Theoretical review of exclusive rare radiative decays

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FPCP 2008, Taiwan

May 2008

Outline

1) $B \to V\gamma$ decays in QCD factorization and SCET

- sample results

2) Improving factorization predictions

- NNLO perturbative corrections
- annihilation at order α_s
- form factor uncertainties

Rare radiative $B \to V\gamma$ decays

Examples: $B \to (\rho, K^*, \omega, \phi)\gamma$ decays

- all involve FCNC
- potential to constrain new physics and CKM parameters

Observables and their relevance

- branching fractions $\leftrightarrow |V_{td}/V_{ts}|$
- CP asymmetries $\leftrightarrow$ new physics, α
- isospin violation $\leftrightarrow$ new physics, γ

Experimental status

Weighted branching fractions in units of 10^{-6}

$$
\begin{aligned}
\mathcal{B}(B \to K^*\gamma) &= 41.8 \pm 1.7 \qquad \text{(HFAG)} \\
\mathcal{B}(B \to (\rho,\omega)\,\gamma) &= 1.28 \pm 0.30 \qquad \text{(HFAG)} \\
\mathcal{B}(B_s \to \phi\,\gamma) &= 57 \pm 22 \qquad \text{(Belle)}
\end{aligned}
$$

These and CP, isospin asymmetries will become more precise at B factories and at LHCb

$\Rightarrow$ improving theory predictions useful and relevant

Theoretical challenge: hadronic matrix elements

Amplitude for $b \to s\gamma$ transitions:

$$\mathcal{A} \sim \langle V\gamma|\mathcal{H}_{\text{eff}}|\bar{B}\rangle \sim \sum_{p=u,c} V_{ps}^* V_{pb} \sum_{i=1}^{8} C_i \langle V\gamma|Q_i^p|\bar{B}\rangle$$

- **Main challenge**: evaluate $\langle V\gamma|Q_i|\bar{B}\rangle$ = hadronic matrix elements

Most important operators:

$$Q_1^p = (\bar{s}\,p)_{V-A}\,(\bar{p}\,b)_{V-A} \qquad Q_2^p = (\bar{s}_i\,p_j)_{V-A}\,(\bar{p}_j\,b_i)_{V-A}, \qquad (p = u, c)$$

$$Q_7 = -\frac{e\,\overline{m}_b}{8\pi^2}\,\bar{s}\,\sigma^{\mu\nu}\,[1+\gamma_5]\,bF_{\mu\nu}, \qquad Q_8 = -\frac{g\,\overline{m}_b}{8\pi^2}\,\bar{s}\,\sigma^{\mu\nu}\,[1+\gamma_5]\,T^a\,bG^a_{\mu\nu}$$

For $b \to d\gamma$ replace $s \to d$

Theoretical approaches

- QCD factorization

  (Ali, Parkhomenko; Bosch, Buchalla; Beneke, Feldmann, Seidel)
- QCD factorization + QCD sum rules

  (Ball, Jones, Zwicky)
- SCET $\simeq$ QCD factorization + resummation

  (Chay, Kim; Grinstein, Grossman, Ligeti; Becher, Hill, Neubert)
- pQCD

  (Keum, Matsumori, Sanda, Yang)

Talk will focus on QCD factorization-based approaches

QCD factorization

Matrix elements of Q_i obtained as a series in $\alpha_s,\ \Lambda_{\rm QCD}/m_b \ll 1$

$$\langle V\gamma\,|Q_i|\,\bar{B}\rangle = t_i^{\rm I}\,\zeta_{V_\perp} + t_i^{\rm II}\star\phi_+^B\star\phi_\perp^V + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

- $\zeta_{V_\perp}$ (form factor) and $\phi^{B,V}$ (LCDAs) are non-perturbative
- $t^{\rm I}$ and $t^{\rm II}$ are perturbative hard-scattering kernels

$$t^{\rm I} = \mathcal{O}(1) + \mathcal{O}(\alpha_s) + \dots \qquad t^{\rm II} = \mathcal{O}(\alpha_s) + \dots$$

