Status of the CKM matrix

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Why precision CKM studies?

- The SM accomodates flavour & CP violation, but ***we have no theory of flavour***
- We have reasons to expect New Physics at the EW scale, and most models predicts additional flavour and CP violation.
- The CKM mechanism is very successful ➠ ***flavour and CP problem*** (NP must preserve agreement with data)
- Need for precision tests of the CKM mechanism, in many ways a challenge for QCD understanding

The CKM matrix

Weak and mass eigenstates

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \qquad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \hat{V}_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\boxed{\hat{V} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\varrho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)}$$

Wolfenstein parameterization $\lambda \sim 0.22$, A, ρ, η *are* $O(1)$

To improve the accuracy, define to all orders in λ

The Cabibbo angle

$$\hat{V} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\varrho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\boxed{\lambda = \sin(\theta_{\text{Cabibbo}}) = V_{us}}$$

Universality of charged currents ⇔ CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |\underbrace{V_{ub}}_{O(10^{-5})}|^2 = 1$$

Comparison between V_{ud}, V_{us} determinations of λ tests unitarity of the first line of V_{CKM}

λ could also be measured from 2nd line, V_{cd} (DIS) at 10%,
W decays at LEP constrains $\Sigma_{ij}|V_{ij}|^2$ at 1.3% V_{cs} at 1.3%

λ from V_{ud}

Superallowed Fermi transitions

($0^+ \to 0^+$ β decay)

$$\langle p_f; 0^+ | \bar{u}\gamma_\mu d | p_i; 0^+ \rangle = \sqrt{2}(p_i + p_f)_\mu$$

extremely precise, 9 expts, δV_{ud}~0.0003 dominated by RC and nuclear structure

Nucleus	ft (sec)	V_{ud}
^{10}C	3039.5(47)	0.97370(80)(14)(19)
^{14}O	3042.5(27)	0.97411(51)(14)(19)
^{26}Al	3037.0(11)	0.97400(24)(14)(19)
^{34}Cl	3050.0(11)	0.97417(34)(14)(19)
^{38}K	3051.1(10)	0.97413(39)(14)(19)
^{42}Sc	3046.4(14)	0.97423(44)(14)(19)
^{46}V	3049.6(16)	0.97386(49)(14)(19)
^{50}Mn	3044.4(12)	0.97487(45)(14)(19)
^{54}Co	3047.6(15)	0.97490(54)(14)(19)
Weighted Ave.		0.97418(13)(14)(19)

Nuclear structure *Coulomb distorsion* *RC*

$$V_{ud} = 0.97418(27) \text{ (superallowed)}$$

neutron β decay *not pure vector, needs g_A/g_V but no nuclear structure. δV_{ud}~0.002, will be improved at PERKEO, Heidelberg. Recent measurement of n lifetime (many σ away) serious problem!*

$$V_{ud} = 0.9746(4)_{\tau_n}(18)_{g_A}(2)_{\rm RC}$$

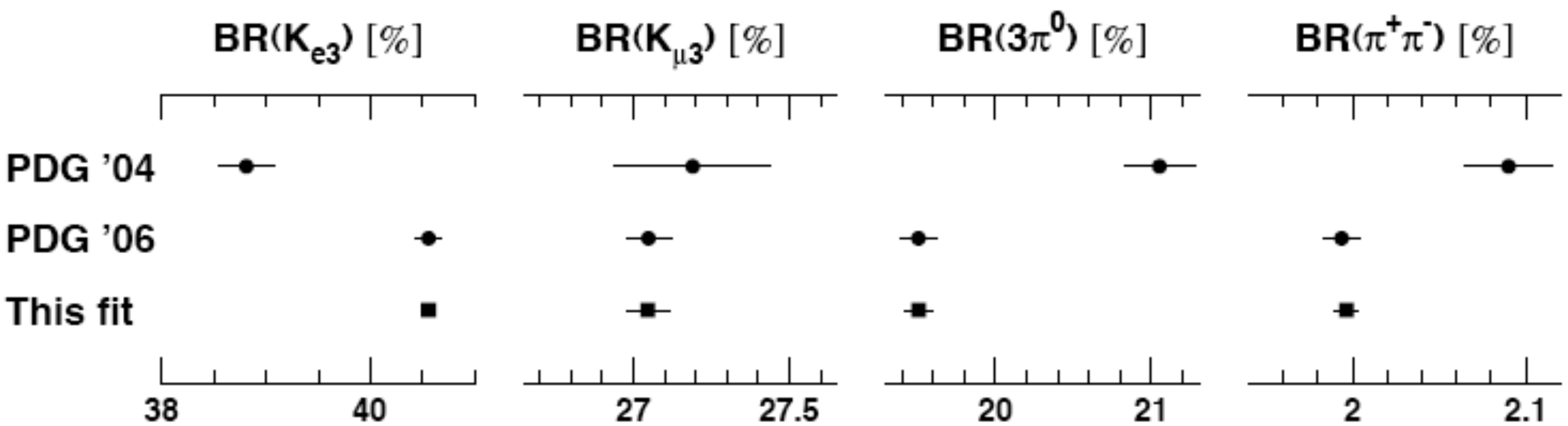
π^+ decay to $\pi^0 e\nu$ *th cleanest, promising in long term but BR~10^{-8} PIBETA at PSI has δV_{ud}~0.003*

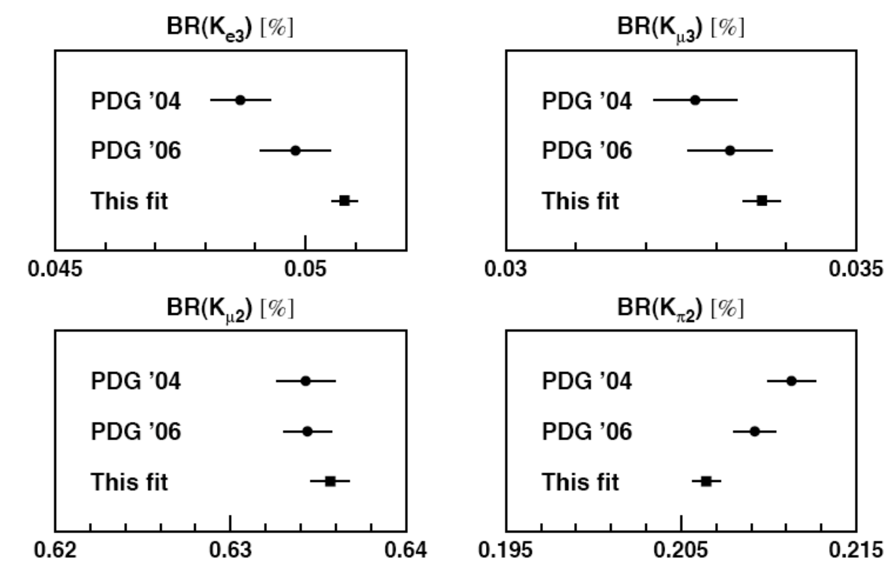
$$V_{ud} = 0.9749(26)\left[\frac{BR(\pi^+ \to e^+\nu_e(\gamma))}{1.2352 \times 10^{-4}}\right]^{\frac{1}{2}}$$

λ from K_{l3} - Experimental progress

➠talk by Wanke

$$\Gamma_{K\ell3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW}(1+\delta_K^\ell+\delta_{SU2})C^2 \left|V_{us}\right|^2 f_+^2(0) I_K^\ell.$$

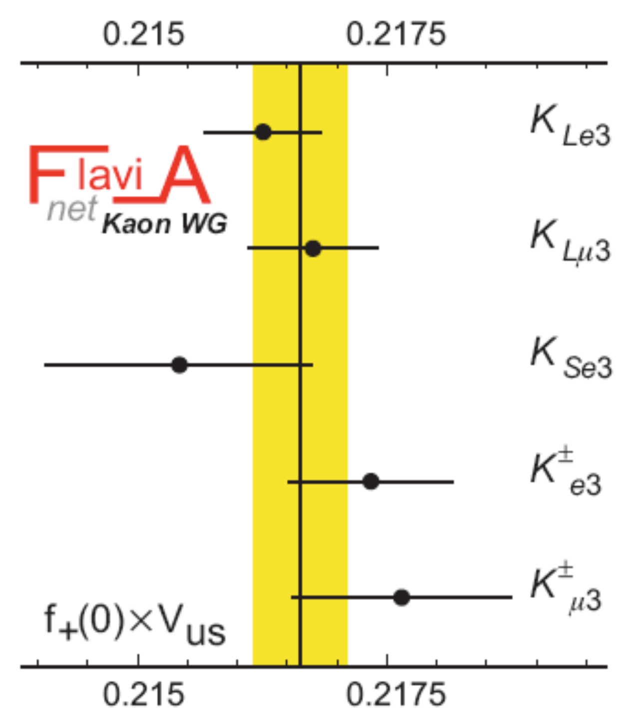




$$|V_{us}| \times f_+(0) = 0.21664(48)$$

0.25% accuracy!

