Theoretical tools -- Perturbative QCD approach

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Outline

- Theoretical framework /comparison
- Direct /mixing induced CP asymmetry
- Next-to-leading order calculations
- *Bs decays
- Summary

Factorization

- **Factorization is essential for hadronic B decays**
- **Is process dependent part calculable?**
- $A(B \to M_1 M_2) \propto f_{M2} F^{B \to M1} a_i$
- **The kinematics (a_i) for different processes are perturbative, factorizable from hadronic inputs**

Different 4-quark operators

Factorization assumption vs factorization theorem

- **Naïve F.A.**
- **of process**
- $A(B \to M_1 M_2) \propto f_{M2} F^{B \to M1}$
- Vacuum$\to M_2$, $B \to M_1$

- **F.T.**
- **of dynamics**
- $\phi_B \otimes x\ \phi_{M1} \otimes x\ \phi_{M2} \otimes x\ H$
- Nonpert perturb

Factorizable: **in the above form**

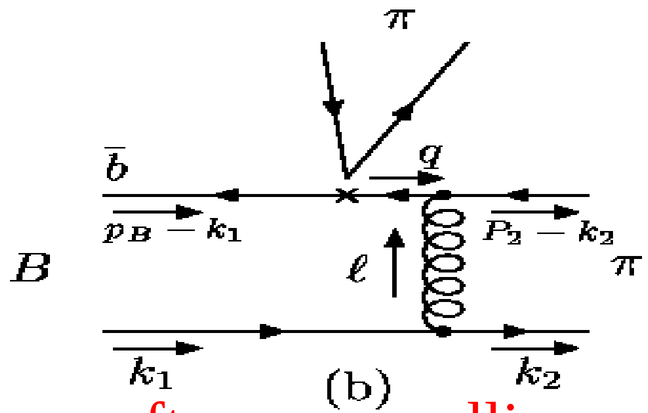
Nonfactorizable: **not in the above form**

Collinear and k_T factorization

- **Collinear Fac**
- **BBNS SCET**
- **k_T factorization**
- **pQCD**

- There are two kinds of expansion series:
- One is α_s , one is power expansion $1/m_B$
- α_s expansion is controllable,
- $1/m_B$ expansion is not controllable , annihilation, charming penguin …

Feynman Diagram Calculation



soft → collinear

$$\int d^4k_1 d^4k_2 \frac{i}{(k_1-k_2)^2} tr\left[\gamma_\alpha \Phi_\pi \gamma^\alpha \Phi_B \gamma^\mu (1-\gamma_5) \frac{i}{\not k_1 - \not p_2}\right]$$

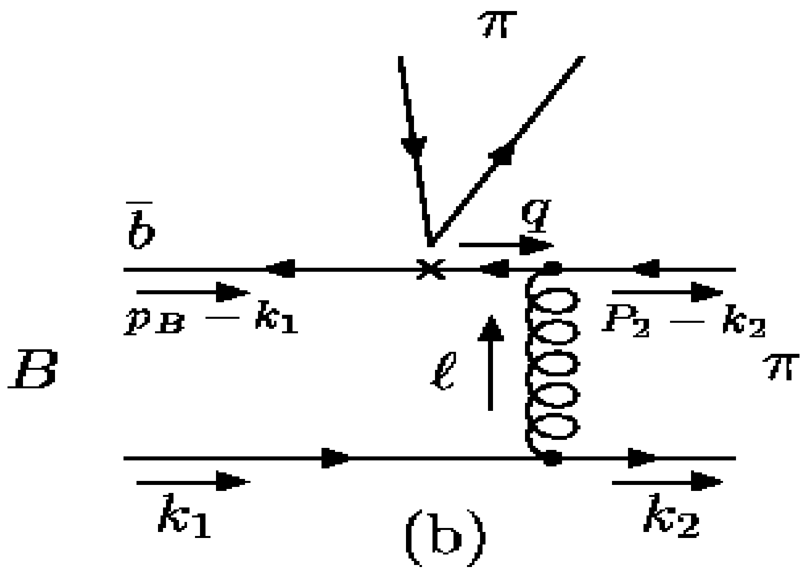
Wave function

$$\frac{i}{(k_1-k_2)^2} = \frac{i}{k_1^{\,2}+k_2^{\,2}-2k_1\cdot k_2} = \frac{i}{-2xym_B^2}$$

$k_2=m_B(y,0,\underline{k_2}^T)$, $k_1=m_B(0,x,\underline{k_1}^T)$

$k_2\cdot k_1= k_2^+k_1^- - k_2^T\cdot k_1^T \approx m_B^2xy$

Endpoint Singularity



The gluon propagator

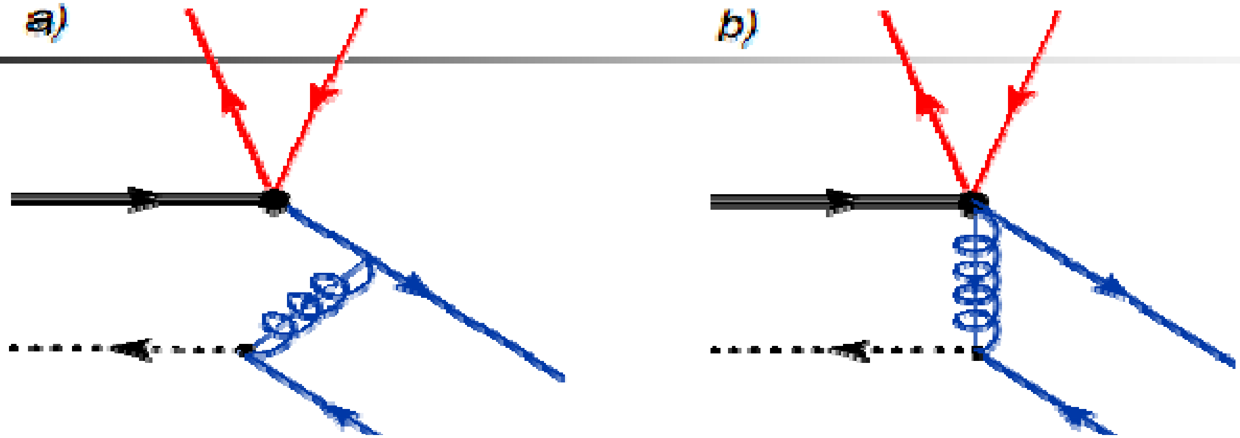
$$\frac{i}{(k_1-k_2)^2} \approx \frac{i}{-2xym_B^2}$$

- **x,y are integral variables from 0→1, singular at endpoint**
- **In fact, transverse momentum at endpoint is not negligible**

$$\frac{i}{(k_1-k_2)^2} = \frac{i}{-2xym_B^2 - (k_1^T - k_2^T)^2}$$

then no singularity

Soft collinear effective theory (SCET)



