

Theoretical Review on CP Violation in Rare B decays

Cheng-Wei Chiang

Department of Physics and Center for Mathematics and Theoretical Physics, National Central University, Chungli, Taiwan 320, R.O.C. and
Institute of Physics, Academia Sinica, Taipei, Taiwan 115, R.O.C.

We discuss several issues related to direct CP violation in rare B meson decays. We review the use of CP asymmetries in extracting information of strong and weak phases, how the experimental data fit into the overall picture, and the current status of the $K\pi$ puzzle. We also examine the flavor symmetry assumption using closely related decay modes and extract the weak phase γ from certain $B \rightarrow K^*\pi$ and ρK decays.

1. Importance of CP violation

As pointed out by Sakharov [1] in the 60's, one of the necessary conditions for the observed Universe is CP violation in physical processes. In the standard model (SM) of particle physics, the only source of CP violation is given by the so-called Kobayashi-Maskawa mechanism [2] in the quark sector. In weak transitions, the up-type quarks and the down-type quarks are coupled through the 3×3 Cabibbo-Kobayashi-Maskawa (CKM) matrix [2, 3], which contains a CP-violating phase. Therefore, studying and understanding the origin of such a phase in the SM is crucial to particle physics and cosmology [4]. More importantly, it is possible to shed some light on new physics in such studies.

Due to its hierarchical structure, the CKM matrix has one useful unitarity condition, which connects its first and third column:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \quad (1)$$

This relation has a special status because it renders on a complex plane a triangle that has all sides about the same size (and so are the angles). An important program of current B -factories is to use various processes to overconstrain this unitarity triangle (UT). Through such an exercise, we hope not only to measure precisely the sides and angles of the UT but also to obtain hints of physics beyond the SM that provides additional CP-violating sources.

The indirect CP violation in the B system has been first established in the charmonium modes in 2001, and is now measured at a precision better than 5%. Soon after that first measurement, the direct CP violation in the B system has also been observed in the $B^0 \rightarrow K^+\pi^-$ decay mode in 2004. This is a result of the interference between color-allowed tree and QCD penguin amplitudes. In the following, we concentrate exclusively on the direct CP asymmetries in rare B decays.

2. Direct CP asymmetries in rare B decays

Among all processes, charmless two-body hadronic B decays are often sensitive to the V_{td} and V_{ub} matrix elements that involve CP-violating phases through B - \bar{B} mixing and/or decay amplitudes. With increasing precision on their branching ratios and CP asymmetries, these rare decay modes provide additional useful constraints on the UT.

Consider the decay of a B meson into some final state f and its CP-conjugated one. Assuming the process involves two amplitudes, one has

$$\begin{aligned} A(B \rightarrow f) &= A_1 + A_2 e^{i(\phi+\delta)}, \\ A(\bar{B} \rightarrow \bar{f}) &= A_1 + A_2 e^{i(-\phi+\delta)}, \end{aligned} \quad (2)$$

and

$$\begin{aligned} \mathcal{A}_{CP} &= \frac{\Gamma(\bar{B} \rightarrow \bar{f}) - \Gamma(B \rightarrow f)}{\Gamma(\bar{B} \rightarrow \bar{f}) + \Gamma(B \rightarrow f)} \\ &= \frac{2A_1 A_2 \sin \phi \sin \delta}{A_1^2 + A_2^2 + 2A_1 A_2 \cos \phi \cos \delta}. \end{aligned} \quad (3)$$

In Eqs. (2) and (3), ϕ and δ denote respectively the relative strong and weak phases between the two amplitudes. Therefore, observing a sizable CP asymmetry in the decays requires the interference of at least two amplitudes with large relative strong and weak phases.

Currently, information of direct CP asymmetries in hadronic B decays is collected by the BaBar, Belle, CLEO, CDF and DØ Collaborations. In Table I, we list the asymmetries that deviate from zero at more than 3σ level.