muon channels perfectly consistent

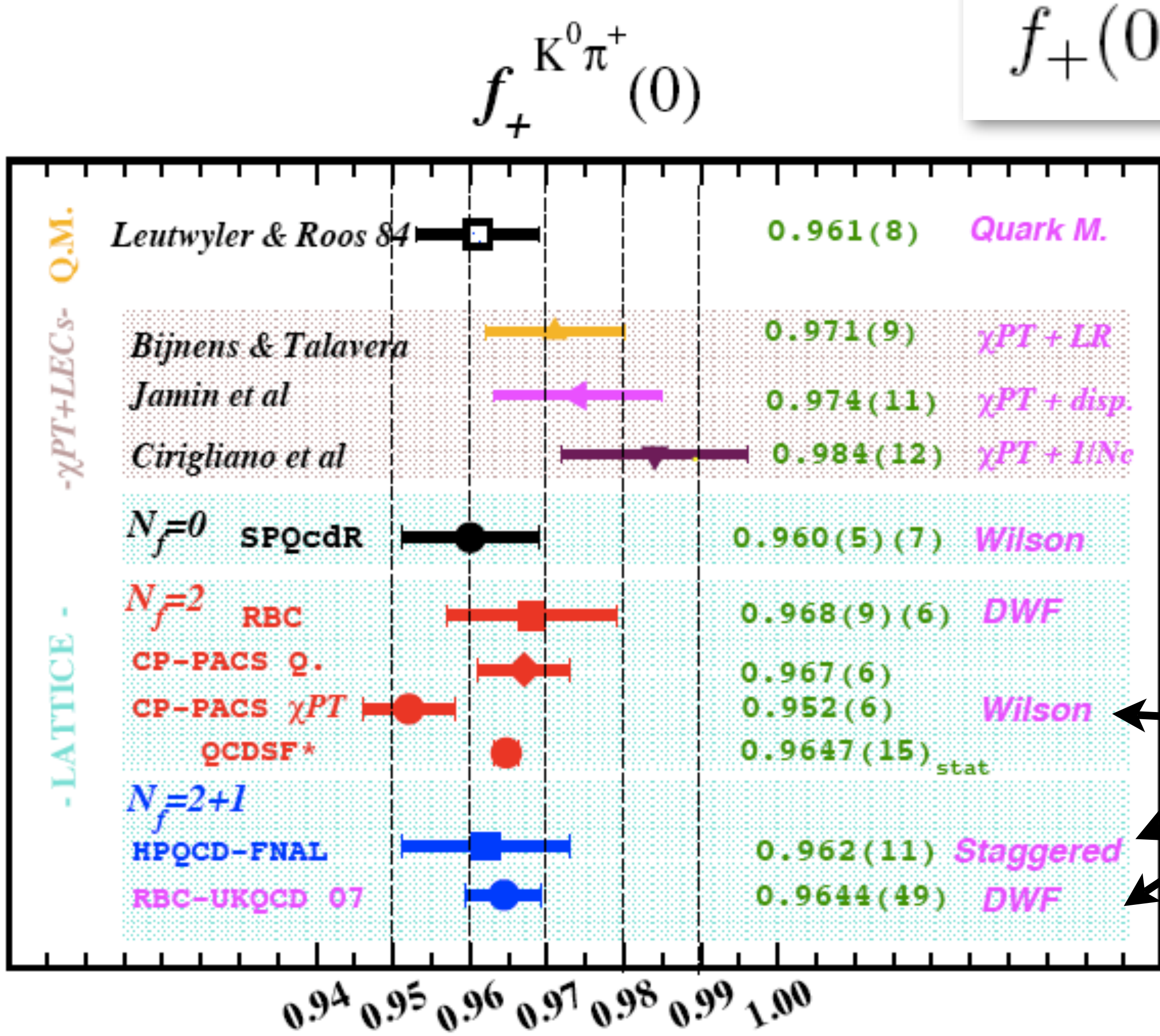


λ from K_{l3} - Theoretical progress

➠talk by Mescia

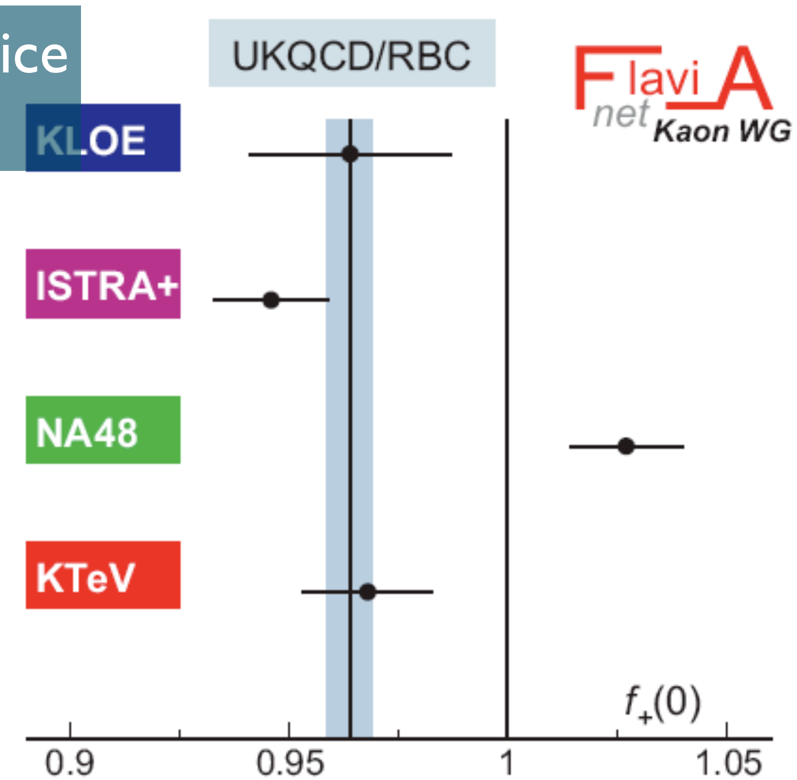
$$f_+(0) = 1 + f_2 + f_4 + \ldots$$

SU(3) symmetry, Ademollo Gatto th



Tests of lattice are now possible from measurements of the shapes

Various lattice actions



$$\boxed{|V_{us}| = 0.2246 \pm 0.0012 \qquad [K_{\ell 3}\ \text{only}]}$$

λ from K_{l2}

$$\frac{\Gamma(K^{\pm}_{\ell 2(\gamma)})}{\Gamma(\pi^{\pm}_{\ell 2(\gamma)})} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{f_K^2 m_K}{f_\pi^2 m_\pi} \left(\frac{1 - m_\ell^2/m_K^2}{1 - m_\ell^2/m_\pi^2}\right)^2 \times (1 + \delta_{\rm em})$$

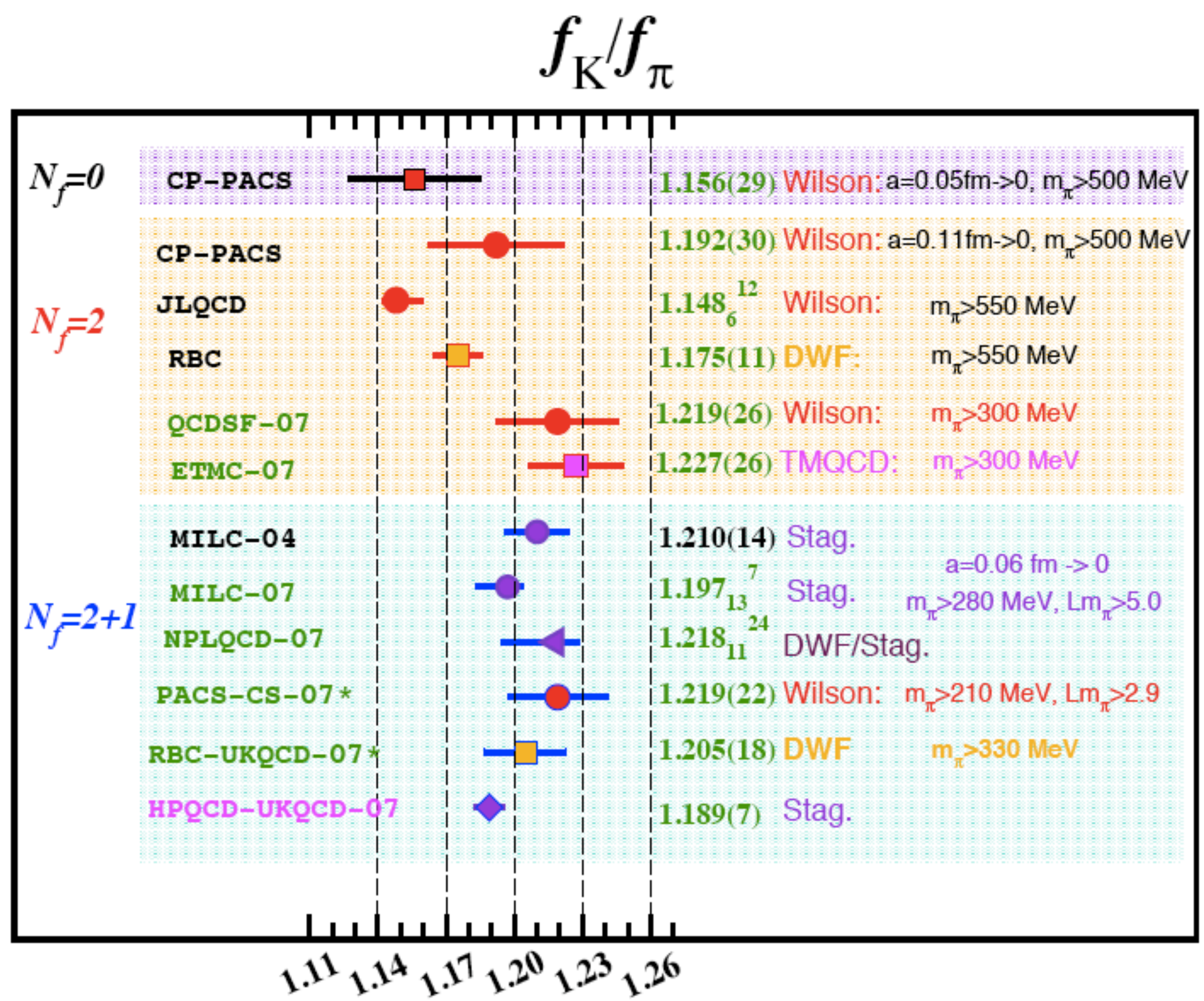
Marciano (2004)

New experimental results by Kloe, NA48/2

$$R_K = (2.457 \pm 0.032) \times 10^{-5},$$

$$\boxed{\left|\frac{V_{us}}{V_{ud}}\right| = 0.2321 \pm 0.0015}$$

Only K_{l2}



Unitarity of the first row

$$|V_{us}| = \sin\theta_C = \lambda = 0.2255(7) \qquad \text{[with unitarity]}$$

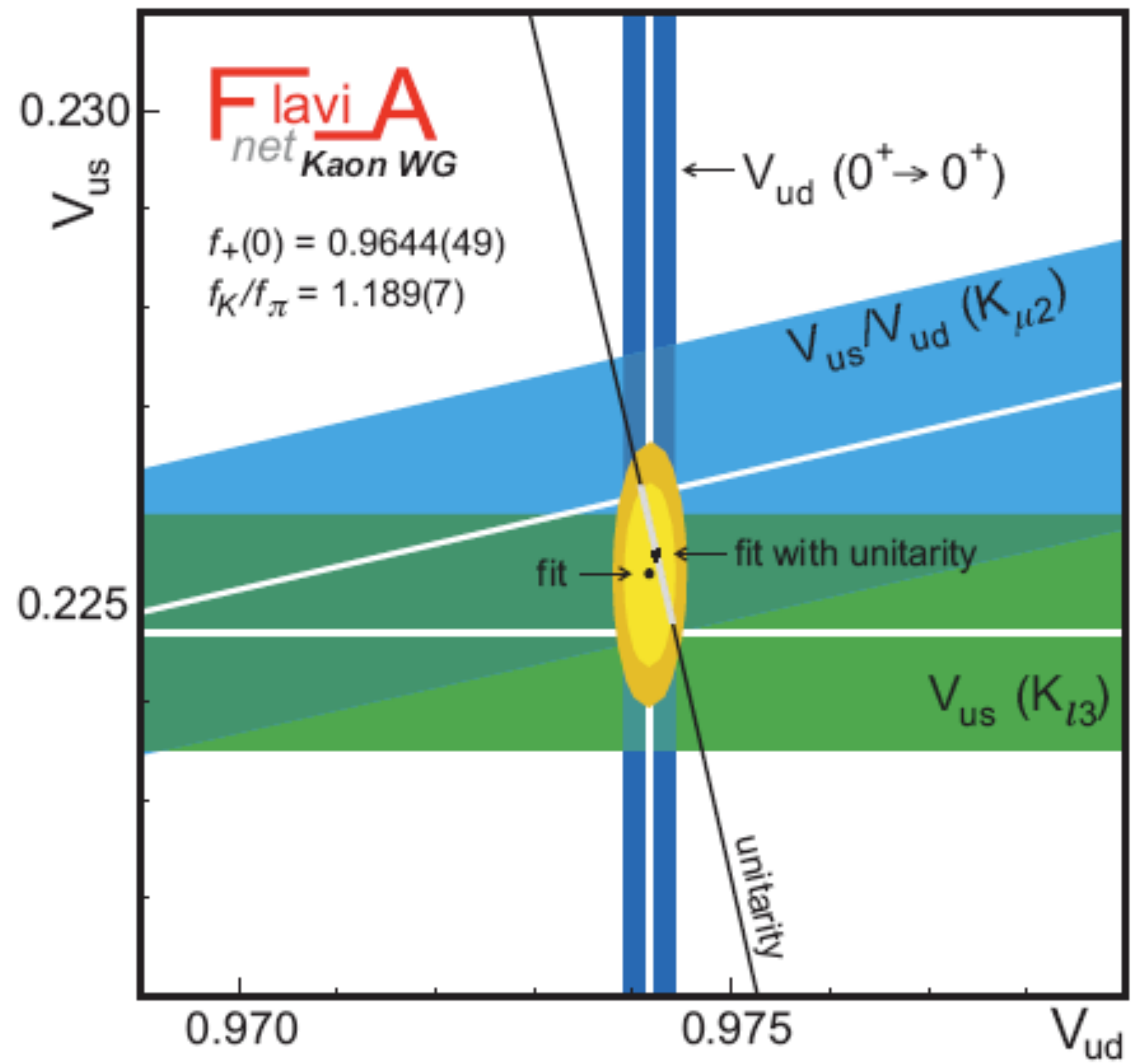
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 =$$

=0.9999±0.0004

Strong constraint on new physics

Tau decays also give
a λ determination with ~1% error.
Preliminary Belle and Babar data suggest
0.2165(26) but there are some doubts
on experimental analysis

