# *Inclusive semileptonic B decays: $|V_{cb}|$ and $|V_{ub}|$*

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*On Behalf of the BaBar and Belle Collaboration*

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# Outline

- *Motivation*
  - *Semileptonic decays*
- *Inclusive* $B \to X_c \ell \nu$
  - *Hadronic moments:* $|V_{cb}|$ *and HQE parameters extraction*
- *Inclusive* $B \to X_u \ell \nu$
  - *BaBar Measurement:* $|V_{ub}|$ *with* $B_{reco}$ *sample*
  - *World Average*
  - $X_u$ *hadronic moments*
  - *Weak Annihilation in* $B \to X_u \ell \nu$ *decays*
- *Conclusions*

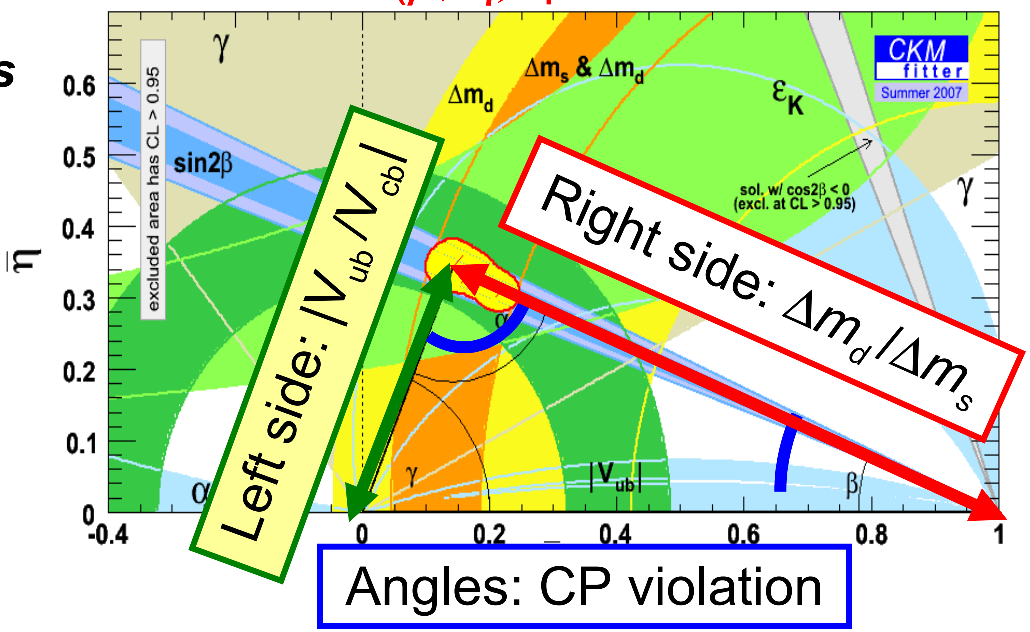
# Motivation

**_CKM description of CP Violation is a success_**

*We are looking for small deviations from SM:*

***need both precision and redundancy***

$(\rho, \eta)$ plane:



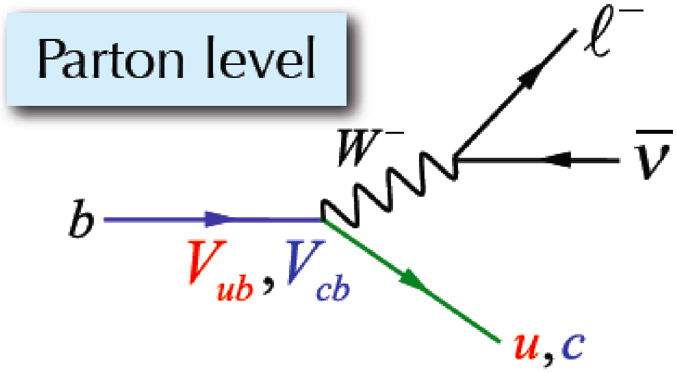

- *Sin2β and $B_{s,d}$ mixing give indirect $|V_{ub}|$ determination from SM loop processes (CKM fitter '07, 5% uncertainty)*
  - $|V_{ub}|=(3.57 \pm 0.17)10^{-3}$
- *Direct determination from inclusive semileptonic decays, only tree level (HFAG '08, ~9% uncertainty):*
  - $|V_{ub}|=(3.99 \pm 0.14 \pm 0.30)10^{-3}$

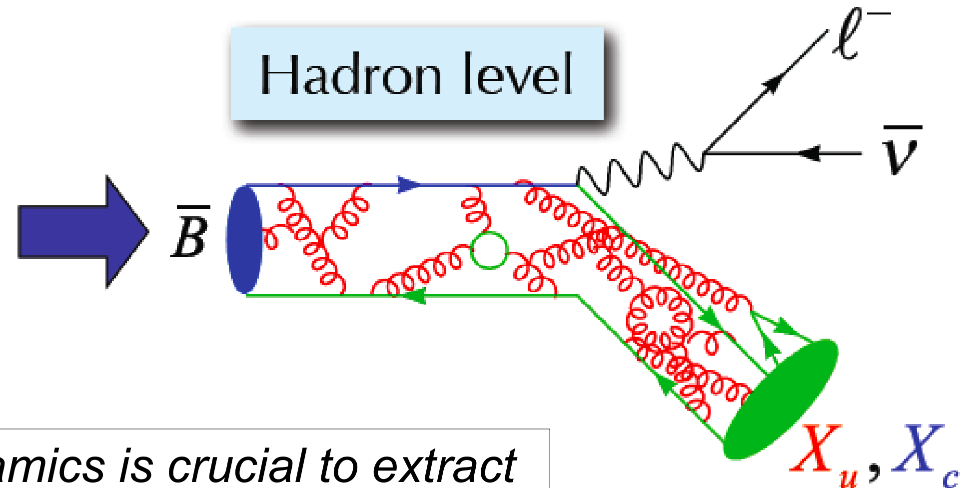
***Precision is improving: was 15% in 2003***