The program of studying CP-violating phases is partly impeded by the lack of full dynamical understanding in hadronic physics, including both strong phases and hadronic matrix elements. A lot of progress in perturbative approaches [5] has been made in recent years. However, it is still a challenging problem to obtain from first principles sufficiently large strong phases, as required by some of the observed large CP asymmetries.

Table I Direct CP asymmetries of rare B decays measured at the 3σ level or more, all quoted from Ref. [6].

DCP	Value	Level
$\mathcal{A}_{CP}(K^+\pi^-)$	-0.097 ± 0.012	8.1σ
$\mathcal{A}_{CP}(\pi^+\pi^-)$	0.38 ± 0.07	5.4σ
$\mathcal{A}_{CP}(K^{*0}\eta)$	0.19 ± 0.05	3.8σ
$\mathcal{A}_{CP}(\rho^0 K^+)$	0.37 ± 0.11	3.4σ
$\mathcal{A}_{CP}(\rho^\pm \pi^\mp)$	-0.13 ± 0.04	3.3σ
$\mathcal{A}_{CP}(\eta K^+)$	-0.27 ± 0.09	3σ

An alternative approach employs the flavor SU(3) symmetry to help relating parameters in amplitudes with the same flavor topology. Strangeness-conserving ($\Delta S = 0$) amplitudes mediating $b \rightarrow q\bar{q}d$ transitions and strangeness-changing ($|\Delta S| = 1$) amplitudes mediating $b \rightarrow q\bar{q}s$ transitions are thus related to each other, where q denotes any of the light quarks. In the symmetry limit, the magnitudes of the two types of amplitudes with the same flavor topology differ only in their CKM factors, and the associated strong phases are taken to be the same.

A few questions naturally arise: (1) Do the CP asymmetries along with the rates of the rare B decays provide a coherent picture? (2) Is it consistent with what we have learned from other processes (*e.g.*, charmed B decays, CP violation in kaon decays, etc)? (3) Is flavor symmetry breaking effects serious for such analyses?

3. Global fits to rare B decay observables

In recent years, several analyses [7, 8, 9, 10] have been performed to obtain a global fit to the measured observables in the $B^{+,0}$ decays to either two pseudoscalar mesons (PP) or one vector meson plus one pseudoscalar meson (VP). One could potentially extend the framework to decay modes with two vector mesons in the final state. However, it involves more amplitudes of different polarizations in each flavor topology, and is beyond the current ability to analyze due to limited available data.

In practice, an SU(3)-breaking factor is often associated to the color-allowed tree amplitude (denoted by T) and color-suppressed tree amplitude (denoted by C) between $\Delta S = 0$ and $|\Delta S| = 1$ transitions, but not for the QCD penguin or electroweak penguin amplitudes (denoted by P and P_{EW} , respectively). This is suggested by the factorizability of T and C amplitudes. In this case, the SU(3)-breaking factor is generally taken to be the appropriate ratio of decay constants. For the PP modes and the VP modes with the spectator quark in B meson going into the vector meson, it is $f_K/f_\pi \simeq 1.22$ [11]. For the VP

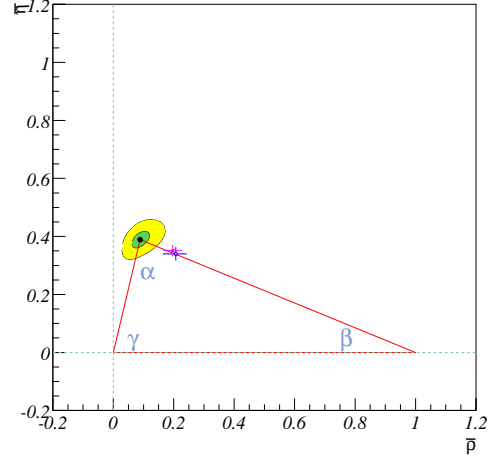


Figure 1: Constraints on the $(\bar{\rho}, \bar{\eta})$ vertex using all the PP mode data [9]. This is consistent with other methods [12, 13] (given by the crosses) within errors.

modes with the spectator quark going into the pseudoscalar meson, it is $f_{K^*}/f_\rho \simeq 1.00$ [11]. Ref. [9] has found that such a breaking scheme leads to better fits than exact symmetry for the PP modes and no additional SU(3)-breaking factor is preferred. However, for the VP modes, it is found that the exact symmetry scheme renders better fits [10]. As a by-product of the fitting, the UT vertex $(\bar{\rho}, \bar{\eta})$ can be constrained, as shown in Fig. 1 using the PP modes for example. It is noted that current analyses have not considered SU(3) breaking in the strong phases.