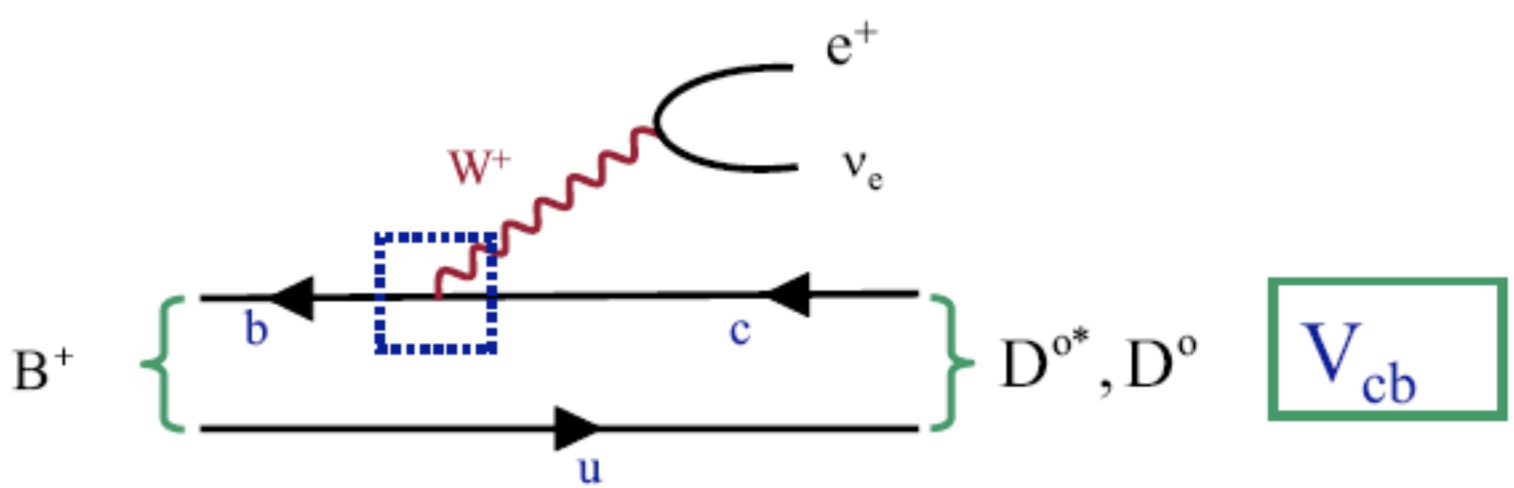
Gamiz et al 2007



Determination of A

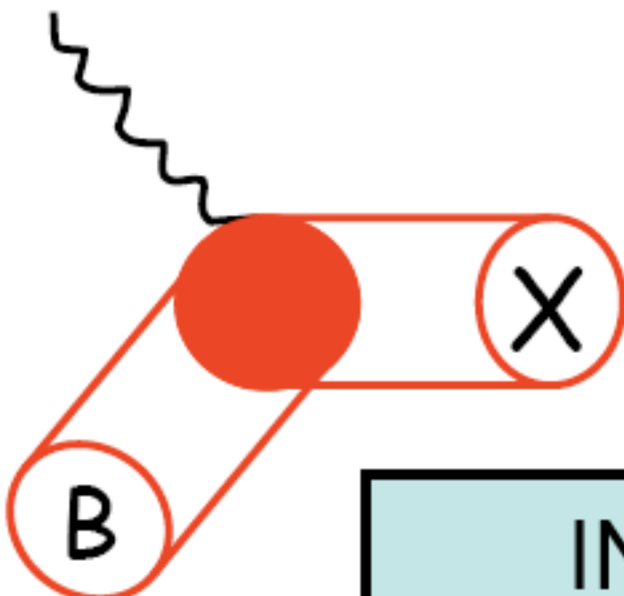
$$\hat{V} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\varrho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

A can be determined from V_{cb} or V_{ts}



Two roads to V_{cb}: inclusive and exclusive

Inclusive vs exclusive B decays



Simplicity: ew or em currents probe the B dynamics

INCLUSIVE	EXCLUSIVE
OPE: non-pert physics described by B matrix elemnts of local operators can be extracted by exp suppressed by $1/m_b^2$	Form factors: in general computed by non pert methods (lattice, sum rules,...) symmetry can provide normalization

Exclusive semileptonic B decays

➠talk by Kakuno

At zero recoil, where rate vanishes, the ff is

$$\mathcal{F}(1) = \eta_A(1 + \delta_{1/m^2})$$

Recent progress in the measurement of slopes and shape parameters Despite extrapolation, exp error ~2%

Main problem is normalization F(1): ***The non-pert quantities relevant for excl decays cannot be experimentally determined***

CKM 2003: $F(1) = 0.91^{+0.03}_{-0.04}$

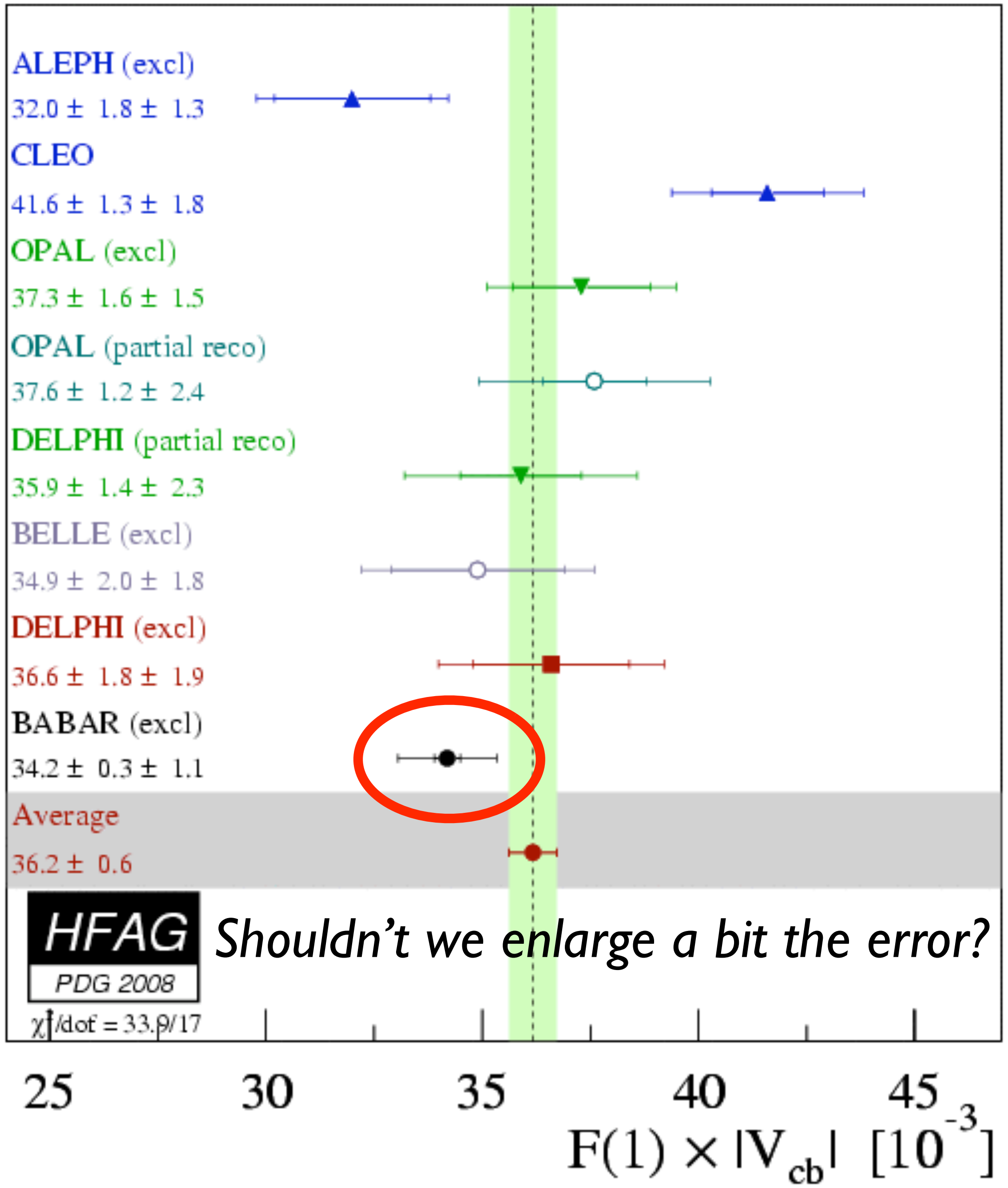
New unquenched Lattice QCD: F(1) =0.924(23)

Laiho et al

$|V_{cb}|=39.2(0.6)(1.0)\times10^{-3}$

~2σ from inclusive determination

Heavy Quark Sum rules give higher $|V_{cb}|$, F(1)=0.89(4)



Shouldn't we enlarge a bit the error?

B→Dlν gives consistent but less precise results; lattice control is better

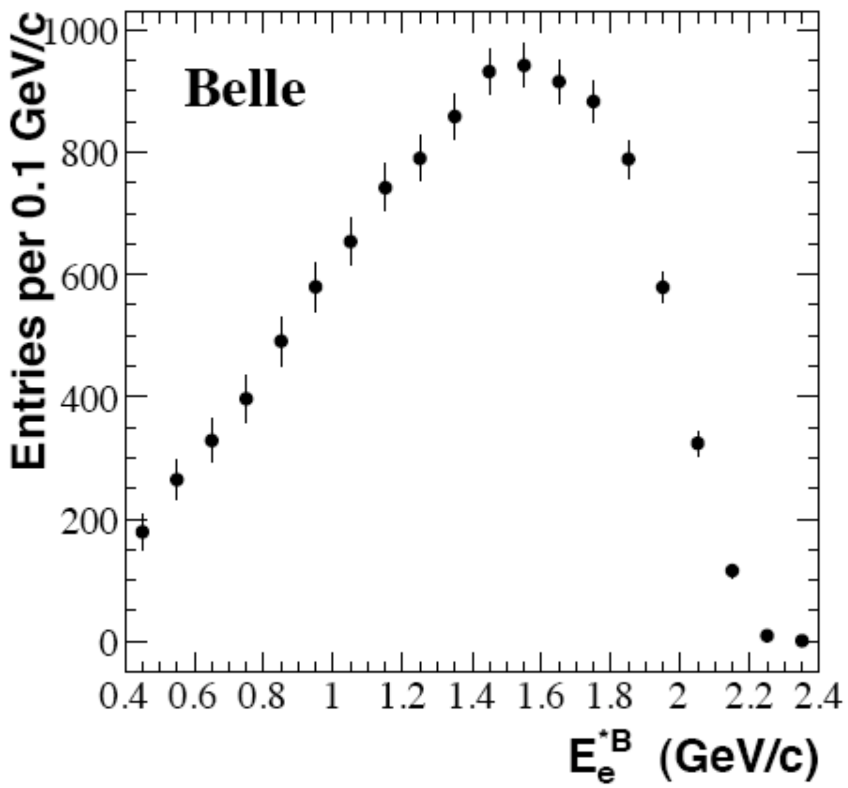
Inclusive $|V_{cb}|$: basic features

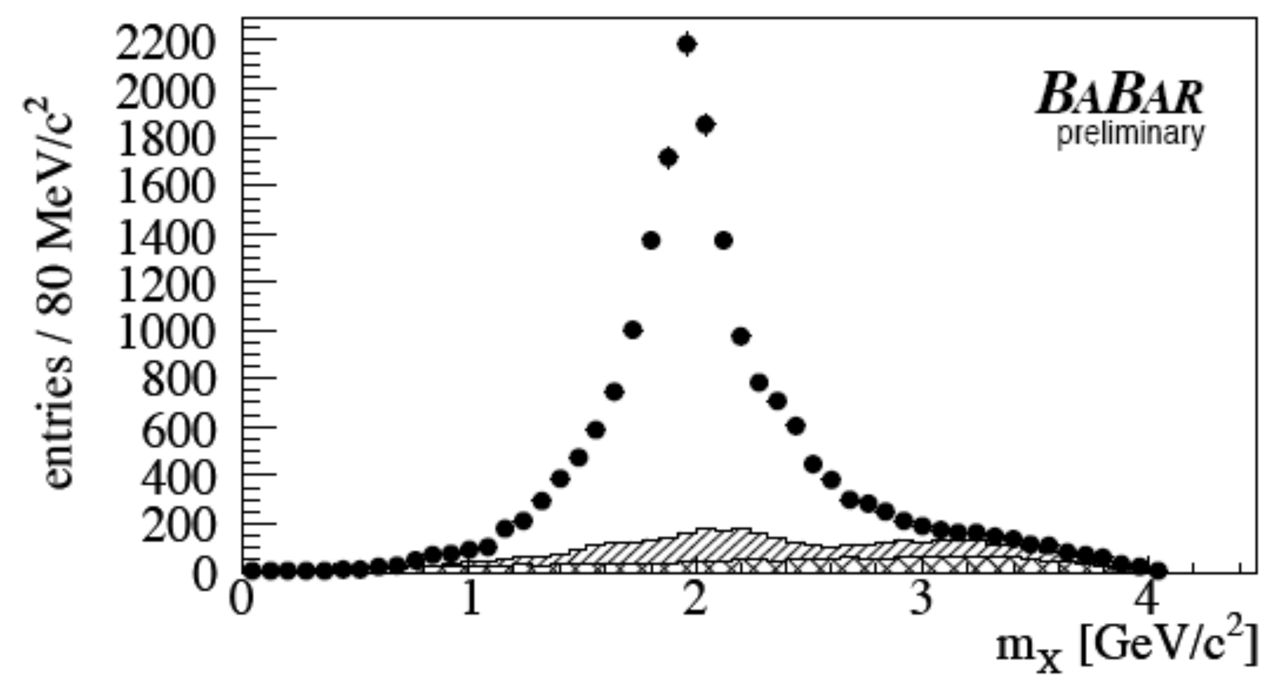
- ***Simple idea:*** inclusive decay do not depend on final state, factorize long distance dynamics of the meson. OPE allows to express it in terms of local operators
- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: ***double series in α_s, Λ/m_b***
- Lowest order: decay of a free *b*, linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

$$\mu_\pi^2(\mu) = \frac{1}{2M_B}\left\langle B \,|\, \bar{b}\,(i\vec{D})^2\, b \,|\, B\right\rangle_\mu \qquad \mu_G^2 = \frac{1}{2M_B}\langle B|\bar{b}\frac{i}{2}\sigma_{\mu\nu}G_{\mu\nu}b|B\rangle$$

Fitting OPE parameters to the moments

➟talk by Rotondo





*Total **rate** gives CKM elmnts; global **shape** parameters (moments of the distributions) tell us about B structure*

HQE parameters describe universal properties of the B meson and of the quarks

***Perturbative scheme**: $O(\alpha_s)$ Wilson coefficients depend on the exact definition of OPE parameters. They should be all short-distance parameters*

***In the kinetic scheme** the contributions of gluons with energy below $\mu \approx 1$GeV are absorbed in the OPE parameters*

NB In the fits “scheme” means also a number of different assumptions and a recipe for theory errors

Global fit (kinetic scheme)

Buchmüller-Flächer, new

New December 2007

with BaBar’s

Breco $B \to X_s\gamma$ and M_n^H

Babar $M_{1,2}^{\gamma} \times 3$ (1.9–2.0 GeV)

Babar $M_{1,2}^{H}$ (0.9–1.5 GeV(?))