# Semileptonic decays

*Semileptonic Decays are a natural probe for |Vub| and |Vcb|*
*We want to study **quark** interactions but measure **hadron** decays*

$$\Gamma(b \to x\ell\nu) \propto |V_{xb}| \cdot m_b^5$$




*Understanding the QCD dynamics is crucial to extract informations on weak interactions*

- *Error on $|V_{ub}/V_{cb}|$ is dominated by $|V_{ub}|$: ~9%*
  - *~7% from the b-quark mass and b momentum inside the B meson*
  - *these parameters are extracted using inclusive $B \to X_c \ell \nu$ decays (and the $B \to X_s \gamma$)*
- *Improve $B \to X_c \ell \nu$ to reduce uncertainty on $|V_{ub}|$*

# *Inclusive $|V_{cb}|$*

- *$|V_{cb}|$ can also be extracted using $B \rightarrow D^* \ell \nu$ (next talk by H. Kakuno)*
  - *Still large uncertainty from Form Factor normalization (Lattice-QCD)*

# Inclusive $|V_{cb}|$: Heavy Quark Expansion

- *HQE connect the inclusive $b \to c \ell \nu$ decay width to $|V_{cb}|$*  $r = m_c/m_b$

$$\Gamma_{SL} \sim |V_{cb}|^2 m_b^5 \left[ z_0(r) + 0/m_b + z_2(r, \mu_\pi^2/m_b^2, \mu_G^2/m_b^2) + z_3(r, \rho_D^3/m_b^3, \rho_{LS}^3/m_b^3) + \ldots \right]$$

- *Similar expressions for moments of various inclusive distributions:*
  - *Hadron mass moments* $\langle M_X^n \rangle_{E>Ecut}$
  - *Lepton energy moments* $\langle E_{lep}^n \rangle_{E>Ecut}$
  - *Photon energy moments in $b \to s\gamma$:* $\langle E_\gamma^n \rangle_{E>Ecut}$

*Extract $|V_{cb}|$, $BR_{cl\nu}$*

*and the HQE parameters*

$m_b$, $m_c$, $\mu_\pi^2$, $\mu_G^2$, $\rho_D^3$, $\rho_{LS}^3$

$$\langle M_X^n \rangle_{E>Eo} = \tau_B \int M_X^n d\Gamma = f\left(E_0, m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3\right)$$

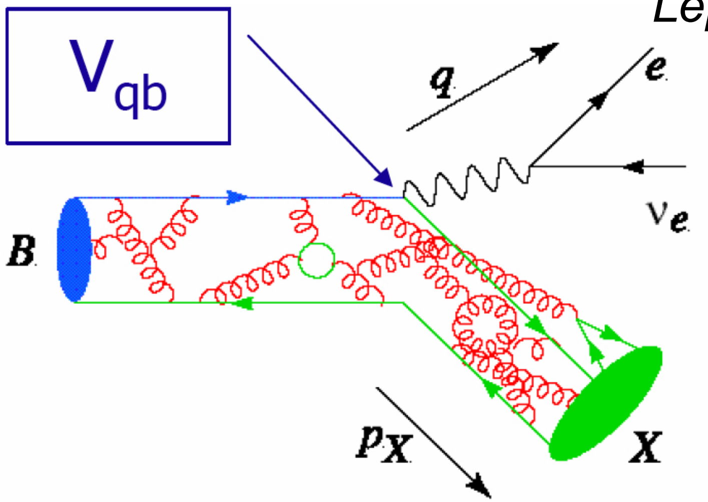
Lepton/$\gamma$ energy cut

Quark masses

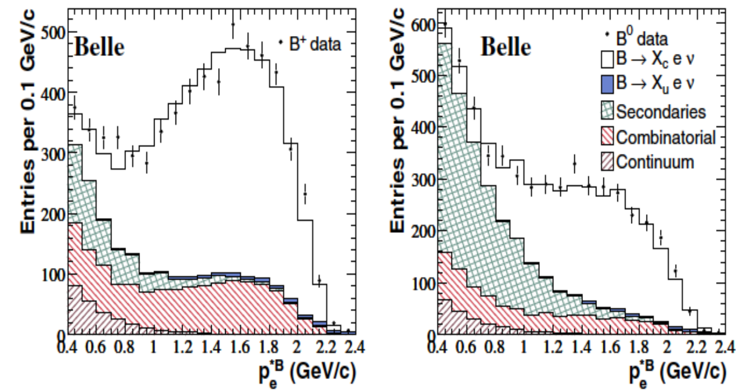
Non-pert. matrix element at order: $1/m_b^2$ & $1/m_b^3$