Charming penguin

$$
\begin{aligned}
\langle M_1 M_2 | O_i | B \rangle &= C_i^I(u) \otimes \Phi_{M_2}(u) \zeta^{B\to M_1}(0) \\
&+ C_i^{II}(u,\tau) \otimes \Phi_{M_2}(u) \otimes \zeta_J^{B\to M_1}(\tau,0) \\
&+ \lambda_c^{(f)} A_{cc}^{M_1 M_2}
\end{aligned}
$$

soft

Perturbative part:

$$
\zeta_J^{B\to M_1}(\tau,0) = J(\tau,\omega,v) \otimes \Phi_B(\omega) \otimes \Phi_{M_1}(v)
$$

$B \to \pi$ form factor

- $F^{B\pi} = \zeta^{B\pi} + \zeta_J^{B\pi}$
- $\zeta^{B\pi}$: nonfactorizable (soft,with singularity)
- $\zeta_J^{B\pi}$: factorizable (hard)
- In SCET, $\zeta^{B\pi}$ *and* $\zeta_J^{B\pi}$ are at the same order
- In QCDF, $\zeta_J^{B\pi}$ is negligible
- In PQCD (k_T factorization), Both $\zeta^{B\pi}$ and $\zeta_J^{B\pi}$ are factorizable, same order in α_s

Penguin over tree

- $B^0 \rightarrow K^+\pi^-$ **and** $B^0 \rightarrow \pi^+ \pi^-$ **are dominated by penguin (P) and tree (T) operators, respectively**
- **In leading power,**
- $|P/T| \sim |f_K/f_\pi| \ * \ |V_{ts}/V_{ub}| \ * \ |a4/a1|$

  $= 158/132 * 41.61/3.96 * 0.045/1.05 = 0.54$

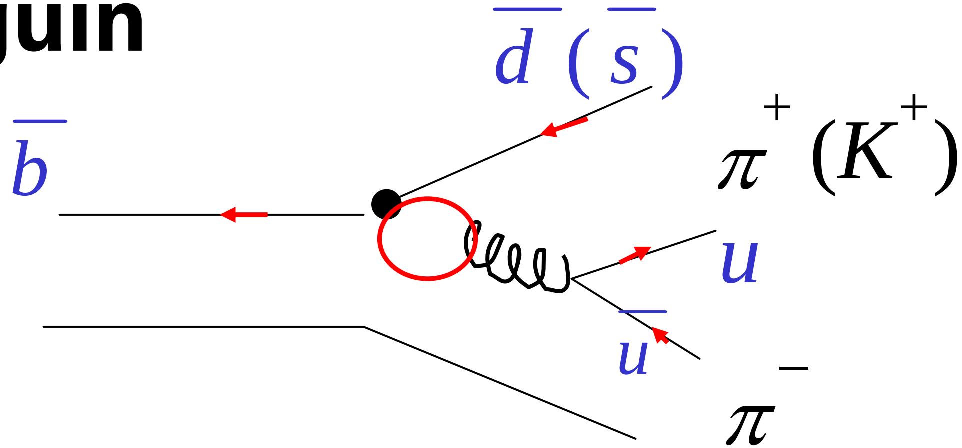
Exp: $B(B^0 \rightarrow K{+}\pi{-})/B(B^0 \rightarrow \pi{+}\pi{-}) = 18.2/4.6 = 4$

Penguin over tree

- **(V-A)(V+A) operator O_6 can be chirally enhanced when doing Fierz transformation in QCDF and pQCD.**
- **a_6 only slightly larger than a_4, QCDF needs very large chiral factor $m_0 = m_K^2/m_s$, small m_s.**
- **pQCD has additional chirally enhanced space like penguin contribution O_6, does not need small m_s**
- **SCET/BPRS without a_6, needs very large charming penguin**

Charming penguins in SCET

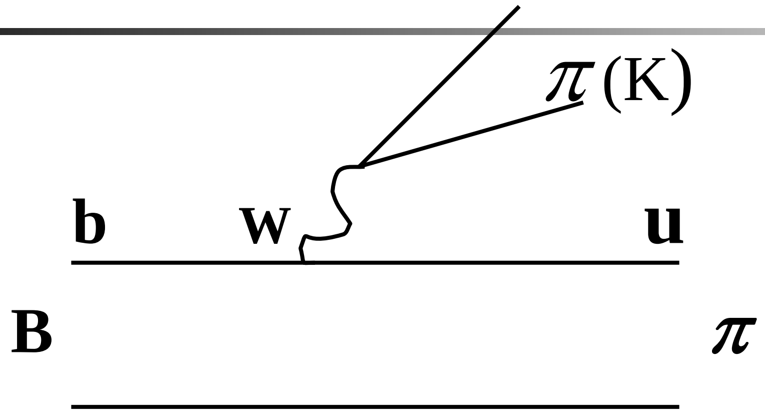
- has the same topology as **chiral enhanced penguin**
- **Charming penguin** appear always together with chiral enhanced penguin



Importance of power corrections

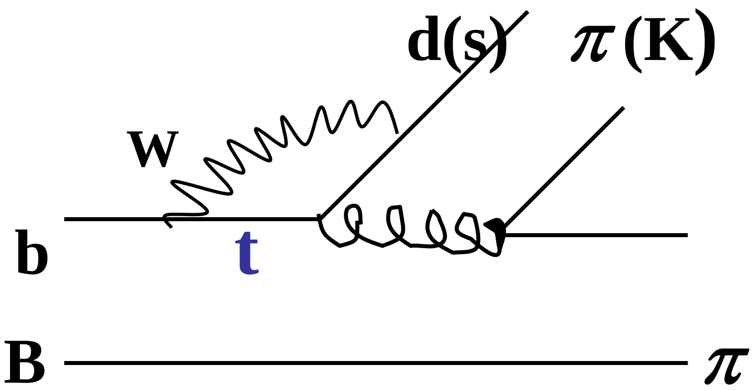
- Most of the branching ratios agree well with experiments – leading power
- Difficult to distinguish between approaches
- but CP / polarization, suppressed channels require strong phase, sensitive to weak phase, power corrections will be different