Babar $M_{0,1,2,3}^{\ell}$ (0.6–1.5 GeV)

Belle $M_{1,2}^{\gamma}$ (1.8–2.0 GeV)

Belle $M_{1,2}^{H}$ (0.7–1.3 GeV)

Belle $M_{0,1,2,3}^{\ell}$ (0.6–1.4 GeV)

CLEO M_{1}^{γ} (2.0 GeV)

CLEO $M_{1,2}^{H}$ (1.0–1.5 GeV)

DELPHI $M_{1,2,3}^{H}$ (0 GeV)

DELPHI $M_{1,2,3}^{\ell}$ (0 GeV)

CDF $M_{1,2}^{H}$ (0.7 GeV)

old HFAG $\mathcal{B}$ (0.6 GeV)

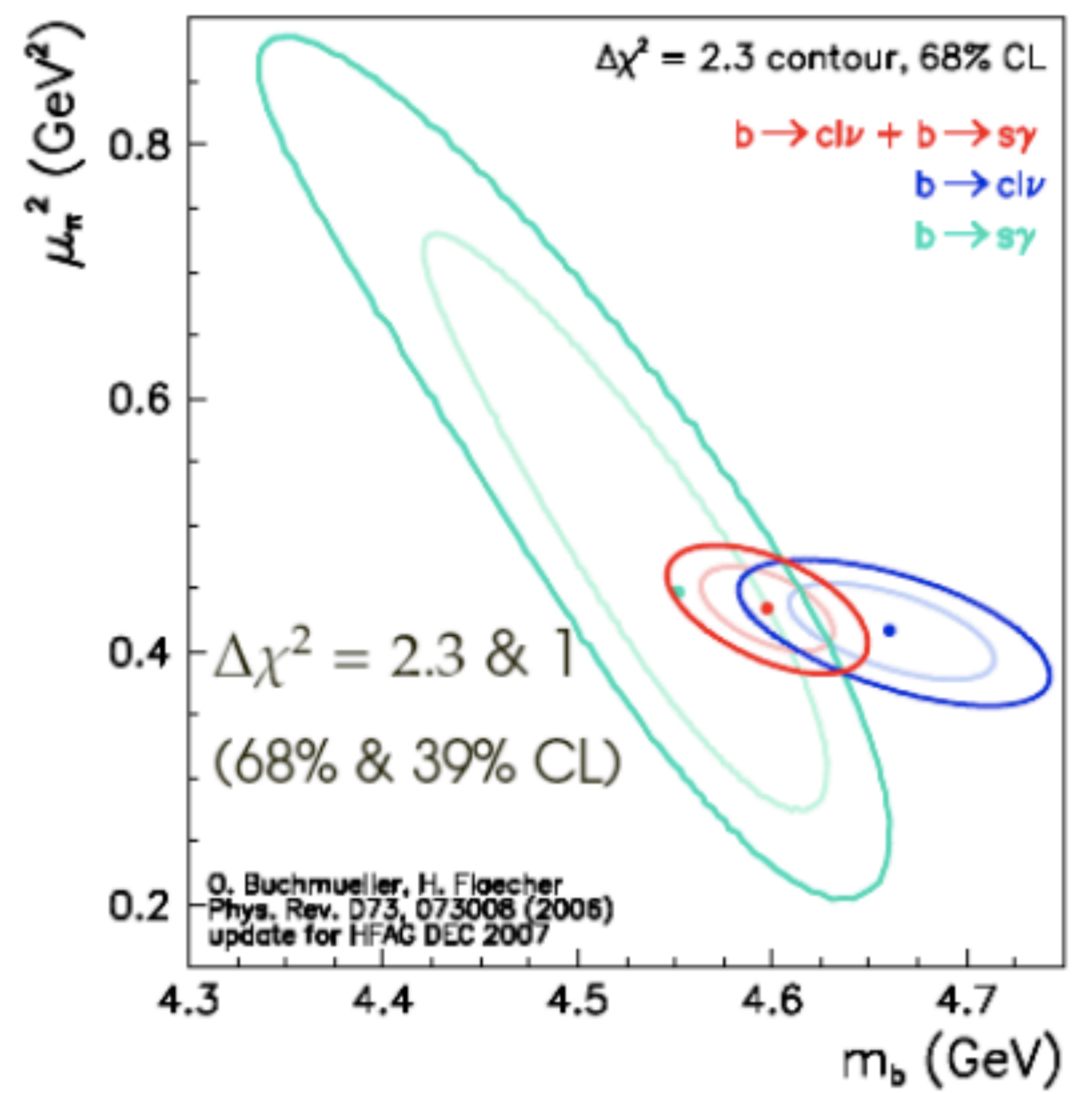
$$|V_{cb}| = (42.04 \pm 0.34_{\rm fit} \pm 0.59_{\Gamma_{\rm sl}}) \times 10^{-3}$$

$$m_b^{\rm kinetic} = 4.597 \pm 0.034_{\rm fit}\ \text{GeV}$$

$$m_c = 1.1634 \pm 0.051_{\rm fit}\ \text{GeV}$$

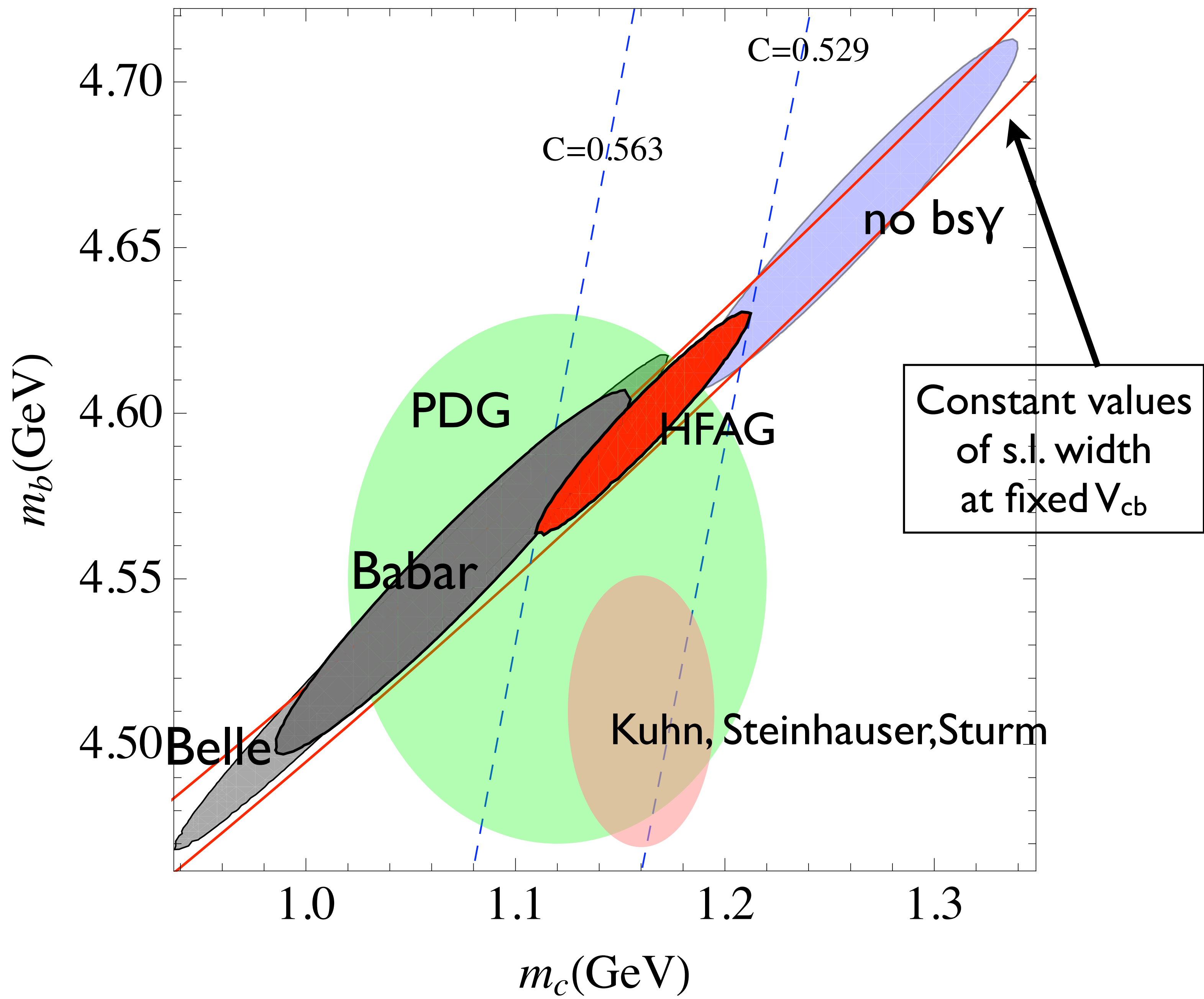
$$\mu_\pi^2 = 0.4341 \pm 0.033_{\rm fit}\ \text{GeV}^2$$

$$\rho_D^3 = 0.2927 \pm 0.020_{\rm fit}\ \text{GeV}^2$$



Fits & Quark Masses

- Assumes duality but it self-consistently checks it
- Very close results for $|V_{cb}|$ in 1S scheme (Bauer et al).
- **Higher order** power corr. under control Mannel et al
- new pert $O(\alpha_s^2)$ ⇒**-0.5% in $|V_{cb}|$**
  Melnikov, Czarnecki, Pak
- part of $O(\alpha_s/m_b)$ Becher et al
- **Fitted $|V_{cb}|$ stable, not so the masses**
- In the global HFAG fit the $B \to X_s \gamma$ moments **change significantly $m_{b,c}$** determinations, but not in Babar & Belle fits... Without radiative moments the masses are too high! Radiative moments have subleading contributions without OPE!



