# **Theory Review of Kaon Physics**

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Focus on recent theoretical calculations and New-Physics possibilities through clean observables in Kaon

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#### 1) Tree-level mediated decays -> CKM unitarity



Hadronic uncertainties from  $\langle \pi | \bar{s} \gamma^{\mu} u | K \rangle \Leftrightarrow f_{+,0}(q^2)$ 

**Novelty:** estimate of  $f_+(0)$  on the Lattice at ~0.5%. (RBC/UKQCD)

2) Elicity suppressed decays -> sensitivity to the Higgs sector



#### **Novelty:**

- recent improving on f<sub>k</sub>/f<sub>π</sub> by Lattice (HPQCD/UKQCD)
- bounds on scalar couplings competitive to B

New Golden mode =>  $K_{e2}/K_{\mu 2}$ 

3) <u>FCNC processes</u> -> SUSY, Little Higgs ....



Improved theoretical estimate of SM contributions
 "New Physics" model studies with all the constraints



## V<sub>us</sub> determination and the CKM Unitarity



 $G_{ew}$ = 1.1678(26) 10<sup>-5</sup> GeV<sup>-2</sup>

V<sub>us</sub> below 1% makes CKM unitarity competitive to Electro-Weak Precision Test





 $V_{\mu s}$  from  $K_{\ell 3}$  decays

with  $K = K^+$ ,  $K^0$ ; l = e,  $\mu$  and  $C_{\kappa^2} = 1/2$  for  $K^+$ , 1 for  $K^0$ 

## Inputs from experiment:

 $\Gamma(K_{l3(\gamma)})$  Branching ratios with well determined treatment of radiative decays; lifetimes

 $I_{Kl}(\lambda)$ 

Phase space integral: λs parameterize form factor dependence on t:

 $K_{e3}$ : only  $\lambda_{+}$  (or  $\lambda_{+}' \lambda_{+}''$ )  $K_{\mu3}$  : need  $\lambda_+$  and  $\lambda_0$ 

**Extraordinary experimental progress:** 2004-06 PDG average superseded by the new results:



**R.** Wanke's talk

 $V_{us}$  from  $K_{\ell 3}$  decays

$$\Gamma(K_{l3(\gamma)}) = \frac{C_{K}^{2} G_{F}^{2} M_{K}^{3}}{192\pi^{3}} S_{EW} |V_{us}|^{2} |f_{+}^{K^{0}\pi^{-}}(0)|^{2} I_{Kl}(\lambda) (1 + 2\Delta_{K}^{SU(2)} + 2\Delta_{Kl}^{EM})$$

with  $K = K^+$ ,  $K^0$ ; l = e,  $\mu$  and  $C_K^2 = 1/2$  for  $K^+$ , 1 for  $K^0$ 





Average from *Flavianet KW* [arXix:0801.1817]  $|V_{us}| f_{+}(0) = 0.2166(5) \implies \sigma_{rel} \sim 0.21\%$ 

The  $|V_{us}|f_+(0)$  ratio among Neutral and Charged modes:

ſ	Δ <sup>SU(2)</sup> exp	<sub>p</sub> = 2.86(38)%	-	success o calcula	f CHPT ations	$[\Delta^{SU}]$	<sup>(2)</sup> th =	2.31(	(22)%]
	0.215	0.2175			% err	Approx BR	. contr. τ	to % er ð	r from: I <sub>K1</sub>
			K <sub>L</sub> e3	0.2163(6)	0.28	0.09	0.19	0.15	0.09
	· · · · · · · · · · · · · · · · · · ·	<mark>•-</mark>	<i>K<sub>L</sub></i> μ3	0.2168(7)	0.31	0.10	0.18	0.15	0.15
		+ !	K <sub>s</sub> e3	0.2154(13)	0.67	0.65	0.03	0.15	0.09
l		<b></b> -	K±e3	0.2173(8)	0.39	0.26	0.09	0.26	0.09
	f <sub>+</sub> (0)×V <sub>us</sub>		<i>K</i> ±µ3	0.2176(11)	0.51	0.40	0.09	0.26	0.15
	0.215	0.2175							

Average from *Flavianet KW* [arXix:0801.1817]  $|V_{us}| f_{+}(0) = 0.2166(5) \implies \sigma_{rel} \sim 0.21\%$ 