$B \to \pi\pi$, πK Have Two Kinds of Diagrams with different weak phase



O_1, O_2

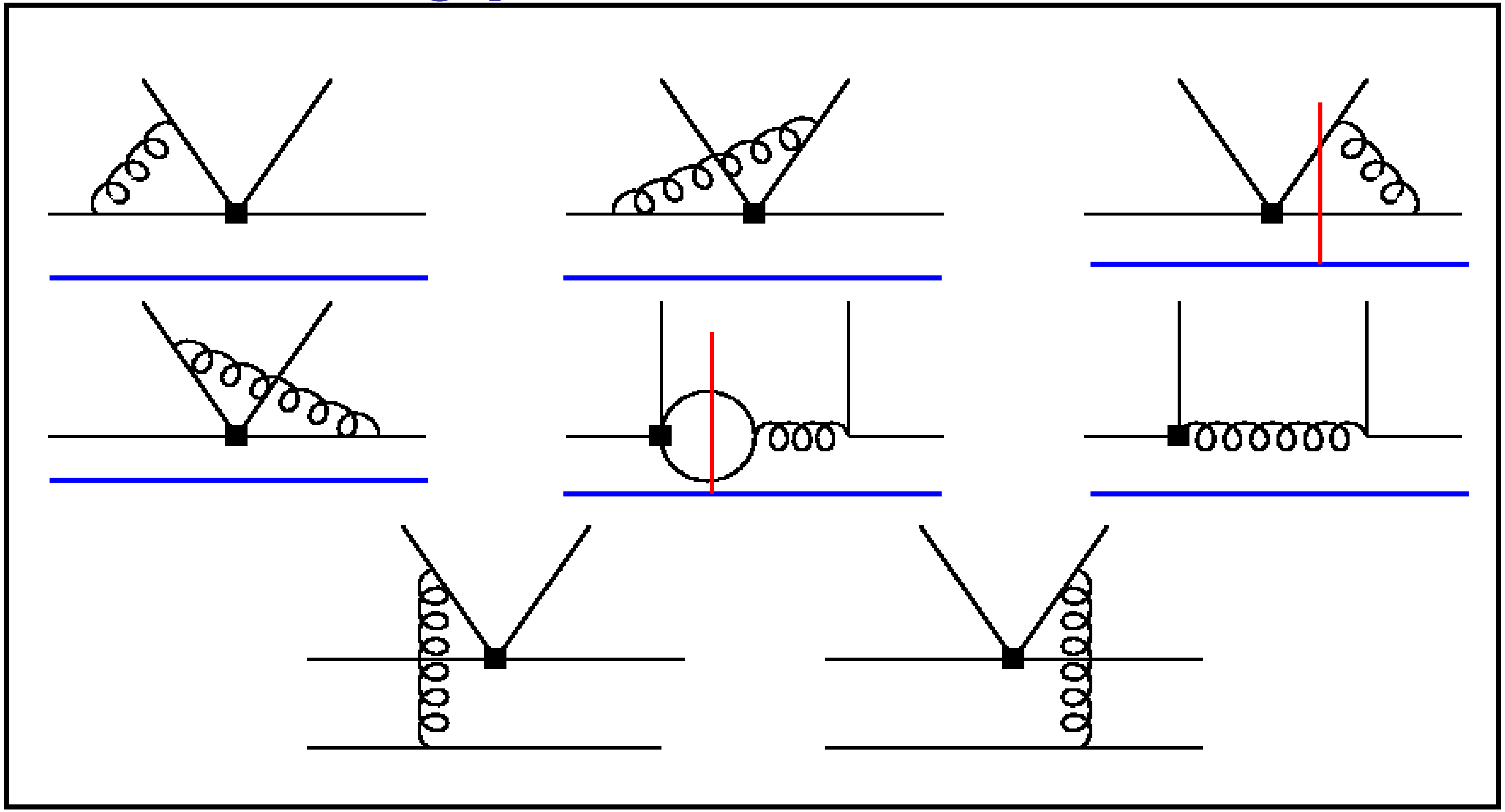
Tree $\propto V_{ub} V_{ud}^{*}{}_{(s)}$



O_3, O_4, O_5, O_6

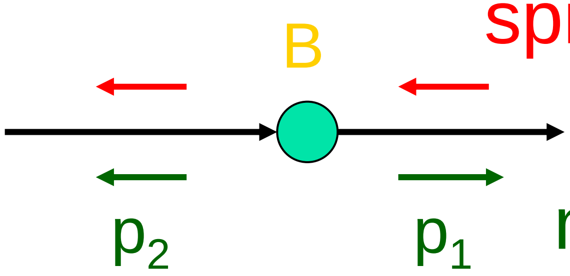
Penguin $\propto V_{tb} V_{td}^{*}{}_{(s)}$

QCD corrections are at α_s order, strong phase too small



annihilation penguin can provide a large strong phase

spin (this configuration is not allowed)



fermion flow

momentum

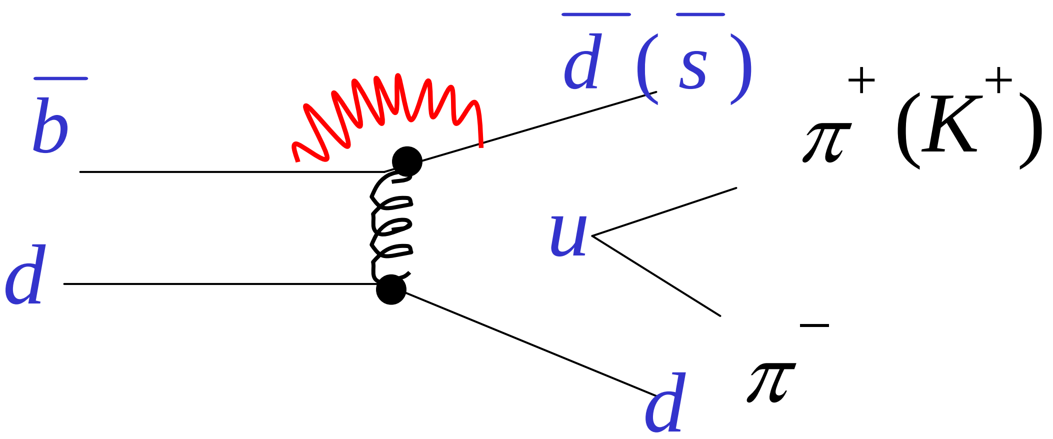
Like $B \rightarrow e\ \nu_e$

pseudo-scalar B requires spins in opposite directions, namely, helicity conservation

Annihilation suppression ~ $1/m_B$ ~ 10%

No suppression for O_6

- Space-like penguin (annihilation)
- Become (s-p)(s+p) operator after Fiertz transformation   Chirally enhanced
- No suppression, contribution "big" (20-30%)