One salient conclusion from the analyses is large relative size (~ 0.6) an strong phase ($\sim -56^\circ$) between the C and T amplitudes for the PP decays. This result is largely driven by the large branching ratio of the $B^0 \rightarrow \pi^0\pi^0$ mode and the fact that $\mathcal{A}_{CP}(B^+ \rightarrow \pi^0 K^+)$ and $\mathcal{A}_{CP}(B^0 \rightarrow \pi^+ K^-)$ differ too much. Interestingly, the C_V and T_V amplitudes also has a ratio about 0.6 and a large relative strong phase in the VP decays. In comparison, the ratio $|C_P/T_P|$ is only about 0.2 - 0.3. Here the subscript V (P) indicates that the spectator quark in the B meson ends up in the vector (pseudoscalar) meson in the final state.

It is worth pointing out some of the CP asymmetries predicted based upon the best fit. The direct CP asymmetry $\mathcal{A}_{CP}(B^0 \rightarrow \pi^0\pi^0)$ is expected to be at the order of 0.5 or more, a result of comparable QCD penguin and color-suppressed tree amplitudes and a non-trivial strong phase between them. The $B_s \rightarrow \pi^+ K^-$ and $K^+ K^-$ modes involve the same flavor amplitudes as the $B^0 \rightarrow \pi^+ \pi^-$ and $\pi^+ K^-$ decays, respectively. Therefore, they are expected to have sizable CP asymmetries due to the interference between color-allowed tree and QCD penguin amplitudes. The CP asymmetry of $B_s \rightarrow \pi^+ K^-$ is predicted to be

about 0.3, which agrees well with the latest measurement of 0.39 ± 0.17 by the CDF Collaboration [14]. Moreover, the $B_s \rightarrow \pi^0 K_s$ decay involves the same flavor diagrams as the $B^0 \rightarrow \pi^0 \pi^0$ mode. Therefore, its CP asymmetries should be roughly the same as their counterparts in $B^0 \rightarrow \pi^0 \pi^0$.

To account for the branching ratios of B decays involving η or η' in the final state (particularly the $\eta' K$ and ηK^* decays), one possible solution is to have a large singlet penguin amplitude [15, 16, 17, 18]. Moreover, it has a trivial strong phase with respect to the QCD penguin amplitudes in order to produce maximal constructive or destructive interference. However, such large singlet amplitudes remain difficult to accommodate in the perturbative framework [19, 20].

4. The $K\pi$ puzzle

Experimental data of the following $B \rightarrow K\pi$ modes have aroused a lot of interest in recent years:

$$\begin{aligned} A(B^+ \rightarrow K^0 \pi^+) &= P' , \\ \sqrt{2}A(B^+ \rightarrow K^+ \pi^0) &= -(P' + T' + C' + P'_{EW}) , \\ A(B^0 \rightarrow K^+ \pi^-) &= -(P' + T') , \\ \sqrt{2}A(B^0 \rightarrow K^0 \pi^0) &= P' - C' - P'_{EW} , \end{aligned} \quad (4)$$

where the primes in the flavor amplitude decompositions denote $|\Delta S| = 1$ transitions. A perplexing fact is first observed a few years ago by noticing that [26, 27, 28] the ratios of averaged decay widths

$$R_c \equiv \frac{2\bar{\Gamma}(B^+ \rightarrow K^+ \pi^0)}{\bar{\Gamma}(B^+ \rightarrow K^0 \pi^+)} \text{ and } R_n \equiv \frac{\bar{\Gamma}(B^0 \rightarrow \pi^- K^+)}{2\bar{\Gamma}(B^0 \rightarrow \pi^0 K^0)}$$

are quite different. However, they should be about the same if the C and P_{EW} amplitudes are negligible, as one would naïvely expect in the SM. This puzzle is disappearing as the two values currently become 1.12 ± 0.07 and 0.98 ± 0.07 , respectively, and differ only by 1.4σ .