- *Fit predicted moments of inclusive $b \to s\gamma$ and $b \to c \ell \nu$ for different cuts on kinematics variables*

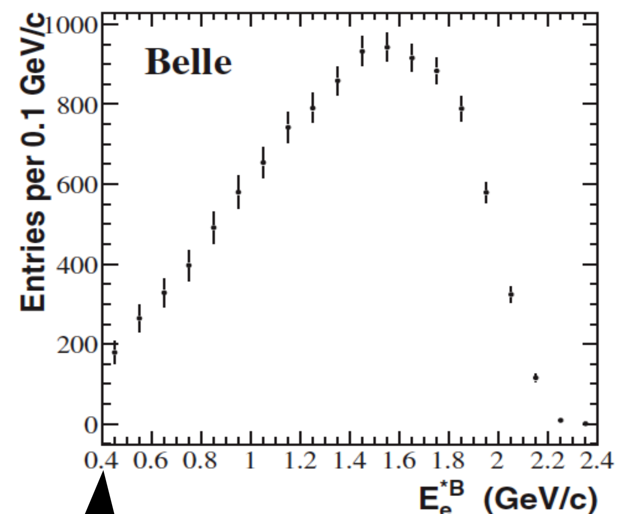
# Observables




*Lepton Momentum*

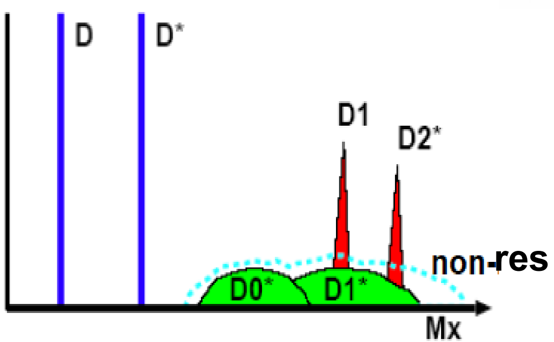


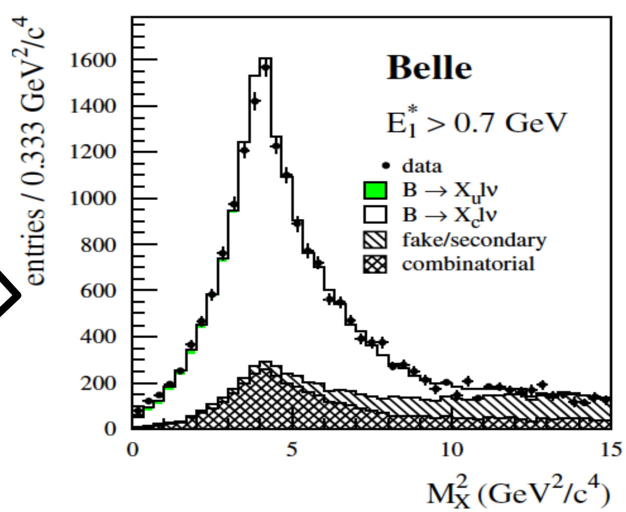

Unfolding




$E_e > 0.4$ GeV

*Hadron Mass Spectra*



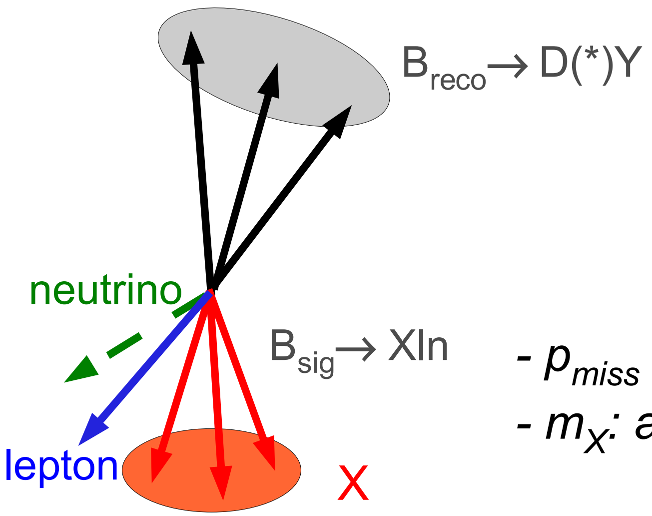




PRD75,032005(2007)
154 M BB

PRD75,032001(2007)
154 M BB

# Hadronic mass moments

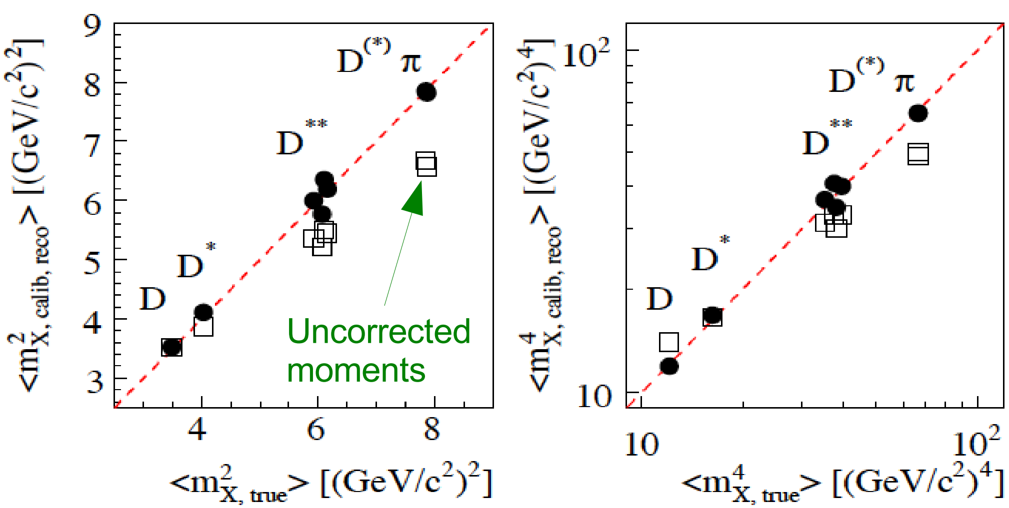
ArXiv: 0707.2670[hep-ex]
232 M BB