Calculable in pQCD approach

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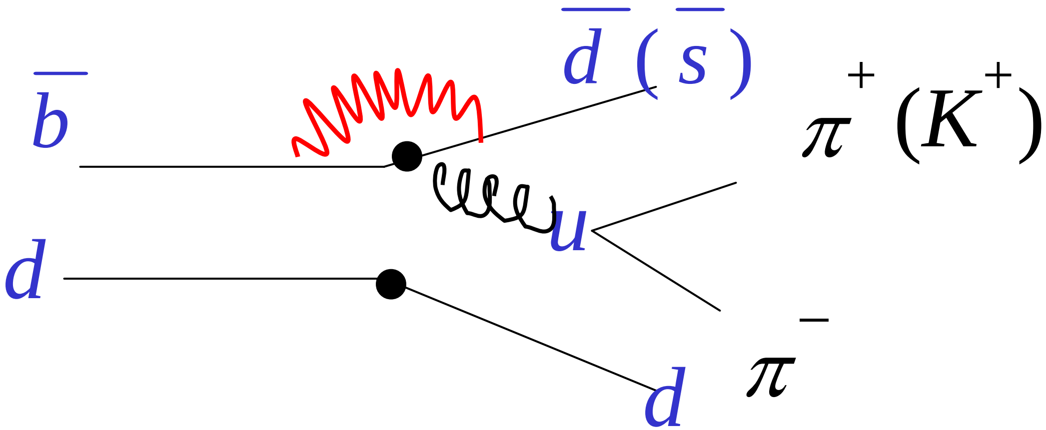
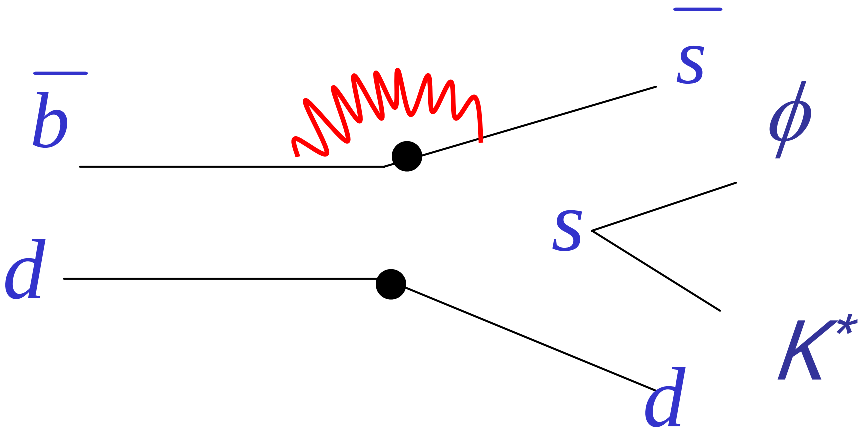


Calculable in pQCD approach

Large transverse component in $B \to \phi K^*$ decays

Annihilation can enhance transverse contribution: R_L = 59% (exp:50%)

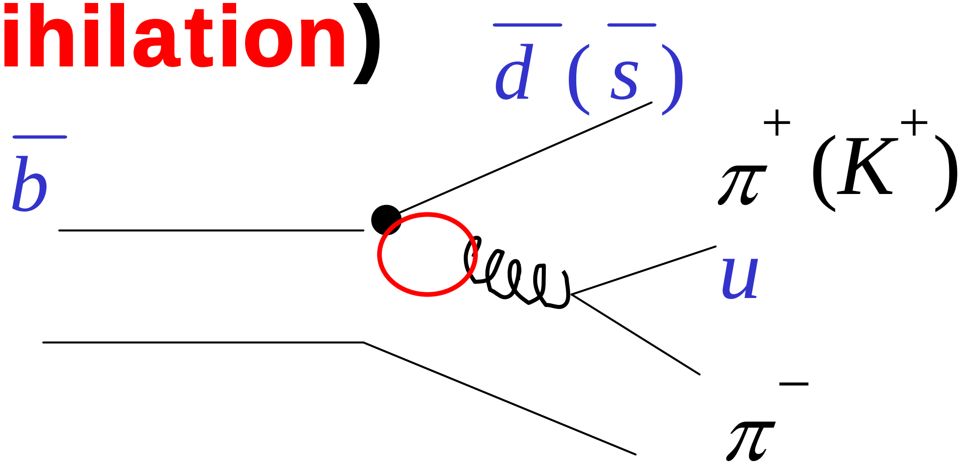
and also right ratio of $R_{=}$, $R_{\perp}$ and right strong phase $\phi_{=}$, $\phi_{\perp}$, charming penguin in SCET



H-n Li, **Phys. Lett. B622, 68, 2005**

Charming penguins in SCET

- **Play the similar role at SCET, but not calculable**
- **Charming penguin appear always together with space like penguin (annihilation)**



SCET

- χ^2 **Fit from experiments requires a large charming penguin, it even become the most important contribution in $B \to K\pi$ decays**
- **It is essential to provide a large strong phase for direct CP asymmetry**

Williamson, Zupan, Phys.Rev.D74:014003,2006,

Wang[2],Yang,Lu, arXiv:0801.3123

Comparison

	charm loops	leading annihilation
BBNS/ QCDF	perturbative	nonperturbative model parameters, large phases
pQCD	perturbative	perturbative, large phases
BPRS/ SCET	nonperturbative fit parameters, large phases	perturbative

B→ Kπ puzzle

- $K^+\pi^-$ and $K^+\pi^0$ differ by subleading amplitudes P_{ew} and C. Their CP are expected to be similar.
- Their data differ by 5σ! A puzzle!

$$A_{CP}(K^+\pi^-) = (-9.7 \pm 1.2)\%$$

$$A_{CP}(K^+\pi^0) = (5.0 \pm 2.5)\%$$

Amplitude parametrization

$$A(B^+ \to K^0\pi^+) = P',$$

$$A(B_d^0 \to K^+\pi^-) = -P'\left(1 + \frac{T'}{P'}e^{i\phi_3}\right),$$

$$\sqrt{2}A(B^+ \to K^+\pi^0) = -P'\left[1 + \frac{P'_{ew}}{P'} + \left(\frac{T'}{P'} + \frac{C'}{P'}\right)e^{i\phi_3}\right],$$

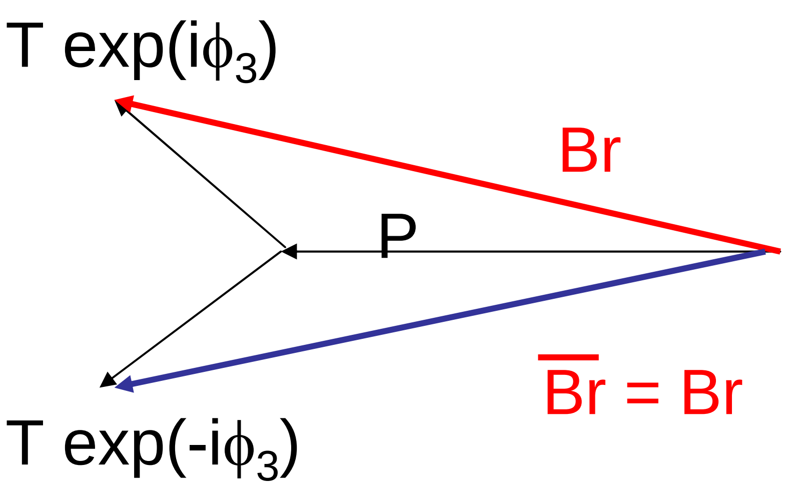
$$\sqrt{2}A(B_d^0 \to K^0\pi^0) = P'\left(1 - \frac{P'_{ew}}{P'} - \frac{C'}{P'}e^{i\phi_3}\right),$$

$$\frac{T'}{P'} \sim \lambda, \quad \frac{P'_{ew}}{P'} \sim \lambda, \quad \frac{C'}{P'} \sim \lambda^2$$