Nevertheless, a more serious new puzzle is emerging, for it occurs in the CP asymmetries. Within the SM, the difference between $A_{CP}(K^+ \pi^0)$ and $A_{CP}(K^+ \pi^-)$ is generally expected to be small, but turns out to be appreciably different (with a difference of 0.147 ± 0.028). There are two possible explanations for this. One is a sizable C' with a large strong phase relative to T' , as found in Refs. [8, 9] through global fits in the flavor SU(3) framework. This is also favored by the large branching ratio of $B^0 \rightarrow \pi^0 \pi^0$. Part of this is also justified in pQCD analysis [29]. The other is a sizable new physics contribution with a new weak phase entering through the electroweak penguin loop [9, 27, 30, 31, 32].

Either of the above-mentioned solutions poses challenges for theorists. The former requires our better

understanding of strong dynamics in the SM. The latter calls for more explorations in justifying the effects of physics beyond the SM. For example, new physics contributions are likely to affect the VP counterparts of the $K\pi$ modes as well.

5. Tests of the flavor symmetry

In addition to performing global fits and checking their quality as mentioned above, one can also examine the flavor symmetry principle by paying attention to some closely related decay modes. For example, a simple test can be done by comparing the magnitudes of QCD penguin amplitudes, *i.e.*, $|P|$ obtained from $B^0 \rightarrow K^0 K^0$ and $B^+ \rightarrow K^+ K^0$ against $|P'|$ from $B^+ \rightarrow K^0 \pi^+$. In this case, one finds that the ratio is consistent with the ratio of CKM factors involved in these modes, $|V_{cd}/V_{cs}|$. This partly justifies our use of $SU(3)_F$ as the working assumption and that f_K/f_π is not applicable to QCD penguin amplitudes.

A more sophisticated comparison can be made for the following set of decay modes:

$$A(B^+ \rightarrow K^0 \pi^+) = P , \quad (5)$$

$$A(B^0 \rightarrow K^+ \pi^-) = T e^{i(\delta_d + \gamma)} + P , \quad (6)$$

$$\xi A(B_s \rightarrow K^- \pi^+) = \frac{1}{\lambda} T e^{i(\delta_s + \gamma)} - \tilde{\lambda} P , \quad (7)$$

which are related by U-spin symmetry. Here $\lambda \equiv |V_{us}/V_{ud}| \simeq 0.2317$, and the SU(3)-breaking factor [11, 21], according to factorization,

$$\xi \equiv \frac{f_K F_{B^0 \pi}(m_K^2) m_{B^0}^2 - m_\pi^2}{f_\pi F_{B_s K}(m_\pi^2) m_{B_s}^2 - m_K^2} = 0.97_{-0.11}^{+0.09} \quad (8)$$

corresponds to almost exact symmetry.

It has been proposed [22, 23] to extract the weak phase γ from the branching ratios and CP asymmetries of these modes, assuming the same relative strong phase $\delta_d = \delta_s$. Given the fact that γ has been constrained using other methods (*e.g.*, DK modes) and that the last mode in Eq. (7) is not measured until recently by the CDF Collaboration (see Table II), one can turn the argument around to test the flavor symmetry assumption. Moreover, its branching ratio still has a large uncertainty. Therefore, its central value may change before it completely settles down.