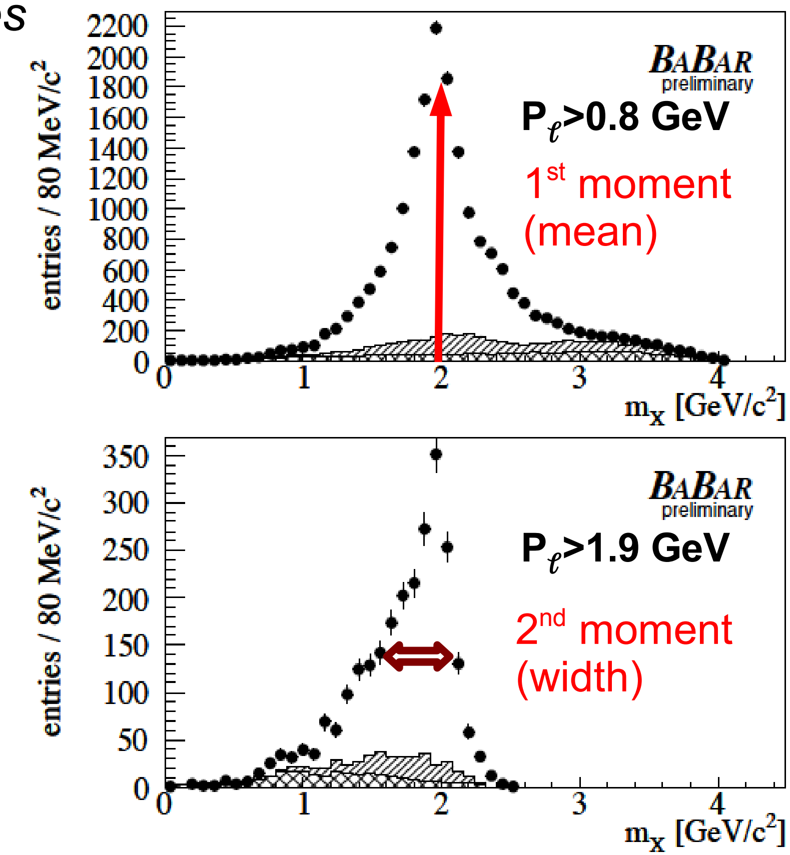



- *Fully reconstruct one B*
- *Measure hadronic moments as function of minimum lepton momentum:*
  - $\langle m_X^1 \rangle$ *to* $\langle m_X^6 \rangle$ *moments for* $E_\ell > 0.8, \ldots, 1.9$ *GeV*

- $p_{miss} = p_{\Upsilon(4S)} - p_{reco} - p_X - p_\ell$
- $m_X$*: all remaining particles*

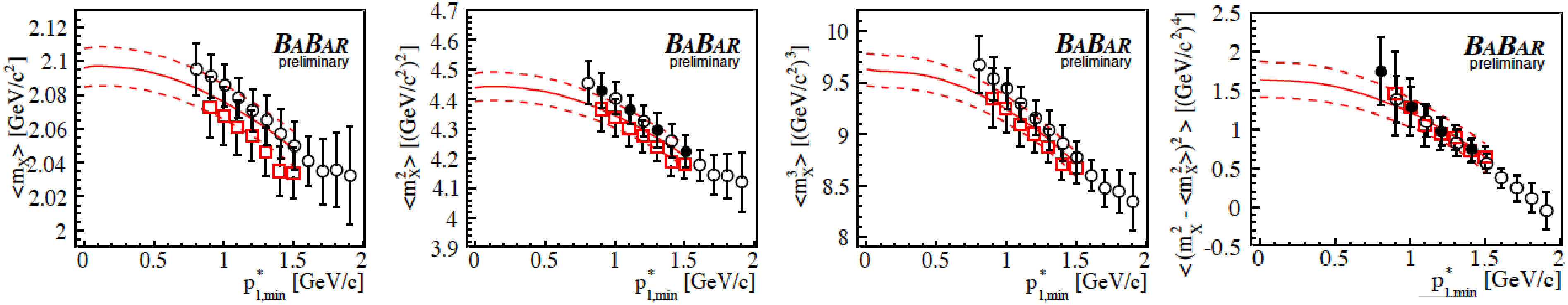
*Event by event* $m_X$ *calibration functions to relate reconstructed* $m_X$ *to true* $m_X$*, in bins of:*
*- X charged tracks multiplicity,* $E_{miss}$*-|*$\mathbf{P}_{miss}$*| and* $P_\ell$