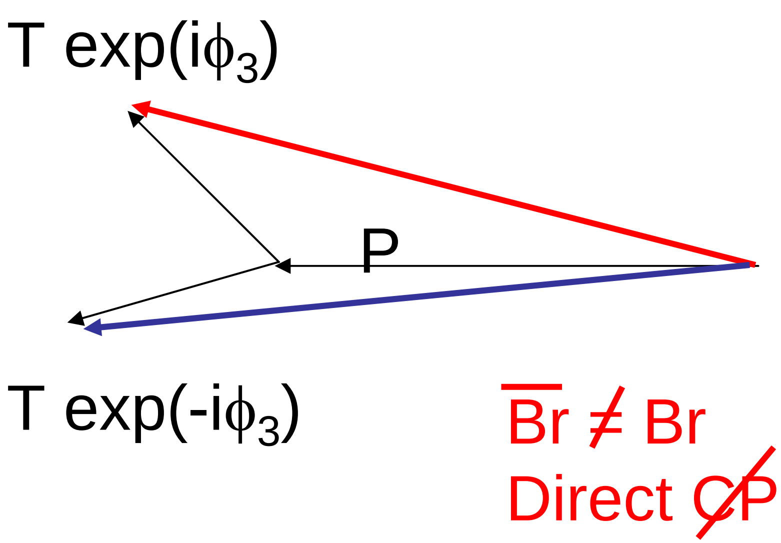
$(C_2/C_4)(V_{us}V_{ub}/V_{ts}V_{tb}) \sim (1/\lambda^2)(\lambda^5/\lambda^2) \sim \lambda$

Direct CP violation

If $\delta_T = 0$



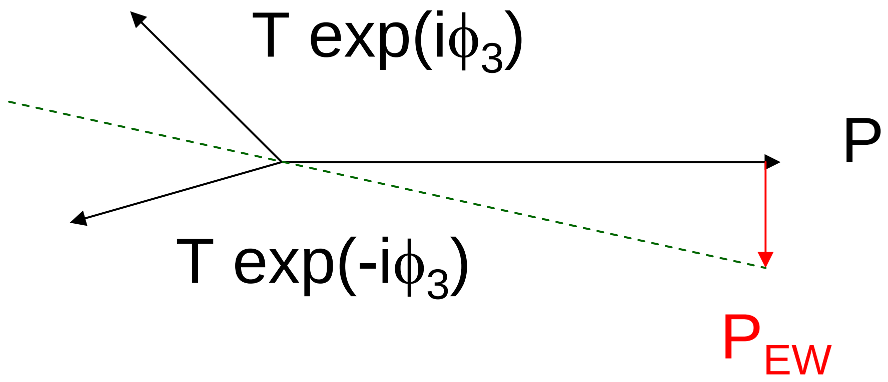
If $\delta_T \neq 0$



Recall $A_{CP} \propto \sin\delta \sin\phi$

Explanation 1

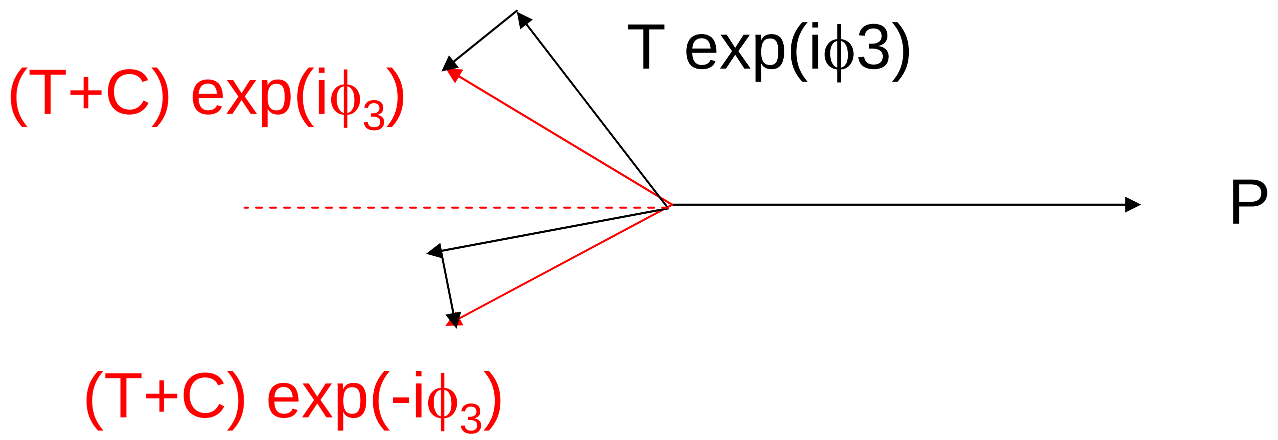
- **Large $K^+\pi^-$ CP implies large δ_T (predicted by PQCD in 2000)**
- **Large P_{EW} to cancel its effect (Buras et al.; Yoshikawa) in $K^+\pi^0$** **new physics?**



$\overline{Br} = Br$

Explanation 2

- **Or large C to cancel its effect (Charng and Li; He and McKellar) in $K^+\pi^0$) mechanism missed in SM calculation?**



$A_{CP}(K^+\pi^0) \neq A_{CP}(K^+\pi^-)$ puzzle?

$A_{CP}(K^+\pi^-) = -0.097 \pm 0.012$ spectator d

difference $= 5\sigma$

$A_{CP}(K^+\pi^0) = 0.046 \pm 0.026$ spectator u

$A(K^+\pi^-) = P + T + \ldots \quad \sqrt{2}A(K^+\pi^0) = P + T + C + \ldots$ (next)

This would be a puzzle if $|C| \ll |T|$ but not if $|C| \sim |T|$

QCD calc. and SU(3) fits (excl. these asym.) find $|C| \sim |T|$

NO PUZZLE

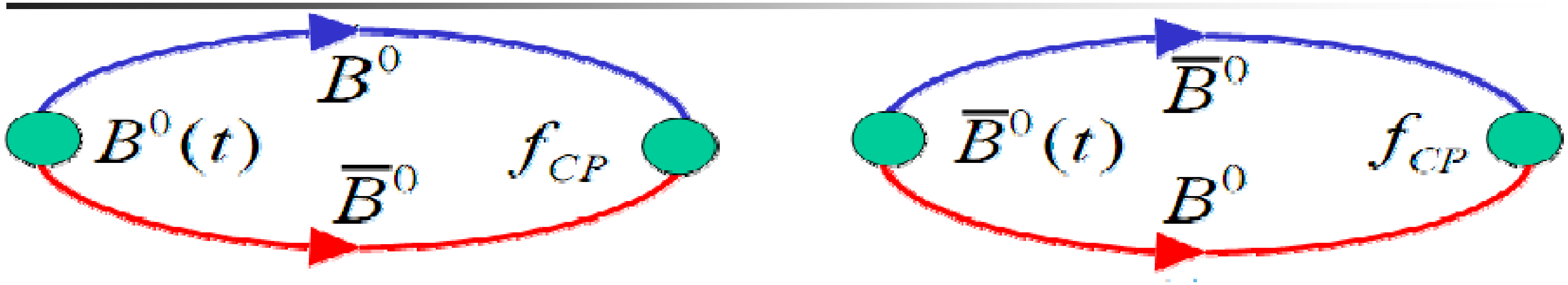
Implication of 2 different asymmetries: $\mathbf{Arg(C/T) < 0}$ large