Upgrade of moments code kin scheme under way

The Unitarity Triangle

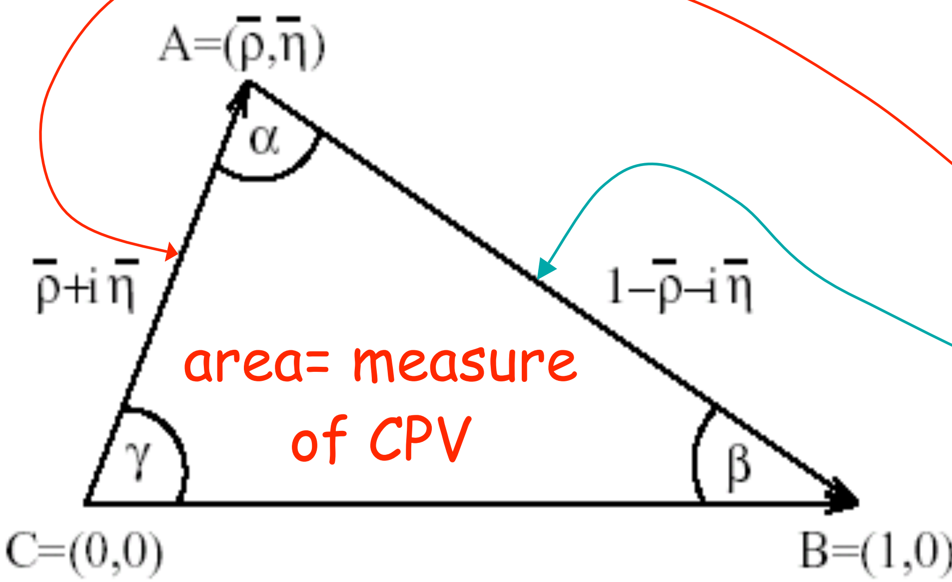
$$L_W = -\frac{g}{2\sqrt{2}} V_{ij}\,\bar{u}_i \gamma^\mu W^+_\mu (1-\gamma^5) d_j + \text{ h.c.}$$

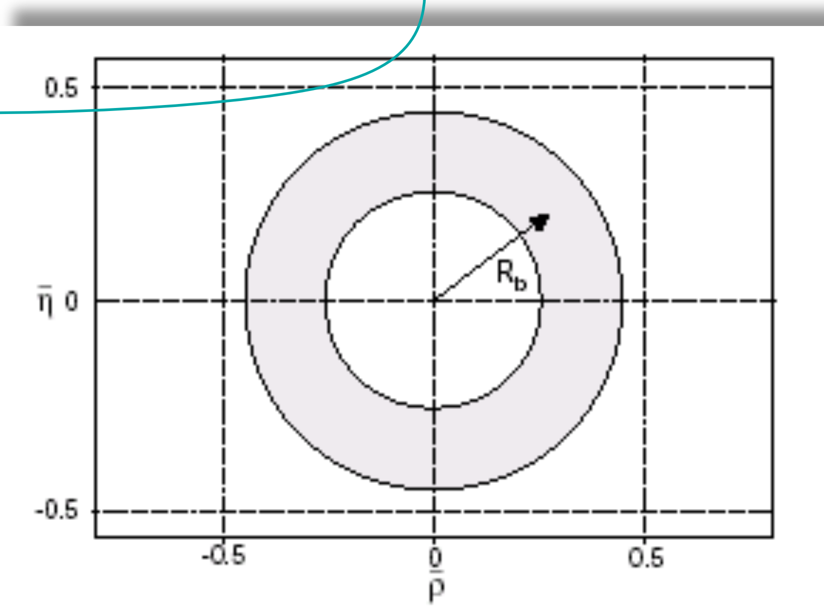
Unitarity determines
several triangles
in complex plane

$$V_{ji} V^*_{jk} = \delta_{ik}$$

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0 \qquad O(\lambda^3)$$

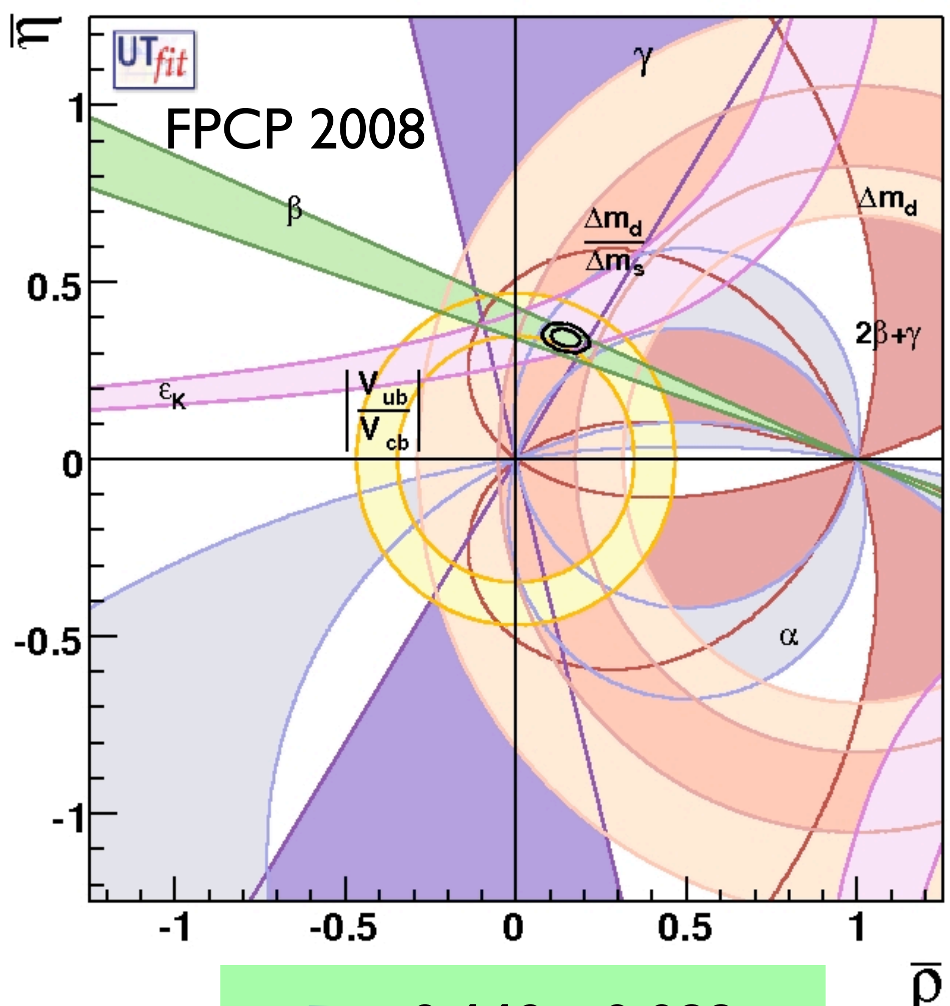
$$1 + \frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}} + \frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}} = 0$$





V_{td} cannot be accessed directly:
we resort to loop transitions
FCNC sensitive to new physics

The Unitarity Triangle

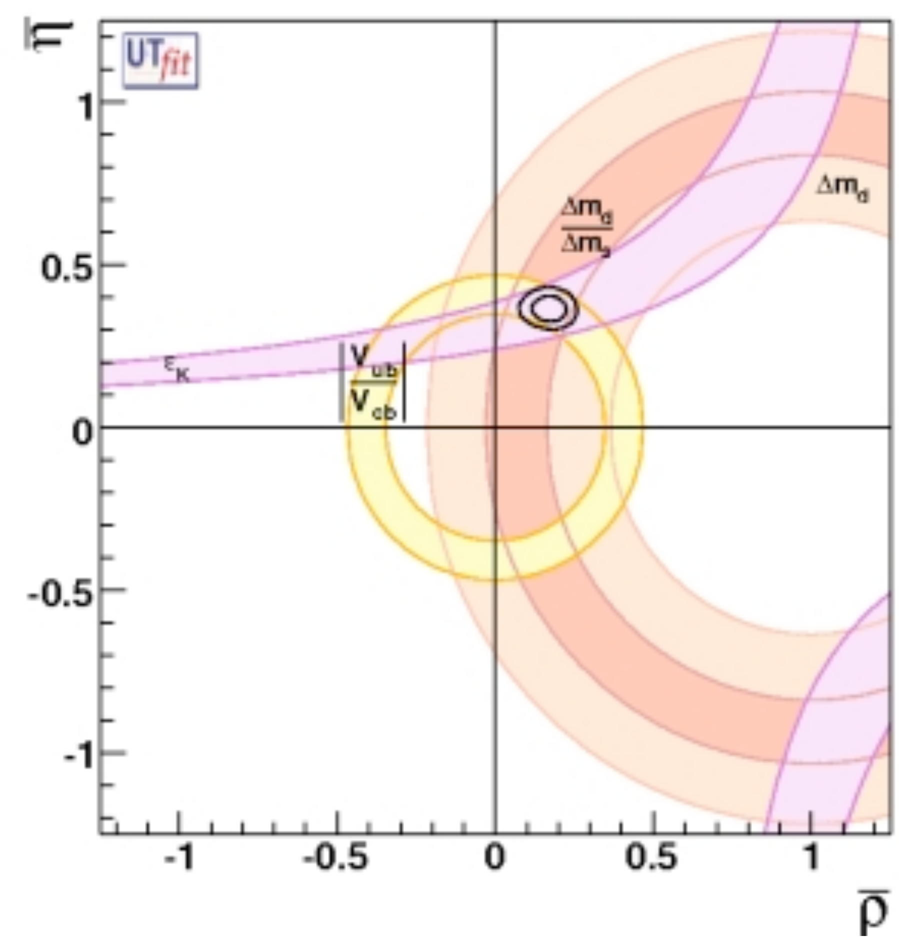


$\bar{\rho}$ **= 0.146 ± 0.028**

$\bar{\eta}$ **= 0.342 ± 0.016**

Almost identical results by CKMfitter @ Moriond 2008

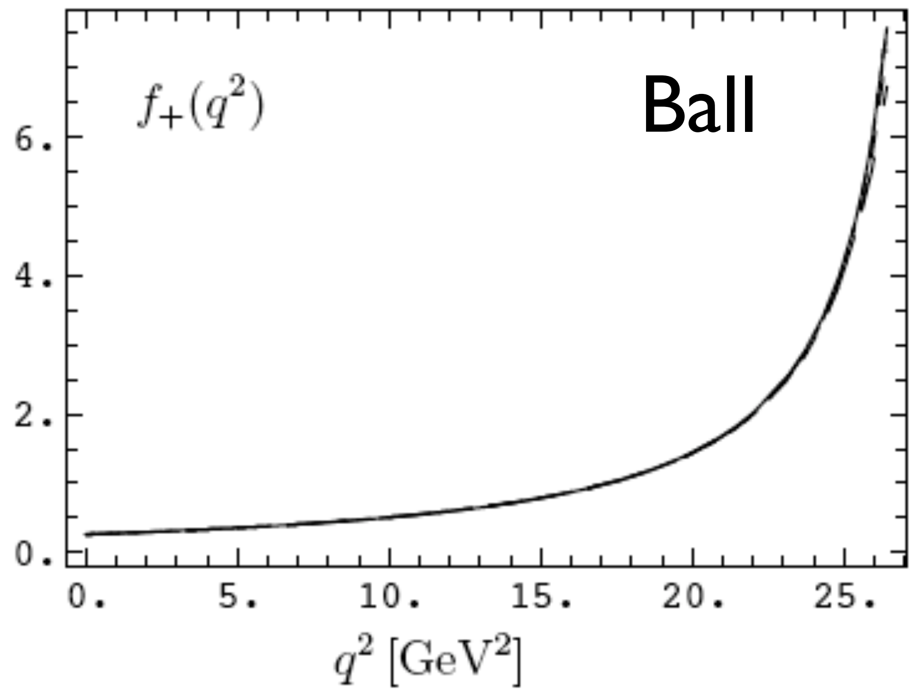
$\sin 2\beta = 0.668 \pm 0.028$



UTfit inputs:
ξ=1.23(6) B_K=0.79(4)(8)