# Hadronic Mass Moments

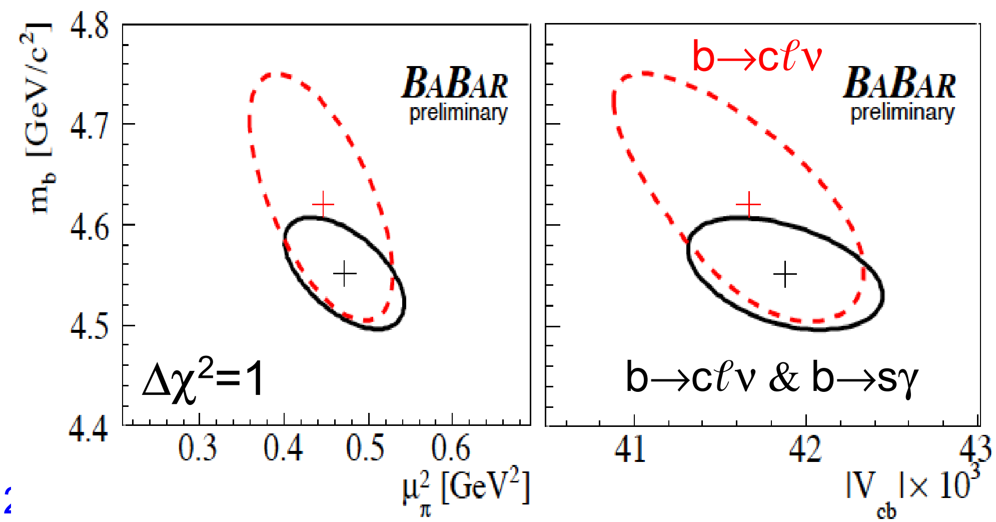
BaBar 2004 This measurement




- *Moments integrated over data for various lepton cuts (all points highly correlated)*
- *Each observable has a different dependence on HQE paramaters and quark masses*
- *Parameters determined by a Global Fit using:*
  - *8 mass moments (only ● are used in the fit, ○ agree well with the fit result)*
  - *10 $E_\ell$ moments*
  - *2 $E_\gamma$ moments*

**$|V_{cb}|$=(41.88±0.44±0.35±0.59)$10^{-3}$**
**$m_b$=(4.552±0.038±0.40) GeV**
**$\mu_\pi^2$=(0.471±0.034±0.62) GeV$^2$**

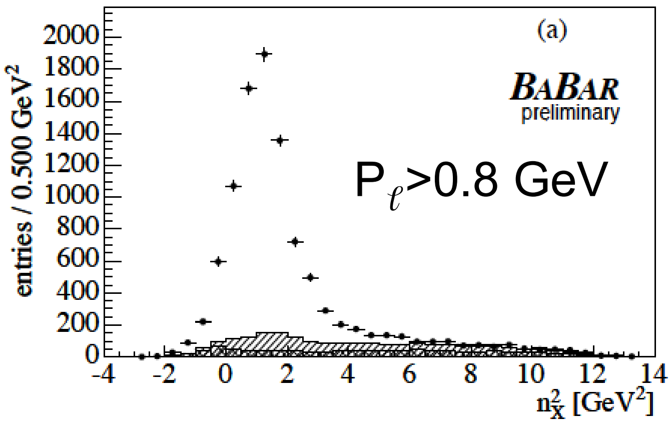
In the Kinetic Scheme
**Gambino, Uraltsev, Eur.Phys.J.C34,181(2004)**

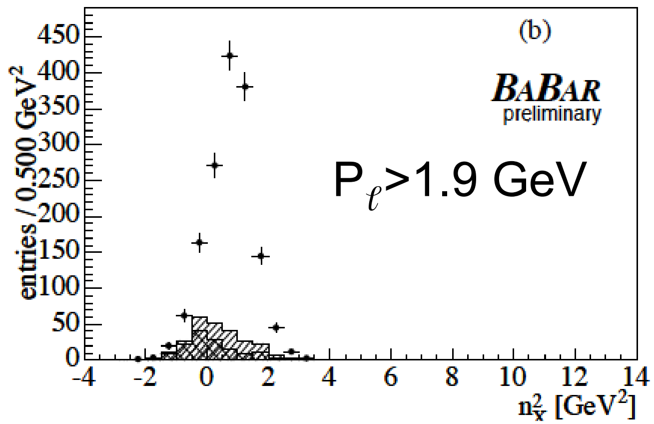


# Hadronic Mixed moments

- *Modified moments: $n_X^2 = m_X^2 - 2\Lambda E_X + \Lambda^2$ (with $\Lambda$=0.65 GeV)*
  - *Combination of Hadronic Mass and Energy Moments*
  - *Expect higher sensitivity to higher order parameters*