seems like a difficulty for QCD-factorization/SCET

NLO direct CP asymmetry

Mode	Data [1]	LO	+NLO
$B^\pm \to \pi^\pm K^0$	0.009±0.025	-0.01	$0.00 \pm 0.00 (\pm 0.00)$
$B^\pm \to \pi^0 K^\pm$	0.050±0.025	-0.08	$-0.01^{+0.03\,(+0.03)}_{-0.05\,(-0.05)}$
$B^0 \to \pi^\mp K^\pm$	**-0.097±0.012**	-0.12	$-0.09^{+0.06\,(+0.04)}_{-0.08\,(-0.06)}$
$B^0 \to \pi^0 K^0$	—	-0.02	$-0.07^{+0.03\,(+0.01)}_{-0.03\,(-0.01)}$
$B^0 \to \pi^\mp \pi^\pm$	0.38±0.07	0.14	$0.18^{+0.20\,(+0.07)}_{-0.12\,(-0.06)}$
$B^\pm \to \pi^\pm \pi^0$	0.06±0.05	0.00	$0.00 \pm 0.00 (\pm 0.00)$
$B^0 \to \pi^0 \pi^0$	0.48±0.32	-0.04	$0.63^{+0.35\,(+0.09)}_{-0.34\,(-0.15)}$

Mixing induced CP violation



$$\Gamma(B^0(t)\to f)=\frac{1}{2}|A_f|^2e^{-\Gamma t}\left\{(1+|\lambda|^2)+(1-|\lambda|^2)\cos\Delta mt-2\mathrm{Im}\lambda\sin\Delta mt\right\}$$

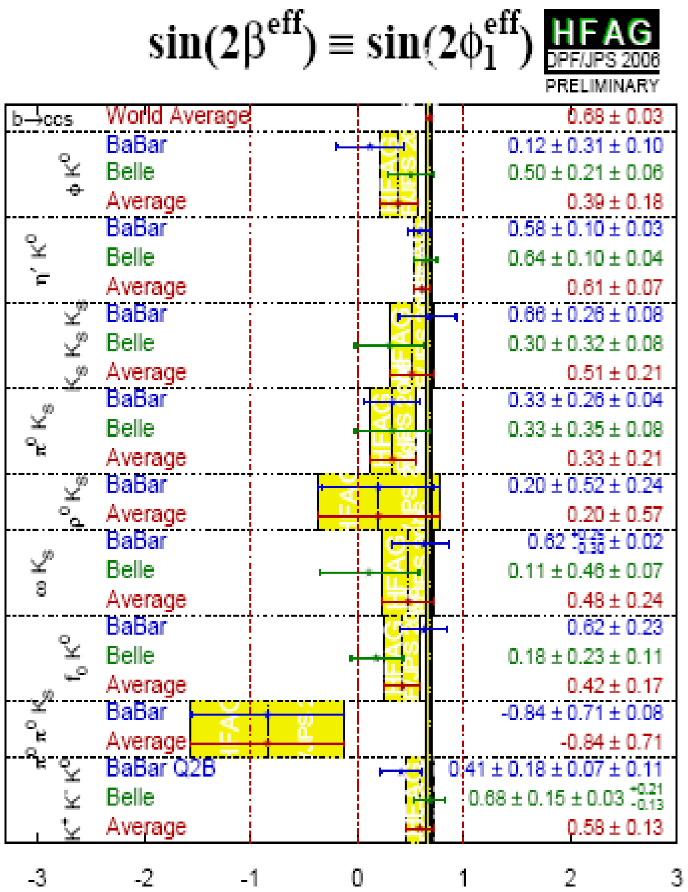
$$\Gamma(\bar{B}^0(t)\to f)=\frac{1}{2}|A_f|^2e^{-\Gamma t}\left\{(1+|\lambda|^2)-(1-|\lambda|^2)\cos\Delta mt+2\mathrm{Im}\lambda\sin\Delta mt\right\}$$

$$\begin{aligned}\mathcal{A}_{CP}(t)&=\frac{\Gamma(B^0(t)\to f)-\Gamma(\bar{B}^0(t)\to f)}{\Gamma(B^0(t)\to f)+\Gamma(\bar{B}^0(t)\to f)}\\&=\frac{|\lambda|^2-1}{|\lambda|^2+1}\cos\Delta mt+\frac{2\mathrm{Im}\lambda}{|\lambda|^2+1}\sin\Delta mt\end{aligned}$$

$$\lambda=\frac{q}{p}\frac{\bar{A}}{A}.$$

Tendency of exp. data is against th.?!

Measured values of $\Delta S_{b\to s}$ are mostly *negative* while many theoretical models predict them *positive*!



In the case of pQCD and QCDF

$$\boxed{\Delta S \simeq 2\epsilon_{KM} \cos 2\phi_1 \sin\phi_3 \cos\delta \left|\frac{A_f^u}{A_f^c}\right|}$$

☞ In perturbative computation, δ is relatively small and $A_f^u/A_f^c \simeq 1 -$ tree/penguin

☞ The sign and size of $-$tree/penguin

$B \to \phi K_S$: zero

$B \to \eta' K_S$: negligible

$B \to \pi K_S$: large positive

$B \to \omega K_S$: large positive

$B \to \rho K_S$: large negative

Mixing Induced CP

- $B \rightarrow \pi^+\pi^-$, ϕK, $\eta' K$, KKK …
- **Dominant by the B-B bar mixing**
- **Most of the approaches give similar results**
- **Even with final state interactions**

For Example:

$\Delta S_{\pi^0 K_S}$ in Factorization-related methods

(From Yossi Nir)

$\Delta S_{\pi^0 K_S}$	Method	hep-ph/	Authors
$+0.06 \pm 0.04$	NF	0503151	Buchalla, Hiller, Nir, Raz
$+0.04 \pm 0.03$	NF+model	0502235	Cheng, Chua, Soni
$+0.07 \pm 0.04$	QCDF	0505075	Beneke
$+0.06 \pm 0.03$	PQCD	0508041	Li, Mishima, Sanda
$+0.08 \pm 0.16$	SCET+SU(3)	0510241	Bauer, Rothstein, Stewart

Bs decays

- **Good test for SU(3) symmetry, U-spin symmetry**
- **CP asymmetry study**
- **CKM angle measurements**
- **… … LHCb**

hep-ph/0703162, PRD76:074018,2007

BR (x 10^{-6})