It is useful to consider the following four quantities:

$$R_d = 1 + r^2 + 2r \cos \gamma \cos \delta_d = 0.899 \pm 0.048 ,$$

$$\xi^2 R_s = \tilde{\lambda}^2 + \left(\frac{r}{\tilde{\lambda}}\right)^2 - 2r \cos \gamma \cos \delta_s = 0.260 \pm 0.059 ,$$

$$\begin{aligned} R_d A_{CP}(B^0 \rightarrow K^+ \pi^-) &= 2r \sin \gamma \sin \delta_d \\ &= 0.087 \pm 0.012 , \end{aligned}$$

$$\begin{aligned} \xi^2 R_s A_{CP}(B_s \rightarrow K^- \pi^+) &= -2r \sin \gamma \sin \delta_s \\ &= -0.101 \pm 0.050 . \end{aligned} \quad (9)$$

Table II Current data of several $B \rightarrow K\pi$ decays. Branching ratios are quoted in units of 10^{-6} .

Observable	Exp. Value	Ref.
$BR(B^+ \rightarrow K^0\pi^+)$	23.1 ± 1.0	[6]
$BR(B^0 \rightarrow K^+\pi^-)$	19.4 ± 0.6	[6]
$A_{CP}(B^0 \rightarrow K^+\pi^-)$	-0.097 ± 0.012	[6]
$BR(B_s \rightarrow K^-\pi^+)$	5.27 ± 1.17	[14]
$A_{CP}(B_s \rightarrow K^-\pi^+)$	0.39 ± 0.17	[14]

Here r denotes the ratio between $|T|$ and $|P|$. The last two in Eqs. (9) imply a simple relation between the strong phases

$$\frac{\sin \delta_d}{\sin \delta_s} = 0.96 \pm 0.54 . \quad (10)$$

Ref. [24] notices that one cannot obtain a solution to Eqs. (9) by assuming $\delta_d = \delta_s$.

When the equality condition on strong phases is relaxed, two sets of solutions are obtained from the four equations in (9) [24], as shown in Fig. 2. One set (upper plot) has very different δ_d and δ_s , suggesting large SU(3) breaking in the strong phases. The other set (lower plot) gives reasonable strong phases and weak phase γ provided $BR(B_s \rightarrow K^-\pi^+)$ is larger than the current value by about 40%. This is possible if either recent evaluations of b quark fragmentation [25] had overestimated the fraction of b quarks ending up as B_s or the SU(3) breaking factor ξ is 20% larger than that given in (8).

6. Extraction of γ from charmless modes

Several methods have been proposed to determine the weak phase γ using the direct CP asymmetries resulted from interference between different amplitudes in $B \rightarrow D^{(*)}K$ decays [33, 34, 35]. However, such early proposals are not completely free from hadronic uncertainties. Recently, a Dalitz plot analysis is used to simultaneously determine γ and other hadronic parameters in the problem [36, 37].

As a complementary means, Refs. [28, 38] suggest to make use of the rates and asymmetries of charmless $B \rightarrow K\pi$ modes. With the input of the color-allowed tree amplitude from $B \rightarrow \pi\ell\nu$ decay, one can obtain a constraint on γ . As data in the charmless VP modes become available, it is possible to use the $K^*\pi$ and ρK final states of the B decays to constrain γ [39, 40].

As shown in Table III, the CP asymmetries of all the listed modes are consistent with zero. In particular, the two neutral B decays imply that the strong phases between the color-allowed tree and QCD penguin amplitudes are trivial. We consider the following