**f**<sub>+</sub>(**0**) - Determinations

#### $f_+(0)$ Chiral Properties:

the Ademollo-Gatto Theorem: chiral corrections  $\propto (m_s - m_u)^2$ 





 $V_{us}$  from  $K_{\ell 2}$  decays



• Being  $f_{\kappa}/f_{\pi}$  better determined than  $f_{\kappa}$ 

$$\frac{\Gamma(\mathbf{K}_{\mu 2(\gamma)})}{\Gamma(\boldsymbol{\pi}_{\mu 2(\gamma)})} = \frac{|\mathbf{V}_{us}|^2}{|\mathbf{V}_{ud}|^2} \times \frac{f_K^2}{f_\pi^2} \times \frac{M_K (1 - m_\mu^2 / M_K^2)^2}{m_\pi (1 - m_\mu^2 / m_\pi^2)^2} \times 1 + \alpha (C_K - C_\pi)$$

#### Inputs from theory: Inputs from experiment: Ratio of pseudoscalar $f_{\kappa}/f_{\pi}$ Rates with well-determined $\Gamma(K_{\mu 2(\gamma)})$ decay constants treatment of radiative decays: $\Gamma(\pi_{\mu 2(\gamma)})$ Branching ratios $C_{K'}C_{\pi}$ Radiative inclusive lifetimes electroweak corrections $1 + \alpha (C_{r} - C_{r}) = 0.9930(35)$ Marciano'04, Cirigliano-Rosell'07 $|V_{us}|/|V_{ud}| \times (f_K/f_\pi) = 0.2760 \pm 0.0006 \implies \sigma_{rel} \sim 0.21\%$

 $V_{us}$  from  $K_{\ell 2}$  decays





• No AdemolloGatto Protection: *chiral dependence*  $\propto$  (m<sub>s</sub>-m<sub>u</sub>)

$$\frac{f_{K}}{f_{\pi}} = 1 + \Delta^{loops}(\mu) - \frac{1}{f_{\pi}^{2}} LEC(\mu)(M_{K}^{2} - M_{\pi}^{2})$$

 sensitivity to the lightest pion simulated (kew point in order to match with ChPt Logs)



2007 Progress

Old simulations with heavy pions =>  $f_{\kappa}/f_{\pi}$  underestimate

Recently, many Lattice calculations, with rather light quark masses,  $m_{\pi} \ge 280$  MeV:

• N<sub>F</sub>=2 and N<sub>F</sub>=2+1;

• First signs of chiral logs

Overall accuracy ~ 1%

• <u>A complete study with</u> <u>all the systematic is</u> <u>still missing</u>

• Interest result from HPQCD: N<sub>F</sub>=2+1, Staggered, m<sub> $\pi$ </sub>  $\ge$  280 MeV  $f_K/f_{\pi}$ =1.189(7) ==>  $\sigma_{rel} \sim 0.6\%$ 

need confirmation!!





## V<sub>us</sub> and CKM Unitarity at Spring 2008

 $f_{+}(0)=0.964(5)$  from UKQCD/RBC'07  $|V_{us}|=0.2246(12)$  from Kl3



 $f_K / f_{\pi} = 1.189(7)$  from HPQCD'07 | $V_{us} / V_{ud}$ |=0.2323(15) from Kl2

## Fit results, no constraint:

 $|V_{ud}|=0.97417(26)$ |V\_{us}|=0.2253(9)  $\chi^2/ndf \ 0.65/1 \ (42\%)$ 

Fit results, unitarity constraint:

 $|V_{us}| = \sin\theta = 0.2255(7)$  $\chi^2/ndf \ 0.80/2 \ (67\%)$ 

 $|V_{ud}|^2 + |V_{us}^{Kl3}|^2 = 0.9995(7)$  This is a highly non-trivial constraint for NP models...



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## New Physics Searches from K<sub>12</sub>

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## Lepton Universality Test $\Rightarrow B(K \rightarrow ev) / B(K \rightarrow \mu v)$

 Leading QCD uncertainties from f<sub>K</sub> cancel out

• Subleading effects calculated in ChPT to  $O(e^2p^4)$ 

Cirigliano Rosell '07

$$\frac{\Gamma(K \to ev_e)}{\Gamma(K \to \mu v_{\mu})} \bigg|_{SM} = \frac{m_e^2}{m_{\mu}^2} \bigg( \frac{m_K^2 - m_e^2}{m_K^2 - m_{\mu}^2} \bigg)^2 \big( 1 + O(e^2 p^n) \big)$$
$$R_K^{SM} : (2.477 \pm 0.001) \to 0.04\% \text{ accuracy}$$

• In MSSM, sizeable LFV couplings at large  $\tan\beta$  enhance  $K \rightarrow ev_{\tau}$ 