Modes	Class	QCDF	SCET	This work	Exp.
$\overline{B}_s^0 \to K^+\pi^-$	T	$10.2^{+4.5+3.8+0.7+0.8}_{-3.9-3.2-1.2-0.7}$	$4.9 \pm 1.2 \pm 1.3 \pm 0.3$	$7.6^{+3.2+0.7+0.5}_{-2.3-0.7-0.5}$	$5.0 \pm 0.75 \pm 1.0$
$\overline{B}_s^0 \to K^0\pi^0$	C	$0.49^{+0.28+0.22+0.40+0.33}_{-0.24-0.14-0.14-0.17}$	$0.76 \pm 0.26 \pm 0.27 \pm 0.17$	$0.16^{+0.05+0.10+0.02}_{-0.04-0.05-0.01}$	
$\overline{B}_s^0 \to K^+K^-$	P	$22.7^{+3.5+12.7+2.0+24.1}_{-3.2-8.4-2.0-9.1}$	$18.2 \pm 6.7 \pm 1.1 \pm 0.5$	$13.6^{+4.2+7.5+0.7}_{-3.2-4.1-0.2}$	$24.4 \pm 1.4 \pm 4.6$
$\overline{B}_s^0 \to K^0\overline{K}^0$	P	$24.7^{+2.5+13.7+2.6+25.6}_{-2.4-9.2-2.9-9.8}$	$17.7 \pm 6.6 \pm 0.5 \pm 0.6$	$15.6^{+5.0+8.3+0.0}_{-3.8-4.7-0.0}$	
$\overline{B}_s^0 \to \pi^0\eta$	P_{EW}	$0.075^{+0.013+0.030+0.008+0.010}_{-0.012-0.025-0.010-0.007}$	$0.014 \pm 0.004 \pm 0.005 \pm 0.004$	$0.05^{+0.02+0.01+0.00}_{-0.02-0.01-0.00}$	< 1000
			$0.016 \pm 0.0007 \pm 0.005 \pm 0.006$		
$\overline{B}_s^0 \to \pi^0\eta'$	P_{EW}	$0.11^{+0.02+0.04+0.01+0.01}_{-0.02-0.04-0.01-0.01}$	$0.006 \pm 0.003 \pm 0.002^{+0.064}_{-0.006}$	$0.11^{+0.05+0.02+0.00}_{-0.03-0.01-0.00}$	
			$0.038 \pm 0.013 \pm 0.016^{+0.260}_{-0.036}$		
$\overline{B}_s^0 \to K^0\eta$	C	$0.34^{+0.19+0.64+0.21+0.16}_{-0.16-0.27-0.07-0.08}$	$0.80 \pm 0.48 \pm 0.29 \pm 0.18$	$0.11^{+0.05+0.06+0.01}_{-0.03-0.03-0.01}$	
			$0.59 \pm 0.34 \pm 0.24 \pm 0.15$		
$\overline{B}_s^0 \to K^0\eta'$	C	$2.0^{+0.3+1.5+0.6+1.5}_{-0.3-1.1-0.3-0.6}$	$4.5 \pm 1.5 \pm 0.4 \pm 0.5$	$0.72^{+0.20+0.28+0.11}_{-0.16-0.17-0.05}$	
			$3.9 \pm 1.3 \pm 0.5 \pm 0.4$		
$\overline{B}_s^0 \to \eta\eta$	P	$15.6^{+1.6+9.9+2.2+13.5}_{-1.5-6.8-2.5-5.5}$	$7.1 \pm 6.4 \pm 0.2 \pm 0.8$	$8.0^{+2.6+4.7+0.0}_{-1.9-2.5-0.0}$	< 1500
			$6.4 \pm 6.3 \pm 0.1 \pm 0.7$		
$\overline{B}_s^0 \to \eta\eta'$	P	$54.0^{+5.5+32.4+8.3+40.5}_{-5.2-22.4-6.4-16.7}$	$24.0 \pm 13.6 \pm 1.4 \pm 2.7$	$21.0^{+6.0+10.0+0.0}_{-4.6-5.6-0.0}$	
			$23.8 \pm 13.2 \pm 1.6 \pm 2.9$		
$\overline{B}_s^0 \to \eta'\eta'$	P	$41.7^{+4.2+26.3+15.2+36.6}_{-4.0-17.2-8.5-15.4}$	$44.3 \pm 19.7 \pm 2.3 \pm 17.1$	$14.0^{+3.2+6.2+0.0}_{-2.7-3.9-0.0}$	
			$49.4 \pm 20.6 \pm 8.4 \pm 16.2$		
$\overline{B}_s^0 \to \pi^+\pi^-$	ann	$0.024^{+0.003+0.025+0.000+0.163}_{-0.003-0.012-0.000-0.021}$	—	$0.57^{+0.16+0.09+0.01}_{-0.13-0.10-0.00}$	< 1.36
$\overline{B}_s^0 \to \pi^0\pi^0$	ann	$0.012^{+0.001+0.013+0.000+0.082}_{-0.001-0.006-0.000-0.011}$	—	$0.28^{+0.08+0.04+0.01}_{-0.07-0.05-0.00}$	< 210

Three measurements of BRs in B_s

	SCET	QCDF	PQCD	EXP CDF
$B_s \to K^- \pi^+$	4.9 ± 1.8	10 ± 6	7.6 ± 3.3	5.0 ± 1.3
$B_s \to K^- K^+$	18 ± 7	23 ± 27	14 ± 9	24 ± 5
$B_s \to \phi\,\phi$		22 ± 30	35 ± 19	14 ± 8

Previous 34

First measurement of CP asymmetry in B_s decays

$B_s \rightarrow K^- \pi^+$

SCET	QCDF	PQCD	EXP
20 ± 26	**-6.7 ± 16**	**30 ± 6**	**$39 \pm 15 \pm 8$**

pQCD agree with EXP in CP

U-spin symmetry (Gronau,Rosner,Lipkin)

$$R_3 = \frac{|A(B_s \to \pi^+ K^-)|^2 - |A(\bar{B}_s \to \pi^- K^+)|^2}{|A(B_d \to \pi^- K^+)|^2 - |A(\bar{B}_d \to \pi^+ K^-)|^2} = -1$$

$$\Delta = \frac{A_{CP}^{dir}(\bar{B}_d \to \pi^+ K^-)}{A_{CP}^{dir}(\bar{B}_s \to \pi^+ K^-)} + \frac{BR(B_s \to \pi^+ K^-)}{BR(\bar{B}_d \to \pi^+ K^-)} \cdot \frac{\tau(B_d)}{\tau(B_s)} = 0$$