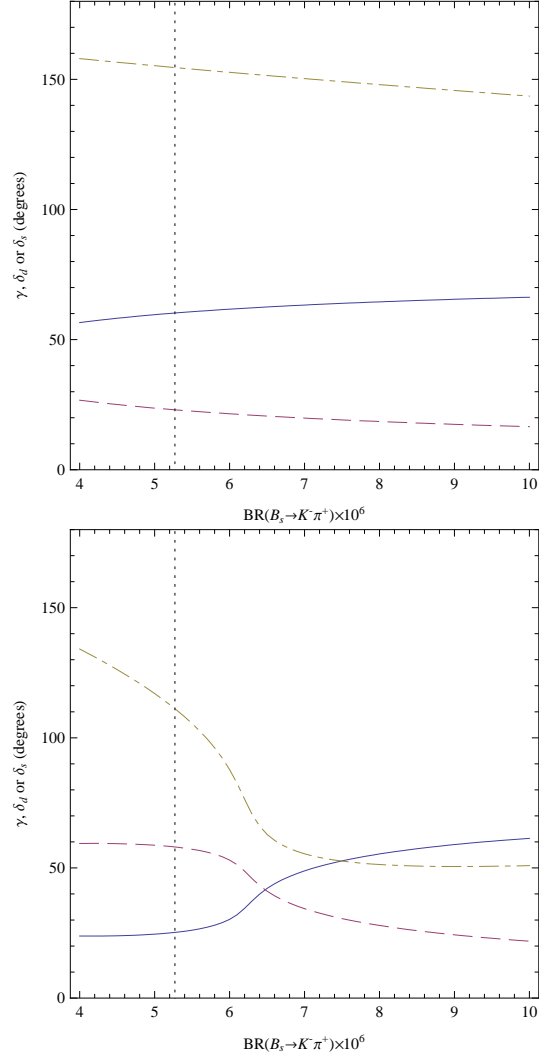


Figure 2: Behavior of solutions as a function of $BR(B_s \rightarrow K^-\pi^+)$ [24]. The solid, dashed, and dash-dotted curves represent γ , δ_d , and δ_s , respectively. The vertical dotted line indicates the current value of $BR(B_s \rightarrow K^-\pi^+)$.

Table III Current rate and asymmetry data of some $B \rightarrow K^*\pi$ and ρK modes.

Mode	Amplitudes	BR ($\times 10^{-6}$)	A_{CP}
$B^+ \rightarrow K^{*0}\pi^+$	P'_P	10.7 ± 0.8	-0.085 ± 0.057
$\rho^+ K^0$	P'_V	8.0 ± 1.5	0.12 ± 0.17
$B^0 \rightarrow K^{*+}\pi^-$	$-(P'_P + T'_P)$	9.8 ± 1.1	-0.05 ± 0.14
$\rho^- K^+$	$-(P'_V + T'_V)$	15.3 ± 3.6	0.22 ± 0.23

four quantities:

$$\begin{aligned}
 R(K^*\pi) &\equiv \frac{\overline{\Gamma}(K^{*+}\pi^-)}{\overline{\Gamma}(K^{*0}\pi^+)} = 1 - 2r_1 \cos \delta_P \cos \gamma + r_1^2 \\
 &= 0.99 \pm 0.13 , \\
 \mathcal{A}_{CP}^{K^{*+}\pi^-} &= -2r_1 \sin \delta_P \sin \gamma / R(K^{*+}\pi^-) \\
 &= -0.05 \pm 0.14 .
 \end{aligned}$$

$$\begin{aligned}
R(\rho^- K^+) &\equiv \frac{\bar{\Gamma}(\rho^- K^+)}{\bar{\Gamma}(\rho^+ K^0)} = 1 + 2r_2 \cos \delta_V \cos \gamma + r_2^2 \\
&= 2.06 \pm 0.61, \\
\mathcal{A}_{CP}^{\rho^- K^+} &= 2r_2 \sin \delta_V \sin \gamma / R(\rho^- K^+) \\
&= 0.22 \pm 0.23,
\end{aligned} \tag{11}$$

where $r_1 \equiv |T'_P/P'_P|$, $r_2 \equiv |T'_V/P'_V|$, and δ_P and δ_V are the corresponding relative strong phases. Instead of treating r_1 and r_2 as independent parameters, one may employ the factorization assumption to obtain

$$\frac{r_2}{r_1} = \frac{f_K A_0^{B\rho}(m_K^2)}{f_{K^*} F_1^{B\pi}(m_{K^*}^2)} \left| \frac{P'_P}{P'_V} \right| = 0.6 - 1.1, \tag{12}$$

where the uncertainty mainly comes from the form factors. In this case, there are now only four parameters for the four observables in Eqs. (11). Solving them gives $\gamma = (65_{-8}^{+10})^\circ$ for $r_2/r_1 = 0.6$ and $(68_{-7}^{+9})^\circ$ for $r_2/r_1 = 1.1$. This result is consistent with other methods [12, 13].