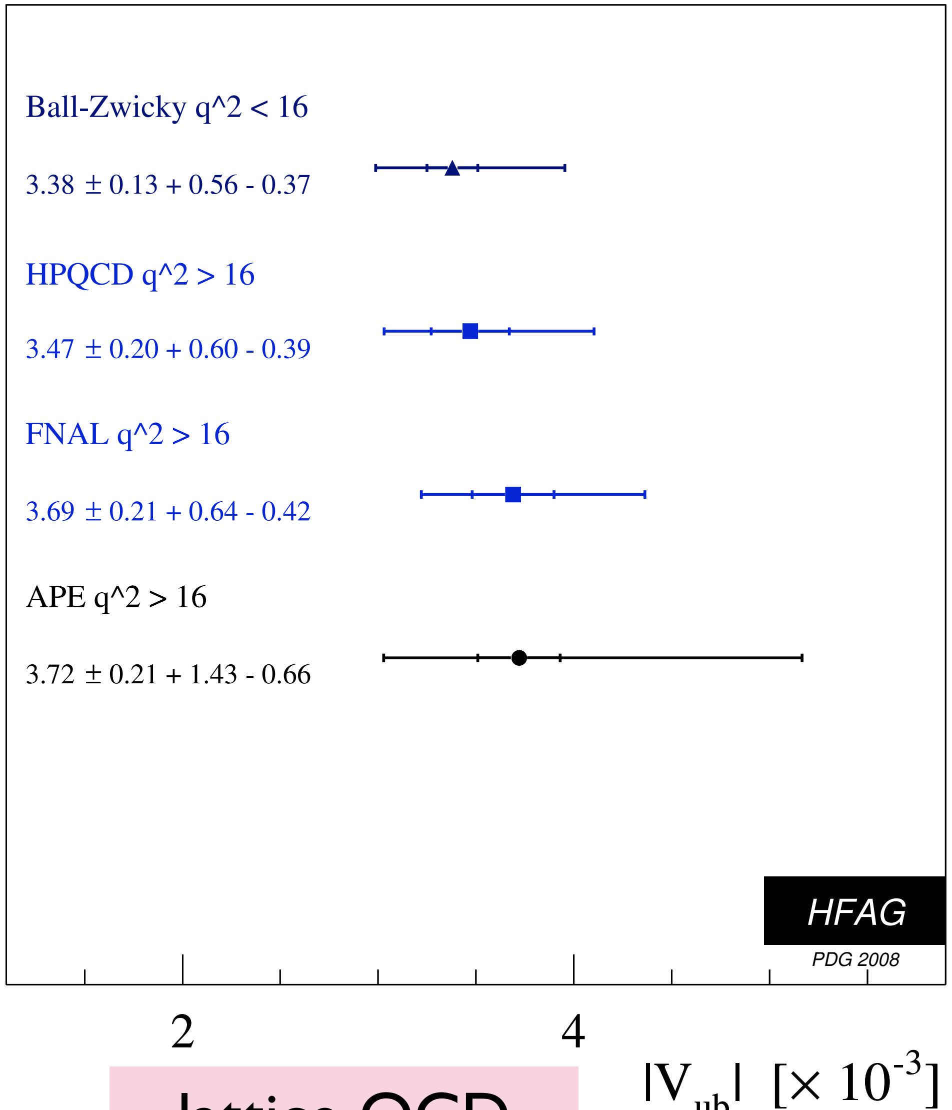
$\hat{B}_K = 0.720(39)$ Juttner LAT07
getting closer to 5% accuracy?
➠ talk by Gamiz

Exclusive $|V_{ub}|$



Various parameterizations based on analyticity etc + experimental data on the q^2 spectrum: model independently

$$|V_{ub}| f_+(0) = (9.1 \pm 0.6(\text{shape}) \pm 0.3(\text{BR})) \times 10^{-4}$$



ff on lattice or with LC sum rules, no symmetry helps
LCSR cannot do much better

lattice QCD
➠ talk by Gamiz

LCSR: $|V_{ub}| = (3.5 \pm 0.4 \pm 0.1) \times 10^{-4}$

Ball-Zwicky

$$|V_{ub}| = \left(3.5 \pm 0.4\big|_{th} \pm 0.2\big|_{shape} \pm 0.1\big|_{BR}\right) \times 10^{-3}$$

Duplancic et al

$|V_{ub}|$ inclusive

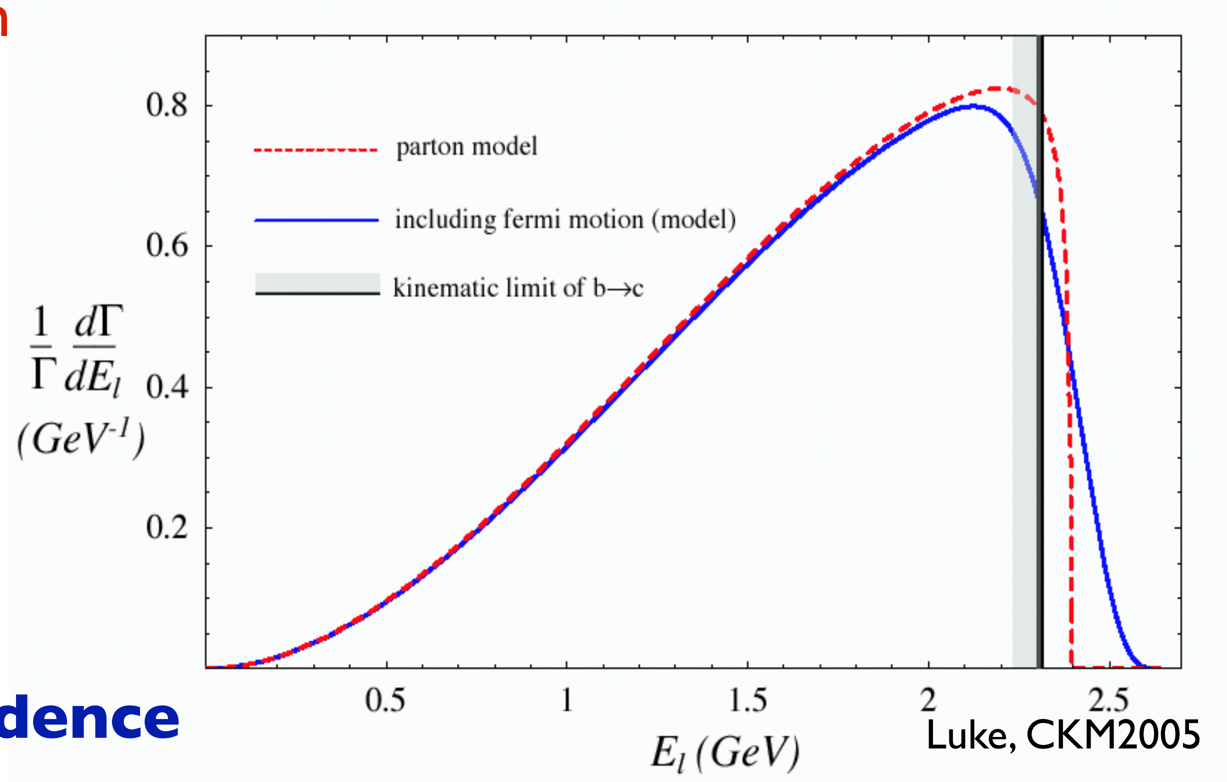
$|V_{ub}|$ from total BR($b \to u l \nu$) like incl $|V_{cb}|$ but we need kinematic cuts to avoid the ~100x larger $b \to c l \nu$ background:

$m_X < M_D$ $E_l > (M_B^2 - M_D^2)/2M_B$ $q^2 > (M_B - M_D)^2$ ...

or combined (m_X, q^2) cuts

The cuts destroy convergence of the OPE that work so well in $b \to c$.
OPE expected to work only away from pert singularities

Rate becomes sensitive to "local" b-quark wave function properties like Fermi motion Dominant non-pert contributions can be resummed into a **SHAPE FUNCTION** f(k+)



Luke, CKM2005

In all approaches strong dependence on input value of m_b

SF from perturbation theory

The photon–energy spectrum: resummed perturbation theory

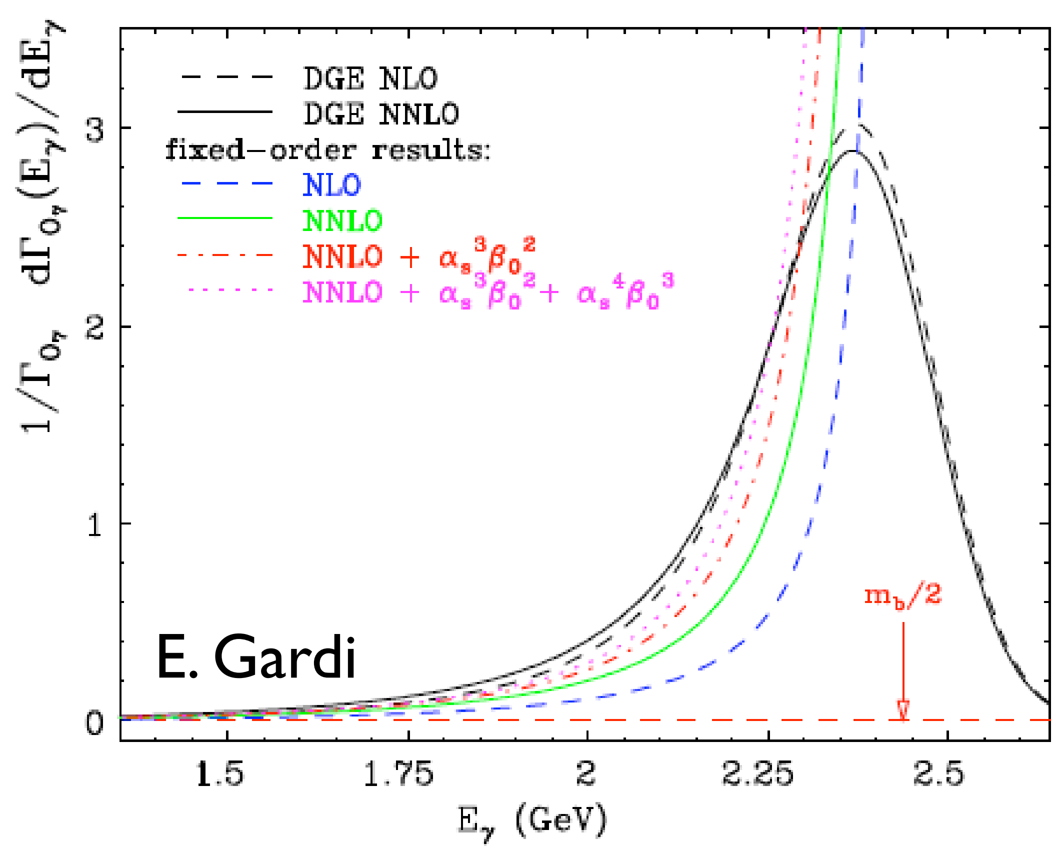
Resummed perturbation theory is qualitatively different: Support properties; stability!

Power corrections are small: resummed perturbation theory yields a good approximation to the meson decay spectrum

b *quark* SF emerges from resummed pQCD but needs an IR prescription and power corrections for b → B

Dress Gluon Exponentiation (DGE) by Gardi et al employs renormalon resummation to define Fermi motion. Power corrections can be partly accomodated.

Aglietti et al use Analytic Coupling (AC) for the IR and no power corrections: it is a model



$|V_{ub}|$ from DGE

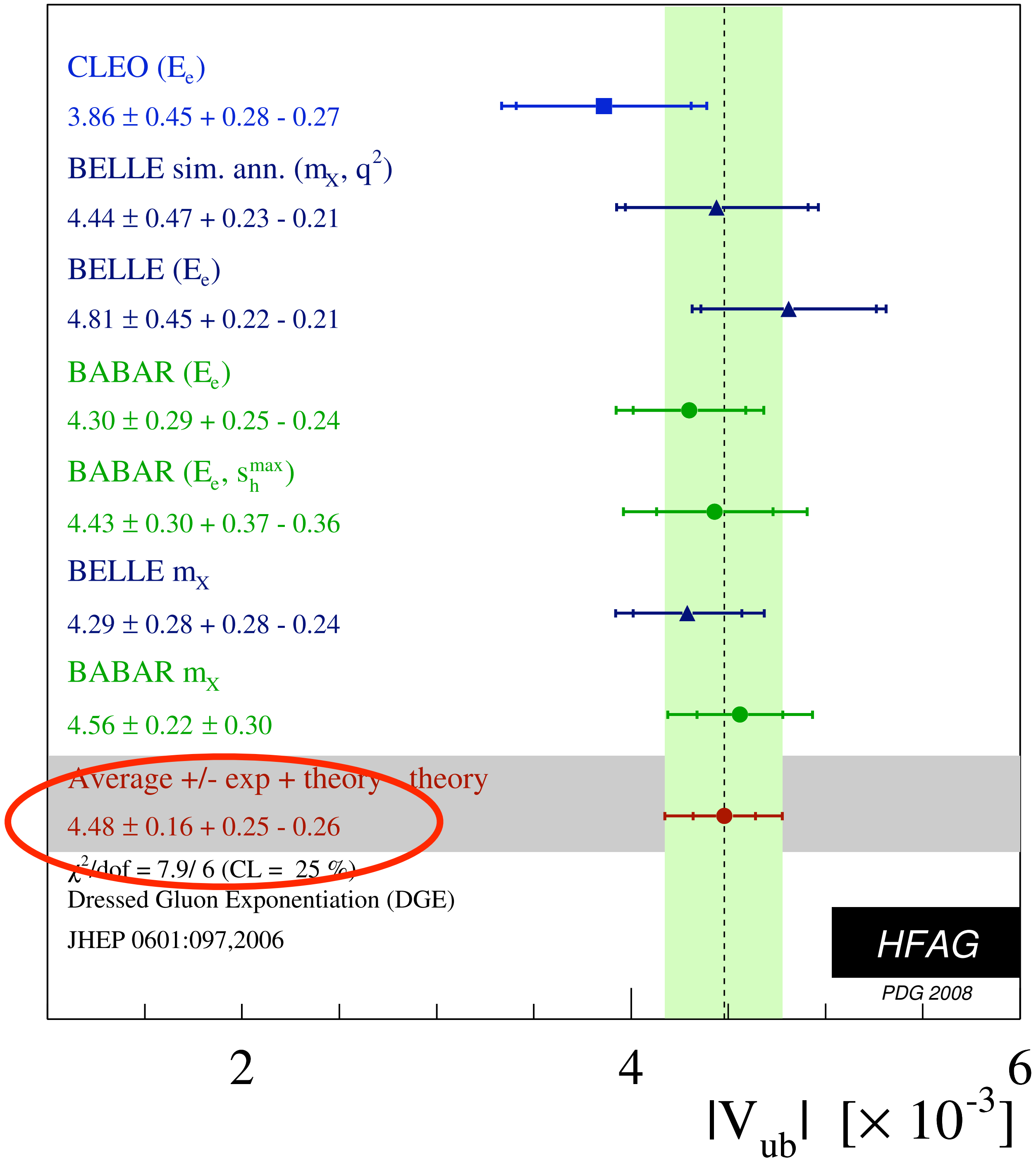
Even in the absence of extra power corrections the main features of the spectra are reproduced ➠$|V_{ub}|$ stable