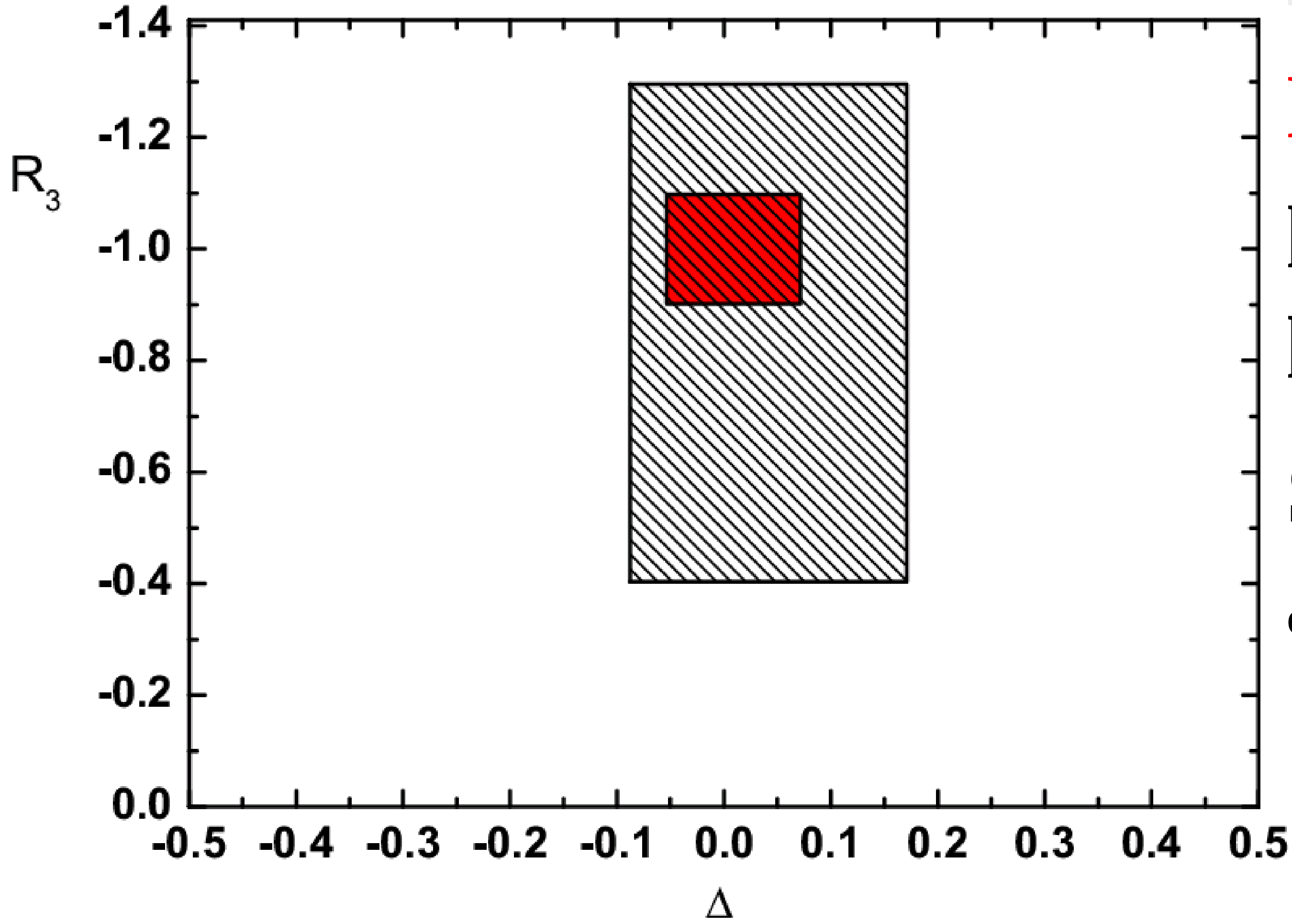
- **Results from pQCD**

$$R_3 = -1.00^{+0.04+0.03+0.09}_{-0.04-0.04-0.08}, \quad \Delta = -0.00^{+0.03+0.03+0.06}_{-0.03-0.02-0.04},$$

- Experimental data

$$R_3 = -0.84 \pm 0.42 \pm 0.15 \qquad \Delta = 0.04 \pm 0.11 \pm 0.08$$

R_3 vs Δ



Red area is pQCD prediction;

Shaded area is exp.

$Bs \rightarrow \rho^0 K_S$

- **If tree dominant (V_{ub}), good for gamma measuremnt.**
- **However, Color suppressed tree is comparable with QCD penguin contribution**
- **Direct CP large**

QCDF	PQCD
$25 \pm 60\ \%$	$97 \pm 30\ \%$

Not good for gamma measurement

Summary / Comment

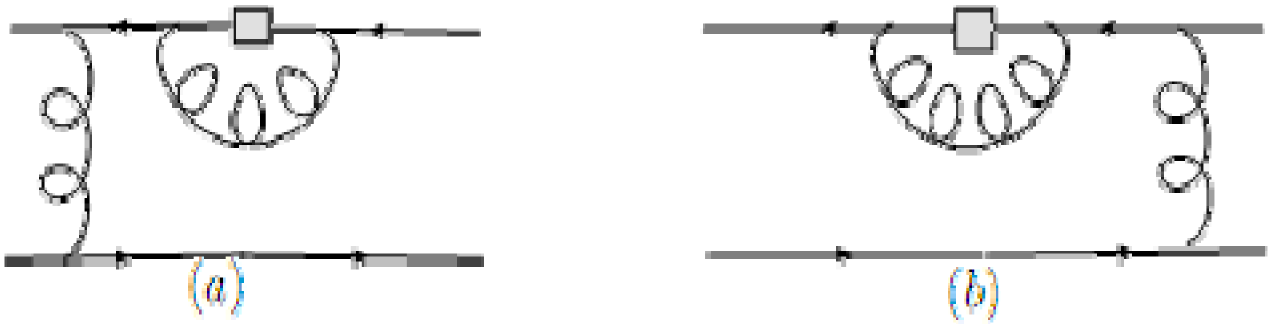
- **Factorization approaches are systematic tools, sometimes have to be used for data fitting (Scenario 1,2,3,4 in QCDF, charming penguin in SCET)**
- **SCET is encouraging, counting rules consistent with pQCD, but need more parameters**
- **NLO, $1/m_B$ corrections not yet fully studied, important for certain channels**

Annihilation Penguin

- The direct CP measurements need a large contribution from annihilation penguin (or charming penguin), with large strong phase
- The large BRs of B→ VP modes also need such annihilation penguin
- Similar in the polarization of B→ VV modes
- Only pQCD approach can predict its size by calculation

Thank you!

Threshold resummation



The radiative corrections of hard part result in double logarithms. The resummation of them give out a jet function

$$S_t^{LL}(x) = -\exp\left(\frac{\pi}{4}\alpha_s C_F\right)\int_{-\infty}^{+\infty}\frac{dt}{\pi}(1-x)^{e^t}\sin\left(\frac{1}{2}\alpha_s C_F t\right)\exp\left(-\frac{\alpha_s}{4\pi}C_F t^2\right)$$

Sudakov factor

The soft and collinear divergence produce double logarithm $ln^2 Pb$,

Summing over these logs result a Sudakov factor. It suppresses the endpoint region

Sudakov factor

- **Exponential suppression at endpoint**