"vertex corrections" "spectator corrections"

- $1/m_b$ power corrections may or may not factorize

SCET approach

SCET factorization formula:

(Chay, Kim '03; Grinstein, Grossman, Ligeti '04; Becher, Hill, Neubert '05)

$$\langle V\gamma\,|Q_i|\,\bar{B}\rangle = t_i^{\mathrm{I}}\,\zeta_{V_\perp} + t_i^{\mathrm{II}} \star \phi_+^B \star \phi_\perp^V + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)$$

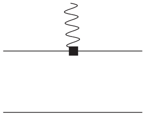
- $\zeta_{V_\perp}, \phi_\perp^V, \phi_+^B$ are matrix elements of SCET operators
- hard-scattering kernels = SCET matching coefficients

$$\begin{aligned} t_i^{\mathrm{I}} &= C_i^A(m_b,\mu) \\ t_i^{\mathrm{II}} &= C_i^{B1}(m_b,\mu) \star j_\perp(m_b\Lambda,\mu) \qquad \text{(subfactorization)} \end{aligned}$$

- large logs in t_i^{II} resummed by solving RG equations

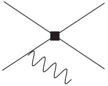
$1/m_b$ power corrections: annihilation

Q_7



LO

$Q_1 \dots Q_6$



Annihilation

$B \to K^*\gamma$ $$\frac{\mathcal{A}_{\text{ann}}}{\mathcal{A}_{\text{LO}}} \sim \frac{2\pi^2}{m_b} * \left(\frac{V_{us}^* V_{ub}}{V_{cs}^* V_{cb}} \sim \lambda^3\right)$$

$B \to \rho\gamma$ $$\frac{\mathcal{A}_{\text{ann}}}{\mathcal{A}_{\text{LO}}} \sim \frac{2\pi^2}{m_b} * \left(\frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}} \sim 1\right)$$

Must understand annihilation to:

- study any observable in $B \to (\rho, \omega)\gamma$
- study isospin and CP asymmetries in $B \to K^*\gamma$

QCDF results 2001-2007

Ali, Lunghi, Parkhomenko; Bosch, Buchalla; Beneke, Feldmann, Seidel, . . .

- form factors and LCDAs from QCD sum rules
- t^{I}, t^{II} known at NLO (α_s)
- annihilation at tree level

Recent analysis in Ball, Jones, Zwicky ’07 in QCDF + sum rules

Will give two sample applications from that paper

Sample application: determination of $|V_{td}/V_{ts}|$

$$\frac{\mathcal{B}(B \to \rho\gamma)}{\mathcal{B}(B \to K^*\gamma)} \propto \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{1}{\xi_\rho^2}\left[1 + \Delta R(\bar{\rho}, \bar{\eta})\right]$$

- $\Delta R \sim 0.1$ can be calculated in QCDF

Result from Ball, Jones, Zwicky using Feb. 2007 HFAG

$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.192 \pm 0.016\,(\mathrm{exp}) \pm 0.014\,(\mathrm{th})$$

- theory errors dominated by form-factor ratio ξ_ρ
- improved lattice results for $f_\rho^\perp$ will reduce error on ξ_ρ

Sample application: Isospin violation in $B \to K^*\gamma$

Isospin asymmetry

$$A_I(K^*) = \frac{\Gamma(\bar{B}^0 \to \bar{K}^*\gamma) - \Gamma(B^- \to K^*\gamma)}{\Gamma(\bar{B}^0 \to \bar{K}^*\gamma) + \Gamma(B^- \to K^*\gamma)}$$

QCDF: $A_I(K^*) = (5.4 \pm 1.4)\%$ (Ball, Jones, Zwicky)

Exp: $A_I(K^*) = (3 \pm 4)\%$ (HFAG 2007)

- sensitive to penguins through Q_6 (Kagan, Neubert)
- to calculate, must understand annihilation
- largest error in QCDF result is μ-dependence