 $\frac{\sum_{i} \Gamma(K \to ev_i)}{\sum_{i} \Gamma(K \to \mu v_i)} \bigg|_{ex}$ 

$$\approx \frac{\Gamma(K \to ev_e) + \Gamma(K \to ev_\tau)}{\Gamma(K \to \mu v_\mu)} = R_K^{SM} \left(1 + \Delta r_K^{LFV}\right)$$

• up to 1% enhancement to the SM

H s,d H  $v_{e}v_{\mu}$   $\delta_{RR}^{\mathfrak{s}\ell} \mathfrak{O}_{\tilde{\ell}_{R}} \tilde{\nu}_{L}$   $\ell_{R}$   $\tilde{\nu}_{L}$   $v_{e}v_{\mu}$ 

key ingredeints for visible effects in SUSY:

• Large tan  $\beta$ ,  $M_H < 1 TeV$ 

• Large LFV slepton minxings,  $\delta_{3j} \sim \mathcal{O}(1)$ ,  $(m_{SUSY} \ge 1 \text{TeV})$ 

Masiero, Petronzio, Paradisi '05

#### Limit on LFV in H<sup>+</sup> coupling $\Rightarrow$





Recent Results at  $O(1-2\%) \implies 0.3\%$  from NA61



 $K_{I2}$ : Higgs effects =>  $|V_{us}|^{KI2}$ ->  $|V_{us}|^{KI2}$  (1 +  $g^H m^2_K / m^2_H$ )

• to test  $g^{H} = f_{K}$  from theory:  $f_{K}/f_{\pi}$  better determined



Bernard, Hirn, Oertel, Passemar & Stern

- "Higgs-Goldstone field" to give mass to W and Z;
- no elementary Higgs particle => higher order operators •

(composite, partial, no Higgs)

#### unbound effects by EWPT on right-handed currents: =>



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#### Rare K Decays

• Unique and precious test of the SM: <u>one-loop FCNC and  $\Delta S=1$  transitions</u>: Hadronic uncertainties are small or under good theoretical control



- Highest CKM suppression  $V_{ts}V_{td} \sim \lambda^5$
- Highly sensitive to New Physics  $\rightarrow$  like  $\epsilon'/\epsilon$

In other words ... as promising as  $sin(2\beta)$  from  $B \rightarrow J/\psi K_S$ 

#### Rare K Decays: the four golden mode

	Short-distance [%] (sensitivity to e.w scale)	Irreducible th. error on the BR
$K_{L} \rightarrow \pi^{0} v \overline{v}$	>99%	< 2%
$K^+ \rightarrow \pi^+ v \overline{v}$	91%	< 5%
$K_{L} \rightarrow \pi^{0} e^{+} e^{-}$	38%	< 10%
$K_L \rightarrow \pi^0 \mu^+ \mu^-$	28%	<10%

Golden Modes	Standard Model	Experiment			CKM
$K^+ \to \pi^+ \nu \overline{\nu}$	$8.22^{\scriptscriptstyle +0.84}_{\scriptscriptstyle -0.84} \times 10^{-11}$	$14.7^{+13.0}_{-8.9} \times 10^{-10}$	<sup>11</sup> E787 E949		$V_{ts}^* V_{td}$
$K_L \to \pi^0 \nu \overline{\nu}$	$2.76^{\scriptscriptstyle +0.40}_{\scriptscriptstyle -0.40} \times 10^{-11}$	$< 6.7 \times 10^{-8}$	E391a		$\operatorname{Im} V_{ts}^* V_{td} \sim \eta$
$K_L \rightarrow \pi^0 e^+ e^-$	$3.5^{+1.0}_{-0.9} \times 10^{-11}$	$< 2.8 \times 10^{-10}$	KTeV		$\operatorname{Im} V_{ts}^* V_{td} \sim \eta$
$K_L \rightarrow \pi^0 \mu^+ \mu^-$	$1.4^{+0.3}_{-0.3} \times 10^{-11}$	$< 3.8 \times 10^{-10}$	KTeV		$\mathrm{Im} V_{ts}^* V_{td} \sim \eta$

But they have to be measured!

