some of the signs are different from the short-distance expectations [30].

V. CONCLUSIONS

Various theoretical approaches and results on $\Delta\mathcal{S}_f$ are briefly reviewed. Considerable progress has been made. From these results we see that the prediction on signs of $\Delta\mathcal{S}_f$ are more or less fluctuating and may be subjected to change when more hadronic contributions are taken into account, on the contrary, the predictions on the sizes of $\Delta\mathcal{S}_f$ should be more robust. Since the predictions on sizes of $\Delta\mathcal{S}_f$, which are not sizable in most cases, have better agreement among various approaches. At the same time for modes with small $\Delta\mathcal{S}_f$ ($\leq 5\%$), we do not expect sizable direct CP violations. Measurements on direct CP violations, some $\Delta S = 0$ rates and three-body rates and spectra can provide useful information that can be used to improve our theoretical predictions on $\Delta\mathcal{S}_f$. To further improve the theoretical accuracy more works are needed to effectively reduce the hadronic uncertainties.

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