Outline

1) $B \to V\gamma$ decays in QCD factorization and SCET

- sample results

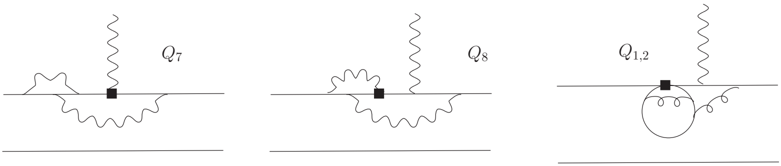
2) Improving factorization predictions

- **NNLO perturbative corrections**
- **annihilation at order α_s**
- **form factor uncertainties**

NNLO perturbative corrections

$$\langle V\gamma \,|Q_i|\, \bar{B}\rangle = t_i^{\mathrm{I}}\, \zeta_{V_\perp} + t_i^{\mathrm{II}} \star \phi_+^B \star \phi_\perp^V$$

Vertex corrections at NNLO



These are virtual corrections to matrix elements in $B \to X_s\gamma$

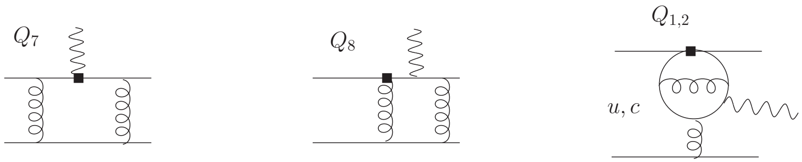
Asatrian, Bieri, Blokland, Czarnecki, Gambino, Greub, Hurth, Misiak, . . .

Status:

- $Q_{7,8}$ known exactly to NNLO (α_s^2)
- $Q_{1,2}$ known at NNLO in large-β_0 limit ($C_F n_f$ terms)
- Can obtain t_i^{I} to same accuracy (Ali, Greub, BP)

Numerics: contributions from Q_1 and Q_7 large, but tend to cancel

Spectator corrections at NNLO



No analog in inclusive decay, must be calculated from scratch

Status:

- $Q_{7,8}$ known to NNLO (α_s^2)
  (Becher, Hill; Beneke, Kiyo, Yang; Ali, Greub, BP)
- $Q_{1,2}$ known only to NLO
  (Bosch, Buchalla; Beneke, Feldmann, Seidel)

Numerics: NNLO corrections from each Q_i individually small

Estimate of branching fractions at NNLO

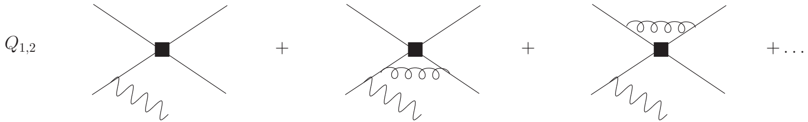
$$10^5 \times \mathcal{B}(B \to K^*\gamma) = 4.6 \pm 1.2\,[\zeta_{K^*}] \pm 0.4\,[m_c] \pm 0.2\,[\lambda_B] \pm 0.1\,[\mu]$$

Matching not complete because of $Q_{1,2}$:

- Assign 100% uncertainty to NLO hard-spectator correction:
  $\Rightarrow \Delta\mathcal{B} \approx \pm 0.1$
- Assign 100% uncertainty to NNLO vertex correction in large-β_0 limit:
  $\Rightarrow \Delta\mathcal{B} \approx \pm 0.5$
- Results for $Q_{1,2}$ beyond large-β_0 limit would reduce errors
  - directly, by eliminating the $\Delta\mathcal{B} \approx \pm 0.5$ above
  - indirectly, by fixing a renormalization scheme for m_c
  - three-loop calculation in progress
    (Boughezal, Czakon, Schutzmeier)

Annihilation at $\mathcal{O}(\alpha_s)$: Two examples

Annihilation in $B \to V\gamma$ with $Q_{1,2}$



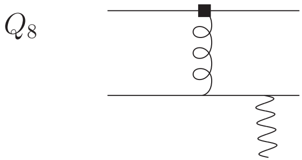
Result to $\mathcal{O}(\alpha_s)$: (Ali, Parkhomenko, BP (in progress))

$$\mathcal{A}_{\text{ann}} \sim \phi_+^B \star t_{\text{ann}}^{\text{II}} \star \phi_\perp^{V,1/m_b}$$