JPARC@ KEK, P326@CERN  $[K_L \rightarrow \pi \nu \nu]$ 

 $[K^+ \rightarrow \pi \nu \nu]$ 

 $K \rightarrow \pi + vv$  and  $K \rightarrow \pi + ll$  : SM Studies

$$H_{eff}\left(s \to d\right) \propto \left( y_{\mathbf{v}}(\overline{s}\gamma_{\mu}d)(\overline{v}\gamma_{L}^{\mu}v) + y_{7V}(\overline{s}\gamma_{\mu}d)(\overline{\ell}\gamma^{\mu}\ell) + y_{7A}(\overline{s}\gamma_{\mu}d)(\overline{\ell}\gamma_{L}^{\mu}\gamma_{5}\ell) \right)$$

1. Top contribution dominant  $\propto G_F^2 m_t^2 \lambda^5 (1 - \rho + i\eta)$ 

2. 
$$\langle \pi | (\overline{s}\gamma_{\mu}d) | K \rangle$$
 related to the  $K_{\ell_3} \langle \pi | (\overline{s}\gamma_{\mu}u) | K \rangle$  ME by isospin (Marciano-Parsa)

Recent Improvement

F.M. and C. Smith '07

$$\mathcal{B}\left(K^{+} \to \pi^{+} \nu \bar{\nu} (\gamma)\right) = \kappa_{\nu}^{+} \left(1 + \underline{\Delta_{EM}}\right) |y_{\nu}|^{2} \qquad \mathcal{B}\left(K_{L} \to \pi^{0} \nu \bar{\nu}\right) = \kappa_{\nu}^{L} \left(\operatorname{Im} y_{\nu}\right)^{2}$$

New analysis: SU(2) corr. at NLO in ChPT and the new KI3 data

 $\kappa_{\nu}^{L} = 3.3624(264) \cdot 10^{-5} \qquad \qquad \kappa_{\nu}^{+} = 0.7867(43) \cdot 10^{-5}$ 

<u>errror reduced by a factor ~4 and ~7 respectively</u>. => CKM uncertainty is the dominant source for the SM predictions.

updating at <u>www.Inf.infn.it/wg/vus</u>



 $K \rightarrow \pi + vv$  and  $K \rightarrow \pi + ll$  : *SM* Studies

$$H_{eff}\left(s \to d\right) \propto \left( y_{\mathbf{v}}(\overline{s}\gamma_{\mu}d)(\overline{v}\gamma_{L}^{\mu}v) + y_{7V}(\overline{s}\gamma_{\mu}d)(\overline{\ell}\gamma^{\mu}\ell) + y_{7A}(\overline{s}\gamma_{\mu}d)(\overline{\ell}\gamma_{L}^{\mu}\gamma_{5}\ell) \right)$$

1. Top contribution dominant  $\propto G_F^2 m_t^2 \lambda^5 (1 - \rho + i\eta)$ 

2.  $\langle \pi | (\overline{s} \gamma_{\mu} d) | K \rangle$  related to the  $K_{\ell_3} \langle \pi | (\overline{s} \gamma_{\mu} u) | K \rangle$  ME by isospin (Marciano-Parsa)

#### Further Improvement

- $K_L \rightarrow \pi^0 v \overline{v}$  CP violating process  $\rightarrow$  fully dominated by the top contribution <u>no further uncertainties!</u>
- $K^+ \to \pi^+ \nu \overline{\nu}$

uncertainties from charm and up contributions accurately estimated (~5%)

NNLO charm effects => Buras, Gorbahn, Haisch and Nierste '06

 $m_t \times V_{ts}$ 

Long Distance effects => Isidori, F.M, C.Smith '05

•  $K_L \rightarrow \pi^0 e^+ e^- (\mu^+ \mu^-)$ 

reached a good theoretical control (~10%)

Isidori,Smith,Underdorfer '04; Friot,Greynat,de Raphael '03; Buchalla,D'Ambrosio,Isidori '03 Two classes of "Beyond SM" scenarios:

1. Minimal Flavour Violation:

flavour breaking induced only by SM Yukawa couplings,  $Y_U \& Y_{D.}$ (Y: Wilson coefficient at  $\Lambda_{flav}$ »1 TeV )

• SM hierarchy of FV couplings:

 $(s \rightarrow d)_{MFV} = O(\lambda^5) \times [SM + new d.o.f]$ 

- Specific realisations in SUSY, UED, LH, EFT
- Small deviations in specific models:  $B(K_L \rightarrow \pi^0 \nu \nu) \le O(20\%-30\%)$
- In specific models, stringent correlations can rise with either B physics  $(B \rightarrow \ell \ell, B \rightarrow X \ell \ell, B \rightarrow X \nu \nu)$  or EWPT  $(\Delta \rho)$