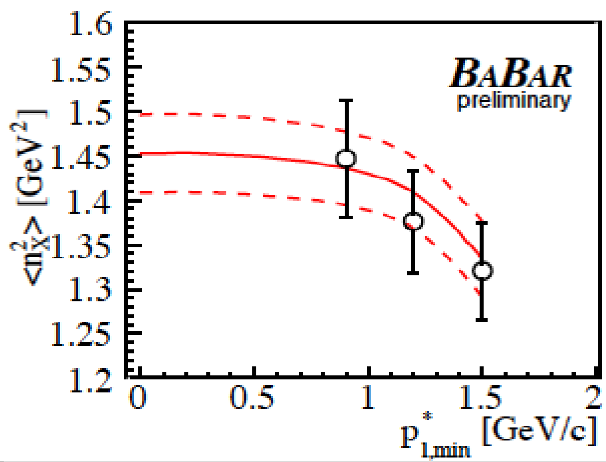
**Gambino, Uraltsev JHEP34,181(2004)**

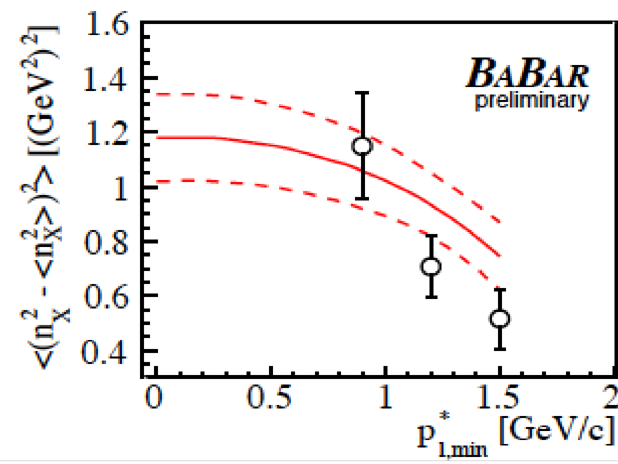



ArXiv: 0707.2670[hep-ex]
232 M BB

*Comparison with theoretical calculation: results of the Global Fit*



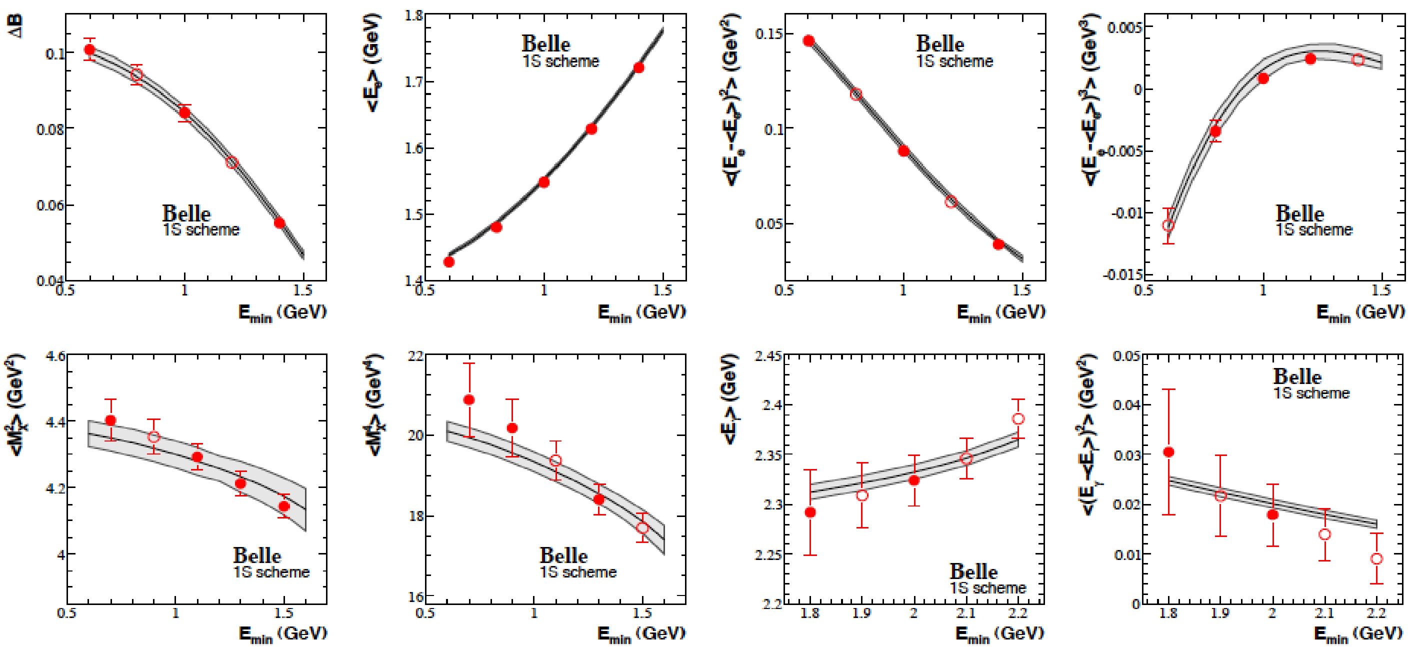



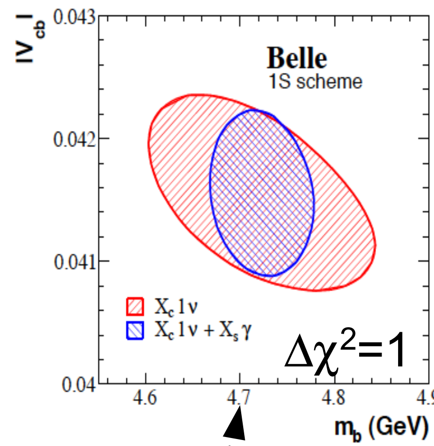
# Belle OPE fit

ArXiv: 0803.2158[hep-ex]