- IR divergences absorbed in LCDAs
- convolution integral converges

Example of factorization at $\mathcal{O}(\alpha_s/m_b)$

Annihilation in $B \to V\gamma$ with Q_8



$$\text{Result at } \mathcal{O}(\alpha_s): \qquad \frac{\mathcal{A}^8_{\text{ann}}}{\mathcal{A}_{\text{LO}}} \sim \frac{\lambda_B}{m_b} \int_0^1 du \, \frac{\phi_\perp^V(u)}{(1-u)^2}$$

- endpoint divergence in convolution integral breaks factorization
- small numerically (Kagan, Neubert)
- a conceptual problem

Endpoint divergences

Possible treatments of endpoint divergences:

- introduce an IR cutoff on u-integral, estimate uncertainty (Kagan, Neubert)
- use zero-bin subtractions (Manohar, Stewart; Arnesen, Ligeti, Rothstein, Stewart)
- introduce subleading form factors that generalize $\zeta_{V_\perp}$

Systematic treatment of $B \to V\gamma$ relies on solving this

Form factor uncertainties

Branching fractions have $\sim 30\%$ form factor uncertainties

To reduce form factor uncertainties, can

- take ratios of branching fractions, estimate SU(3) breaking effects in ratios of form factors with QCD sum rules
- constrain form factors with data, for instance $B \to \rho\ell\nu$ (Bosch, Buchalla)

CP and isospin asymmetries *defined* through ratios

- less sensitive to form factors than branching fractions
- but involve annihilation ...

Summary

Reviewed theory status of $B \to V\gamma$ decays

Systematic studies rely on QCD factorization (or pQCD)

Improving the factorization predictions requires:

- NNLO perturbative corrections
- treatment of power corrections (especially annihilation)
- more precise knowledge of form factors (or SU(3) breaking)

Work on these points in progress

Backup slides

Numerical impact of vertex corrections in $B \to K^*\gamma$

The ratio of NNLO to LO is:

$$\frac{\mathcal{A}_{\mathrm{v}}^{\mathrm{NNLO}}}{\mathcal{A}_{\mathrm{v}}^{\mathrm{LO}}} = 1 + (0.096 + 0.057i)\,[\alpha_s] + (-0.007 + 0.030i)\,\left[\alpha_s^2\right]$$

In terms of individual contributions

$$\Big((0.264 + 0.034i)\,[Q_1] - (0.184)\,[Q_7] + (0.016 + 0.023i)\,[Q_8]\Big)\,[\alpha_s]$$
$$+\Big((0.073 + 0.022i)\,[Q_1] - (0.081)\,[Q_7] + (0.002 + 0.008i)\,[Q_8]\Big)\,[\alpha_s^2]$$

- NNLO correction small due to cancellation between Q_1 and Q_7
- That Q_1 is only large-β_0 limit result can be significant (see branching fractions)

Numerical impact of spectator corrections in $B \to K^*\gamma$

Total corrections:

$$\frac{\mathcal{A}_{\rm hs}^{\rm NNLO}}{\mathcal{A}_{\rm V}^{\rm LO}} = \left(0.11 + 0.05i\right)[\alpha_s] + \left(0.03 + 0.01i\right)[\alpha_s^2]$$

In terms of individual operators:

$$= \Big((0.023 + 0.046i)\,[Q_1] + 0.074\,[Q_7] + 0.010\,[Q_8]\Big)\,[\alpha_s]$$
$$+ \Big((0.004 + 0.003i)\,[Q_1] + 0.025\,[Q_7] + (0.003 + 0.005i)\,[Q_8]\Big)\,[\alpha_s^2]$$

$([Q_1] = \Delta_1 C^{B1(0)} \star j_\perp^{(1)})$

- The NNLO corrections are individually small
- Resummation effects $\sim 10\%$ (but stabilize μ-dependence)