2. <u>New sources of Flavour Symmetry</u> breaking at the TeV scale

• s $\rightarrow$ d new couplings no longer  $O(\lambda^5)$  suppressed

 $(s \rightarrow d)_{BMFV} = O(\lambda^5) \times SM + O(1) \times (new d.o.f)$ 

- Many proposed models already killed from present data (B, K, EWPT & DM)
- One order of magnitude enhancement still possible in MSSM and LHT

 $B(K_L \to \pi^0 v v) \le 510^{-10}$ 

in reach of E391a upgrade

Pattern: effects on  $B(K_L \rightarrow \pi^0 \nu \nu) > B(K^+ \rightarrow \pi^+ \nu \nu) > B(K^+ \rightarrow \pi^+ \ell \ell)$ 

*Peculiarity:*  $K_L \rightarrow \pi^0 \mu \mu - K_L \rightarrow \pi^0 ee$  correlation



### Combining all present th. and exp. information, large deviations on $K \rightarrow \pi v v$ are still possible



rare K decays  $\Rightarrow$  large not covered parameter space!

- complementarity to Atlas/CMS searches  $\Rightarrow$  new particles
- supplementarity to LHcb/SuperB physics  $\Rightarrow$  gluinos

#### What can we ever learn from K-rare?

 $K \rightarrow \pi v v$  are the *best probe* of the flavour structure of the  $A^{U}$  terms ( $\propto M_{t}$ )



Isidori, F.M, Paradisi, Trine, Smith (06)

|A13| or |A23| (GeV)

 $\left| \left( m_U^2 \right)_{_{\boldsymbol{P}\boldsymbol{I}}}^{^{i3}} = \left( A^U - \mu \cot \beta \right)^{^{i3}} M_t \right|$ 



small impact on  $\varepsilon_{K}$  & sin $\beta$ , complementarity to LHCb/SuperB

Isidori, F.M, Paradisi, Trine, Smith (06)

# $K_L \rightarrow \pi^0 \mu \mu - K_L \rightarrow \pi^0 ee$ correlation

1. alike to  $K \rightarrow \pi v v \rightarrow \chi$  contributions by  $\gamma \& Z \text{ peng.}$ ; visible effects on currentcurrent operators 7

6

5

4

3

2

$$H_{eff}^{BSM}(s \to d) = X(\overline{s}\gamma_{\mu}d)(\overline{v}\gamma_{L}^{\mu}v) + y_{7V}(\overline{s}\gamma_{\mu}d)(\overline{\ell}\gamma_{\mu}^{\mu}\ell) + y_{7A}(\overline{s}\gamma_{\mu}d)(\overline{\ell}\gamma_{L}^{\mu}\gamma_{5}\ell)$$

2. contrary to  $K \rightarrow \pi v v \rightarrow$  sensitive to helicity-suppressed operators

$$+ y_{s}(\overline{s}d)(\overline{\ell}\ell) + y_{p}(\overline{s}d)(\overline{\ell}\gamma_{5}\ell)$$

→ H<sup>0</sup> penguins at large  $tan\beta$  0<sup>*L*</sup>. (*as B*→µµ, but different mass insertions)

$$B(K_{L} \rightarrow \pi^{0}\mu^{+}\mu^{-}) \times 10^{-11}$$

$$K_{L} \rightarrow \mu^{+}\mu^{-}$$
bound on  $y_{S,P}$ 

$$FEMP$$

$$V, A \text{ only}$$

$$V, A \text{ only}$$

$$W, W, W, W \text{ only}$$

$$W, W \text{$$

 $y_{S,P} \sim \left(M_W^2/M_A^2\right) \tan^3\beta \left(1 + 0.01 \tan\beta \operatorname{sign}\mu\right)^{-2} \left(\left(\delta_{LL}^D\right)_{12} + 18\left(\delta_{RR}^D\right)_{13}\left(\delta_{LL}^D\right)_{32}\right)$ 

Retico, Isidori (02)/Buras,Chankowaki,Rosiek,Slawianowska(02)/ Foster ,Okumura,Roszkowski (05)

#### **Conclusions from Kaon Physics**

**1.**  $V_{us}$  at 0.1% precision not impossible

 $\Rightarrow$  significant SM test competitive with EWPT

2. A real possibility to discovering SUSY trough Ke2/K $\mu$ 2 mode => in the reach of current experimental runs NA61

**3**. Lattice efforts to improve  $f_{\mathcal{K}}/f_{\pi}$  are welcome

**4.** rare K decays by the E391a upgrade, JPARC and P326 is an unique opportunity for Flavour Physics with LHC running