7. Summary

We review the importance of CP asymmetries in B meson decays. The flavor SU(3) symmetry assumption is employed to analyze charmless two-body modes in a global way. The direct CP asymmetries provide useful information on both weak and strong phases. Currently, there are six direct CP asymmetry observables deviating from zero at 3σ level or more. Sizable relative sizes and strong phases are observed between the color-allowed and color-suppressed tree amplitudes from current data. The puzzle in the CP asymmetry pattern of $B \rightarrow K\pi$ decays can be explained by such a color-suppressed amplitude or some new physics effects with a new weak phase in the electroweak penguin amplitude. The large color-suppressed amplitude explanation requires better understanding of the strong dynamics. The same comment applies to the singlet penguin amplitudes too.

We examine the flavor symmetry principle by scrutinizing a set of $B_{u,d,s} \rightarrow K\pi$ decays. Current data indicate unexpectedly large SU(3) breaking in the strong phases. This may eventually go away if $BR(B_s \rightarrow K^- \pi^+)$ turns out to be 40% larger or the symmetry breaking in amplitude sizes is 20% larger. A definite conclusion on this issue relies on more precise experimental measurements. In addition to the $B \rightarrow D^{(*)}K$ modes, it is useful to constrain γ using charmless decay modes as well. The rates and CP asymmetries of some $K^* \pi$ and ρK modes can be used to determine γ with only a mild assumption of factorization for the tree amplitudes.

Acknowledgments

The author is grateful to many of his collaborators on various aspects of the topic. This work is supported in part by the National Science Council of Taiwan, ROC under Grant No. NSC 96-2112-M-008-001.

References

- [1] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5**, 32 (1967) [JETP Lett. **5**, 24 (1967 SOPUA,34,392-393.1991 UFNAA,161,61-64.1991)].
- [2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
- [3] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
- [4] For a recent attempt to connect current observations in B physics to cosmology, see W. S. Hou, arXiv:0803.1234 [hep-ph].
- [5] See the papers by M. Beneke, C.-D. Lü, and I. Rothstein in this proceedings.
- [6] Experimental data are quoted from the averages done by the Heavy Flavor Averaging Group: <http://www.slac.stanford.edu/xorg/hfag/rare>.
- [7] C. W. Chiang, M. Gronau, Z. Luo, J. L. Rosner and D. A. Suprun, Phys. Rev. D **69**, 034001 (2004) [arXiv:hep-ph/0307395].
- [8] C. W. Chiang, M. Gronau, J. L. Rosner and D. A. Suprun, Phys. Rev. D **70**, 034020 (2004) [arXiv:hep-ph/0404073].
- [9] C. W. Chiang and Y. F. Zhou, JHEP **0612**, 027 (2006) [arXiv:hep-ph/0609128]; and
- [10] C. W. Chiang and Y. F. Zhou, work in preparation.
- [11] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).
- [12] J. Charles *et al.* [CKMfitter Group], Eur. Phys. J. C **41**, 1 (2005) [arXiv:hep-ph/0406184]. Updated results may be found on the web site: <http://ckmfitter.in2p3.fr/>.
- [13] M. Bona *et al.* [UTfit Collaboration], JHEP **0507**, 028 (2005) [arXiv:hep-ph/0501199]. Updated results may be found on the web site: <http://utfit.roma1.infn.it/>.
- [14] M. Morello [CDF Collaboration], Nucl. Phys. Proc. Suppl. **170**, 39 (2007) [arXiv:hep-ex/0612018]; and CDF Public Note 8579v1, described in http://www-cdf.fnal.gov/physics/new/bottom/060921.blessed-bhh_1388_1fb/.
- [15] A. S. Dighe, M. Gronau and J. L. Rosner, Phys. Lett. B **367**, 357 (1996) [Erratum-ibid. B **377**, 325 (1996)] [arXiv:hep-ph/9509428].
- [16] A. S. Dighe, M. Gronau and J. L. Rosner, Phys. Rev. Lett. **79**, 4333 (1997) [arXiv:hep-ph/9707521].