- *Update of ICHEP06 results*
- *7 $m_x$ moments, 11 $E_\ell$ moments, 4 $E_\gamma$ moments*
- *Both 1S scheme and Kinetic scheme*

Kinetic Scheme
**Gambino, Uraltsev, Eur.Phys.J.C34,181(2004)**
1S Scheme
**Bauer, Ligeti, Luke, PRD64, 113004 ((2004)**




*B-quark mass w/wo $b \to s\gamma$ are consistent*

| **1S scheme** | **Kinetic** |
|---|---|
| $\|V_{cb}\|=(41.56\pm0.68\pm0.08)10^{-3}$<br>$m_b^{1S}=(4.723\pm0.055)$ GeV | $\|V_{cb}\|=(41.52\pm0.69\pm0.08\pm0.58)10^{-3}$<br>$m_b^{kin}=(4.543\pm0.075)$ GeV |
| $m_b^{1S}=(4.718\pm0.119)$ GeV | $m_b^{kin}=(4.573\pm0.134)$ GeV |

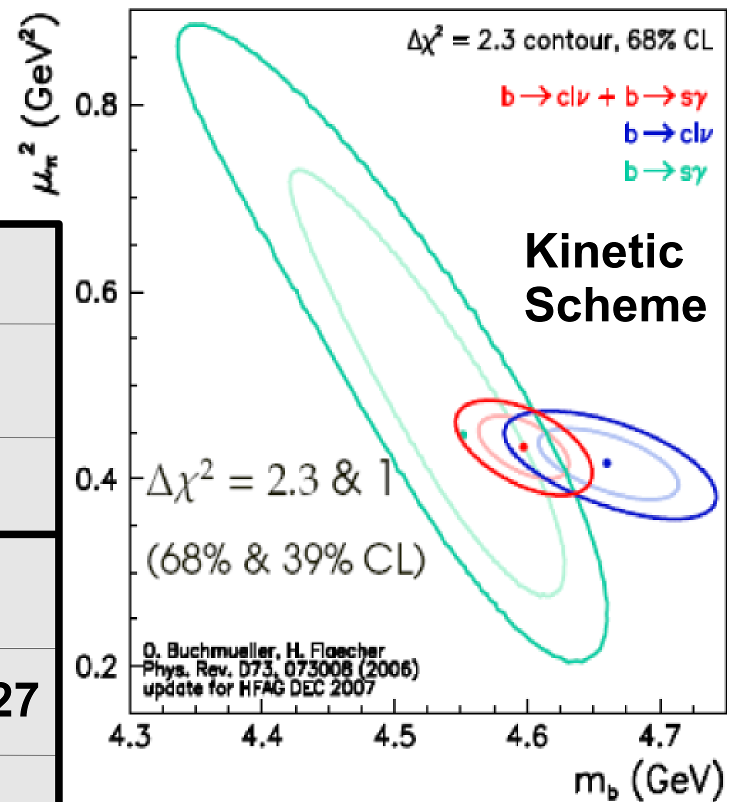
Fit in KS=
Fit in 1S + Translation 1S$\to$KS

*Using only $B \to X_c \ell \nu$*

# Inclusive $|V_{cb}|$: global OPE fit

- *Global fit with all available results*
  - *Babar, Belle, CLEO, CDF, Delphi*

| 1S | $V_{cb}(10^{-3})$ | $m_b$ (GeV) |
|---|---|---|
| | **41.78±0.30±0.08** | **4.701±0.030** |
| **no b→ sγ** | **41.56±0.39±0.08** | **4.751±0.058** |
| **KS** | $V_{cb}(10^{-3})$ | $m_b$ (GeV) |
| | **41.91±0.19±0.28±0.59** | **4.613±0.022±0.027** |
| **no b→ sγ** | **41.68±0.39±0.58** | **4.677±0.053** |




*>100 measurements, different HQE fit implementations, results in very good agreement:*

$$\sigma_{|Vcb|}<2\% \text{ and } \sigma_{mb}<\sim1\%$$

Comparison with exclusive

$|V_{cb}|=(39.2\pm0.6\text{exp} \pm1.0\text{th})10^{-3}$

Inclusive-exclusive:

$\Delta=2.7\pm1.3$ $2.0\sigma$

# *Inclusive $|V_{ub}|$*

- *$|V_{ub}|$ can also be extracted using $B \to \pi \ell \nu$ (next talk by H. Kakuno)*
  - *Still large uncertainty from Form Factor normalization (Lattice-QCD)*
  - *Expected large improvements in the future*