Only input other than α_s $m_b(m_b)$=4.20(7) PDG

small m_b ⇔ high $|V_{ub}|$

central value ~4.15x10^{-3} with moment fit m_b

6.8% total error

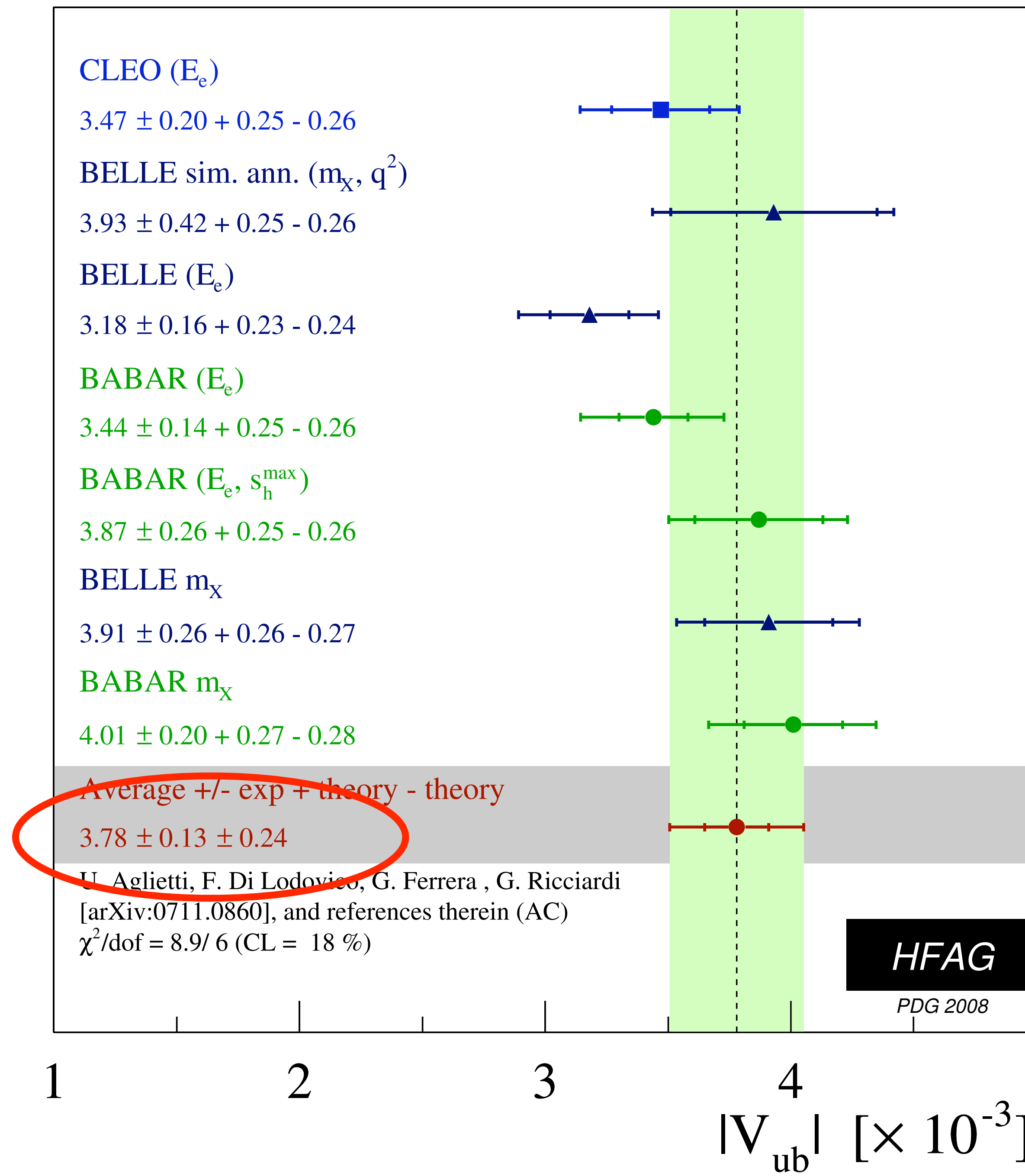


$|V_{ub}|$ from AC

Good consistency also here.
b & c masses from PDG but difficult to compare because normalized to BR($B \to X_c l \nu$)

Look at E_l cuts higher than 2.3GeV because their E_l apparently does not reproduce data

~7% total error
(no model error)



Parameterizing the SF in the OPE

Fermi motion can be parameterized within the OPE like PDFs in DIS. At leading order in m_b only a single function of one parameter enters (SF). Beyond LO, there are more and the q^2 dependence cannot be neglected.

$$\frac{d^3\Gamma}{dq^2\, dq_0\, dE_\ell} = \frac{G_F^2 |V_{ub}|^2}{8\pi^3} \left\{ q^2 W_1 - \left[2E_\ell^2 - 2q_0 E_\ell + \frac{q^2}{2} \right] W_2 + q^2 (2E_\ell - q_0) W_3 \right\}$$

$$W_i(q_0, q^2) = m_b^{n_i}(\mu) \int dk_+ \; F_i(k_+, q^2, \mu) \; W_i^{pert} \left[q_0 - \frac{k_+}{2} \left(1 - \frac{q^2}{m_b M_B} \right), q^2, \mu \right]$$

This factorization formula perturbatively defines the distribution functions

$$\int dk_+ \; k_+^n \; F_i(k_+, q^2) =$$

local OPE prediction $\Leftarrow$ moments fits

needs ansatz for functional form.

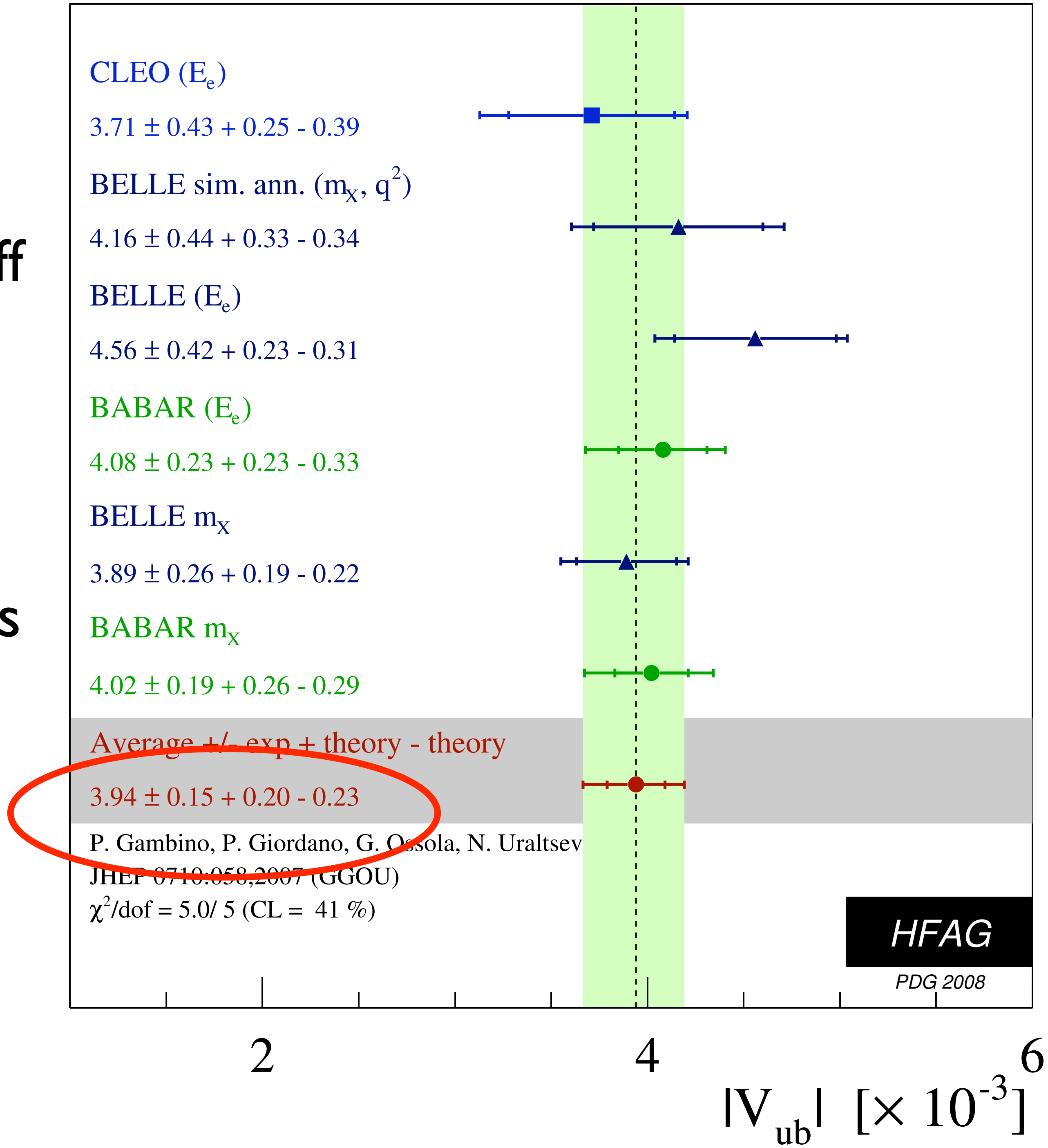
$|V_{ub}|$ in the kinetic scheme -GGOU

PG,Giordano,Ossola,Uraltsev

Good consistency & small th error.
OPE in a scheme with Wilsonian IR cutoff ~1GeV, all subleading $1/m_b$ and $O(\alpha_s^2\beta_0)$ terms consistently included,
careful treatment of high q^2 tail.