- [17] M. Gronau and J. L. Rosner, Phys. Rev. D **53**, 2516 (1996) [arXiv:hep-ph/9509325].
- [18] C. W. Chiang, M. Gronau and J. L. Rosner, Phys. Rev. D **68**, 074012 (2003) [arXiv:hep-ph/0306021].
- [19] M. Beneke and M. Neubert, Nucl. Phys. B **651**, 225 (2003) [arXiv:hep-ph/0210085].
- [20] Y. Y. Charng, T. Kurimoto and H. n. Li, Phys. Rev. D **74**, 074024 (2006) [arXiv:hep-ph/0609165].
- [21] A. Khodjamirian, T. Mannel and M. Melcher, Phys. Rev. D **68**, 114007 (2003) [arXiv:hep-ph/0308297].
- [22] M. Gronau and J. L. Rosner, Phys. Lett. B **482**, 71 (2000) [arXiv:hep-ph/0003119].
- [23] C. W. Chiang and L. Wolfenstein, Phys. Lett. B **493**, 73 (2000) [arXiv:hep-ph/0004255].
- [24] C. W. Chiang, M. Gronau and J. L. Rosner, Phys. Lett. B **664**, 169 (2008) [arXiv:0803.3229 [hep-ph]].
- [25] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. D **77**, 072003 (2008) [arXiv:0801.4375 [hep-ex]].
- [26] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Eur. Phys. J. C **32**, 45 (2003) [arXiv:hep-ph/0309012]; Phys. Rev. Lett. **92**, 101804 (2004) [arXiv:hep-ph/0312259]; Nucl. Phys. B **697**, 133 (2004) [arXiv:hep-ph/0402112].
- [27] T. Yoshikawa, Phys. Rev. D **68**, 054023 (2003) [arXiv:hep-ph/0306147].
- [28] M. Gronau and J. L. Rosner, Phys. Lett. B **572**, 43 (2003) [arXiv:hep-ph/0307095].
- [29] H. n. Li, S. Mishima and A. I. Sanda, Phys. Rev. D **72**, 114005 (2005) [arXiv:hep-ph/0508041].
- [30] W. S. Hou, H. n. Li, S. Mishima and M. Nagashima, Phys. Rev. Lett. **98**, 131801 (2007) [arXiv:hep-ph/0611107].
- [31] S. Baek and D. London, Phys. Lett. B **653**, 249 (2007) [arXiv:hep-ph/0701181].
- [32] T. Feldmann, M. Jung and T. Mannel, arXiv:0803.3729 [hep-ph].
- [33] M. Gronau and D. Wyler, Phys. Lett. B **265**, 172 (1991).
- [34] M. Gronau and D. London., Phys. Lett. B **253**, 483 (1991).
- [35] D. Atwood, I. Dunietz and A. Soni, Phys. Rev. D **63**, 036005 (2001) [arXiv:hep-ph/0008090].
- [36] A. Giri, Y. Grossman, A. Soffer and J. Zupan, Phys. Rev. D **68**, 054018 (2003) [arXiv:hep-ph/0303187].
- [37] A. Poluektov *et al.* [Belle Collaboration], Phys. Rev. D **70**, 072003 (2004) [arXiv:hep-ex/0406067].
- [38] M. Neubert and J. L. Rosner, Phys. Lett. B **441**, 403 (1998) [arXiv:hep-ph/9808493].
- [39] W. M. Sun, Phys. Lett. B **573**, 115 (2003) [arXiv:hep-ph/0307212].
- [40] C. W. Chiang, arXiv:hep-ph/0502183.