Inputs from **global fit** to the moments

+6.3-7.0% total error



$|V_{ub}|$ in BLNP Bosch,Lange,Neubert,Paz

Good consistency. Uses elegant multiscale OPE that resums soft-collinear logs, but plethora of largely unconstrained subleading SFs

$$\boxed{d\Gamma = HJ\otimes \hat{S} + \frac{1}{m_b} H_i' J_i' \otimes \hat{S}_i' + \dots}$$

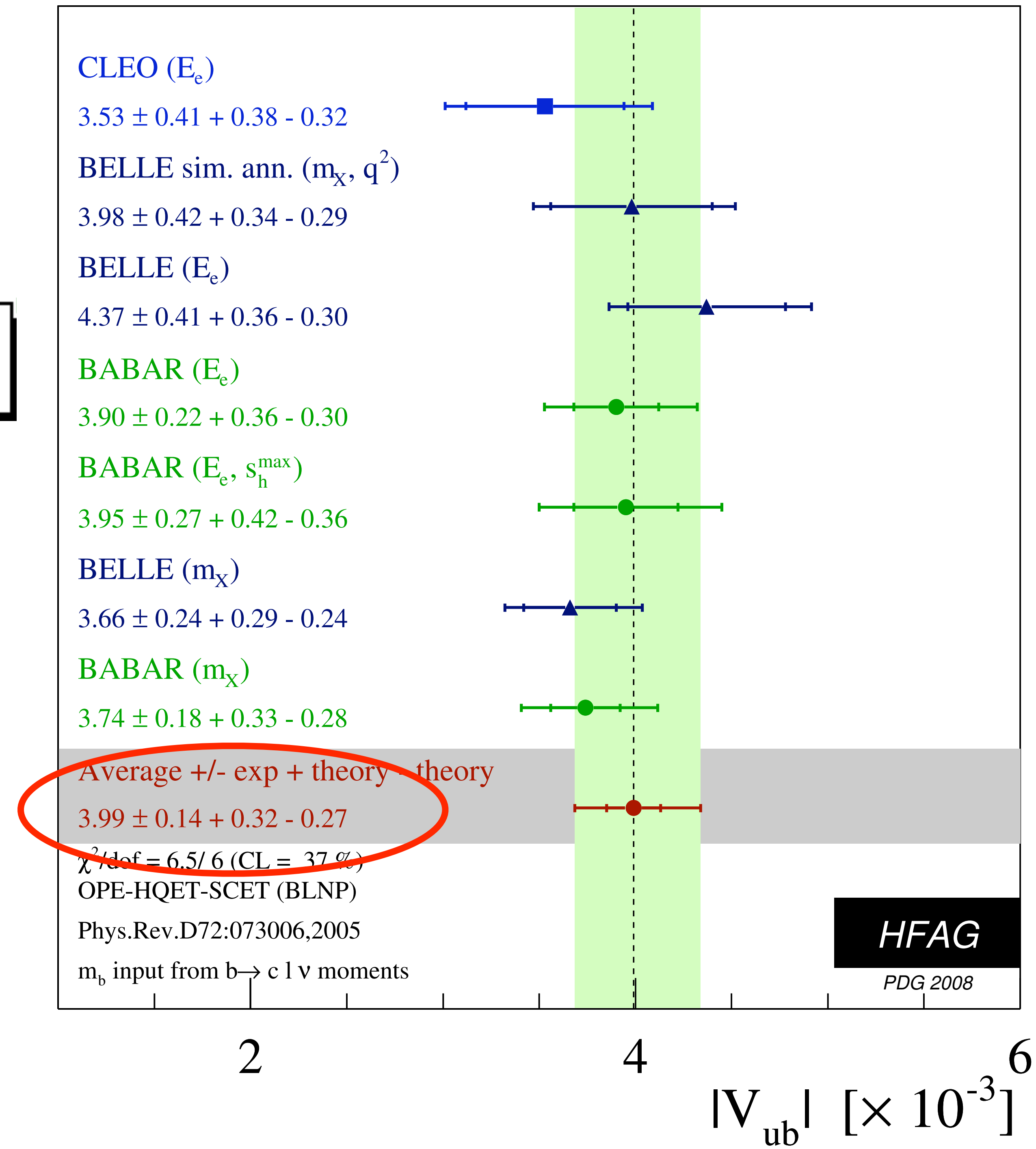
m_b taken from moments fit *without radiative moments*

➠ higher m_b with **larger** error

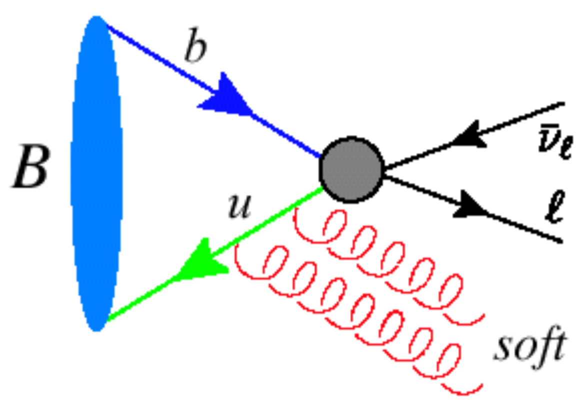
high m_b ⇔ low $|V_{ub}|$

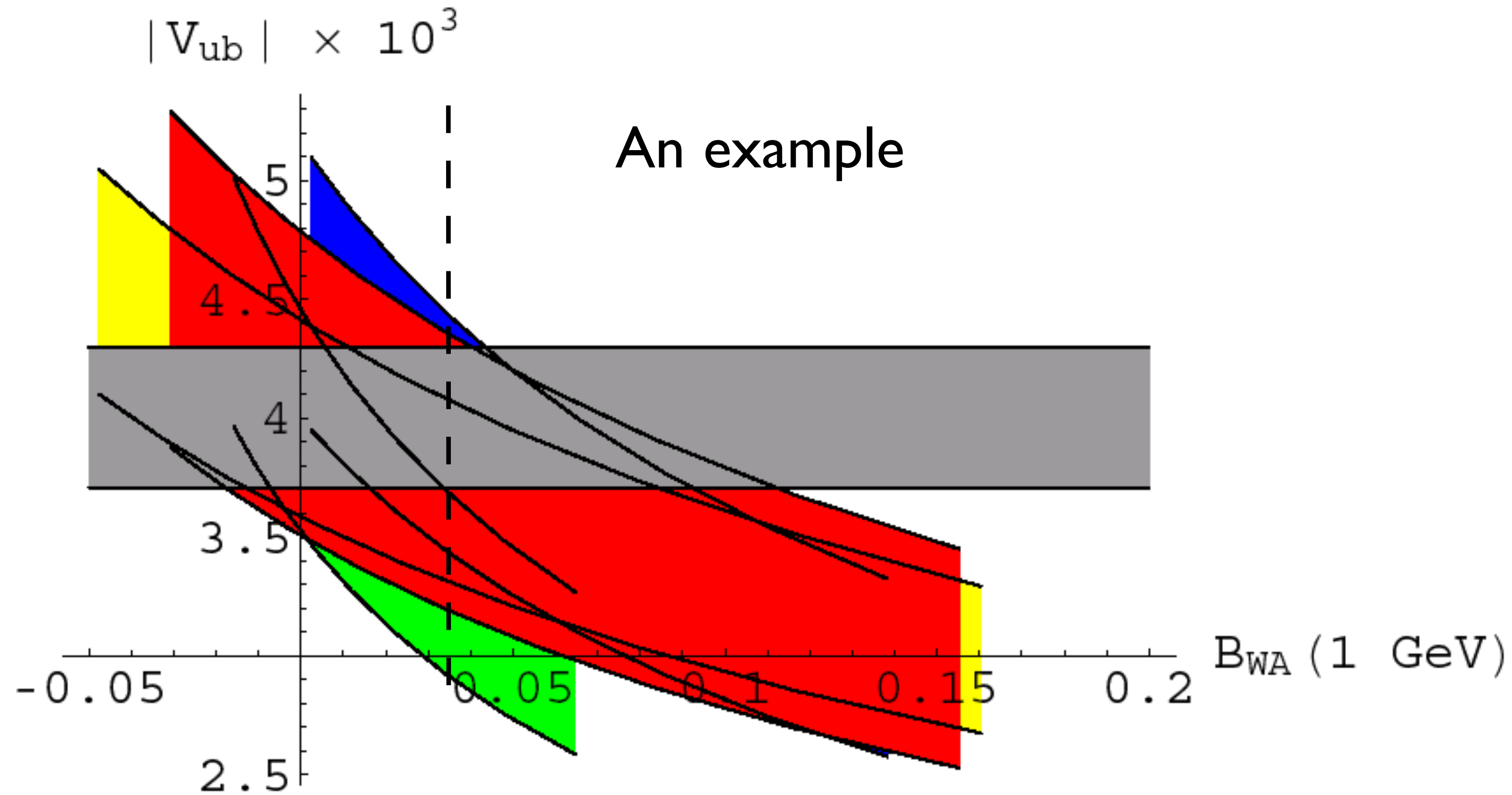
central value ~4.15x10^{-3} with moment fit m_b

~8% total error



Constraining Weak Annihilations





*WA happen at max q^2, may pollute all present estimates, and tend to **decrease** the extracted V_{ub}. Present bounds from CLEO and Babar are not yet at the required level.*

*Analyses with an **upper cut on q^2** are crucial to remove this uncertainty, see Babar*

V_{ub} comments

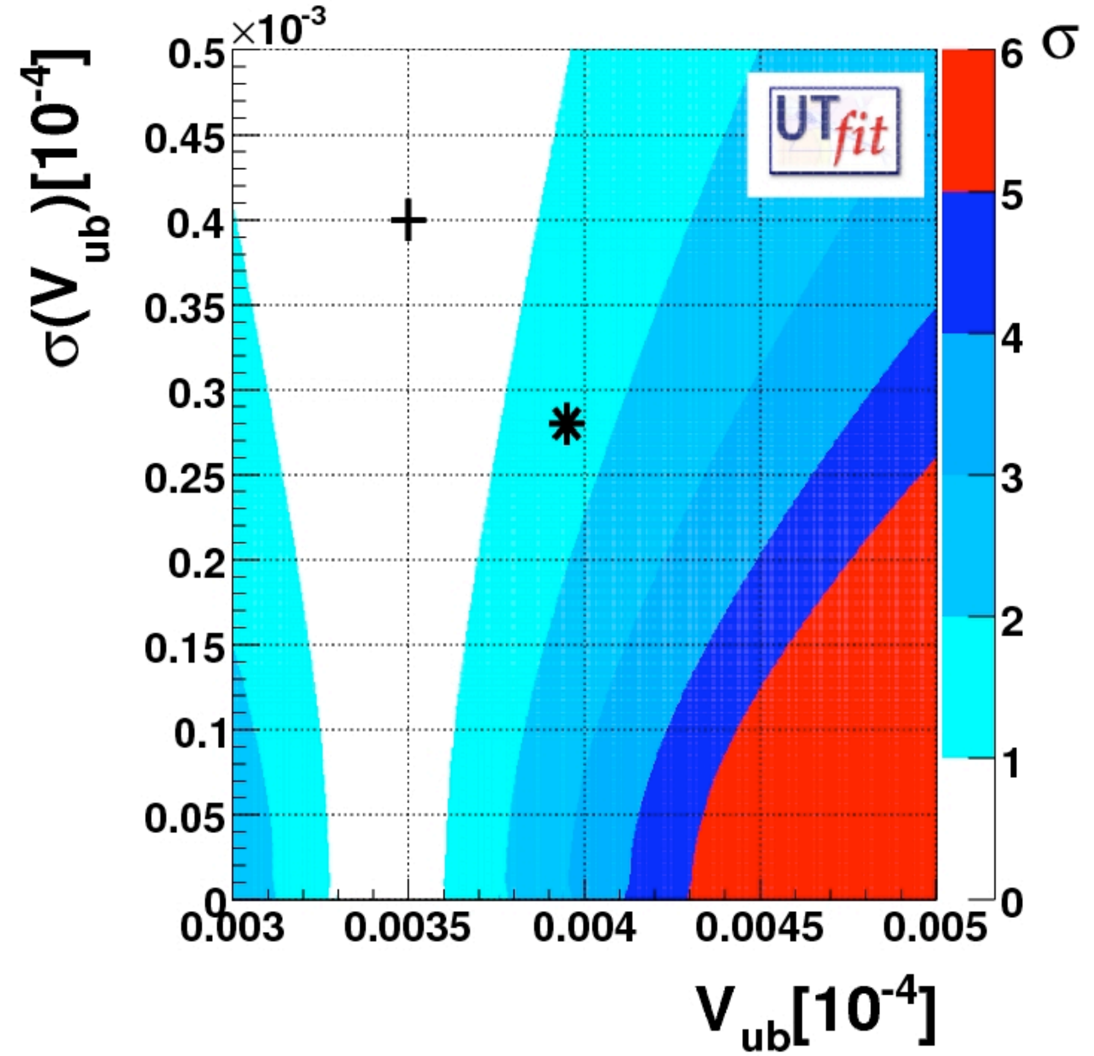
- All frameworks interesting, *not all equivalent* for a precise determination of $|V_{ub}|$
- *Not all observables are equivalent*, some cleaner. For ex high q^2 tail is sensitive to WA: drop it until WA is known!
- Need spectra and/or analysis with varying cuts: only way to test frameworks
- More inclusive measurements decrease the dependence of the results on m_b
- Parametric errors are (largely) experimental: let's agree on which input parameters to use and start quoting them as exp. HFAG has presently 4 incoherent values for V_{ub}

Conclusions

- **CKM is in a great shape!**
- The **K renaissance** has brought us the Cabibbo angle at 0.5%
- **Exclusive and inclusive $|V_{cb}|$** disagree by ~2σ if we take the latest FNAL lattice result (needs confirmation).
- Angles determinations agree well.
- My best $|V_{ub}|$ value (from M_X cut only):

  $|V_{ub}|=(3.95\pm0.17^{+0.20}_{-0.23})\times10^{-3}$

  from GGOU (~BLNP) with m_b=4.61(35)GeV
- ***No discrepancy*** *with exclusive determination,* ***slight tension*** *with UT determination*
- *Need better and safe* ***m_b determinations****, an*
  to use ***data*** *to test theories and models...: information on* ***spectra*** *is crucial*



thanks to Marcella Bona