# Inclusive $B \to X_u \ell \nu$

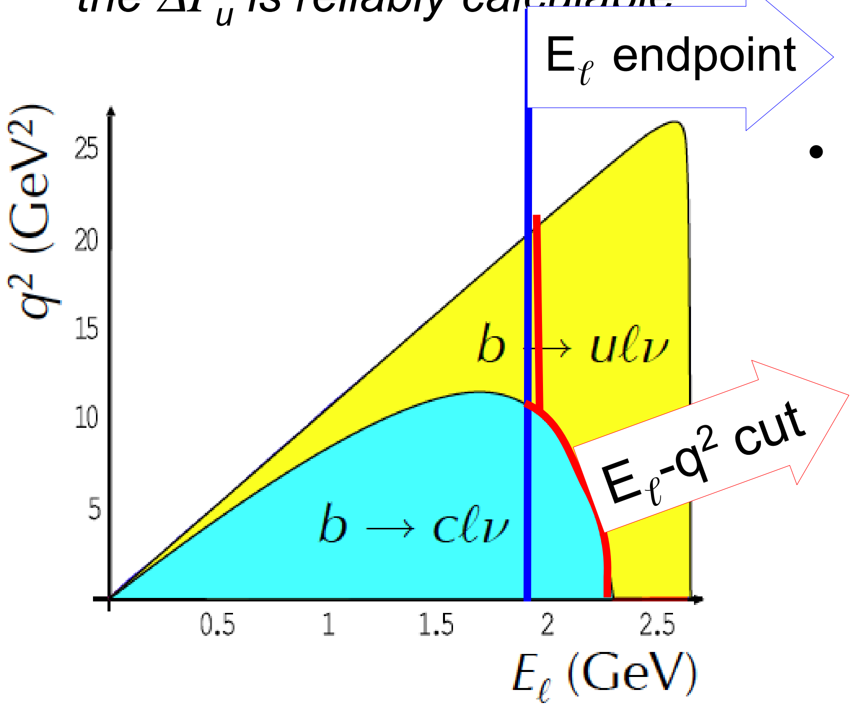
$$\frac{\Gamma(b \to u \ell \nu)}{\Gamma(b \to c \ell \nu)} \approx \frac{|V_{ub}|^2}{|V_{cb}|^2} \approx \frac{1}{50}$$

- *Experiment challenge is to separate $B \to Xc\ell\nu$ from signal*

- *$m_u << m_c$, different kinematics: signal have larger $E_\ell$ and $q^2$*
- *Measure partial $\Delta B(B \to X_u \ell \nu)$ in a region where the S/N is good and the $\Delta\Gamma_u$ is reliably calculable*

$$\Delta B(B \to X_u \ell \nu) = \tau_B |V_{ub}|^2 \zeta_c$$

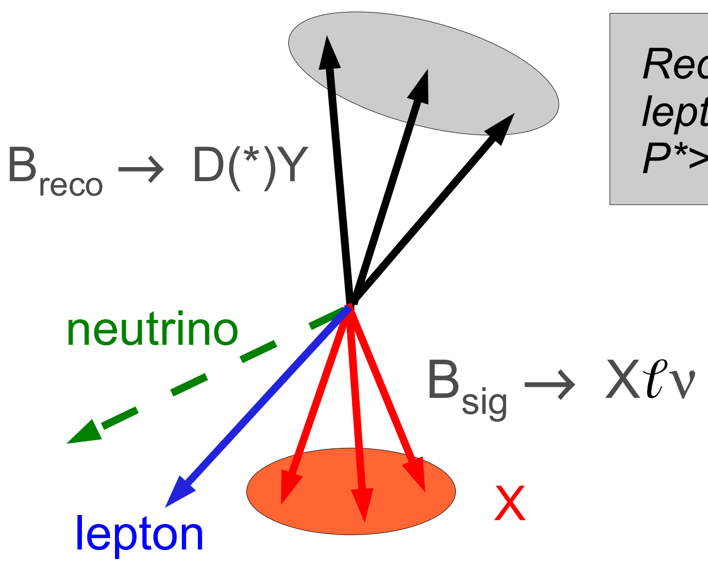
*Cut dependent constant, from Theory (many frameworks available)*



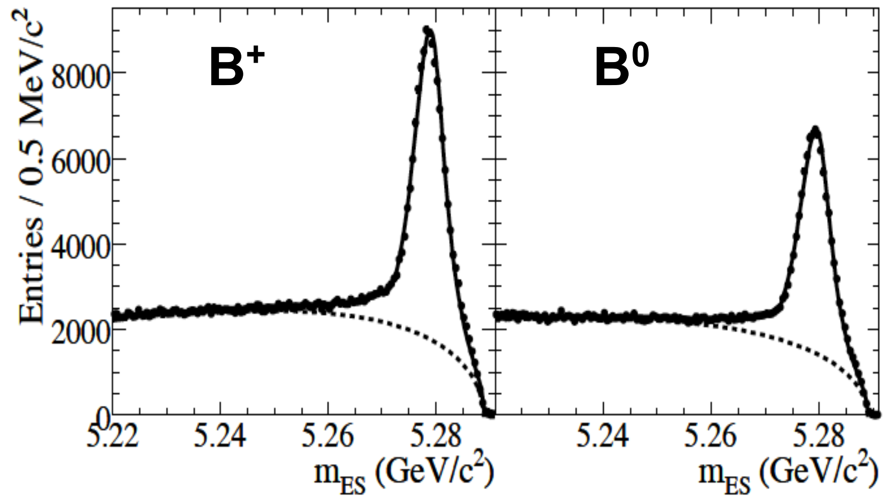

- *OPE convergence is compromised: non perturbative effects at $O(1/m_b)$*
  - *Light cone distribution of b-quark (**Shape Function**) is needed*
  - *Detailed shape not known, in particular the tail, but mean and r.m.s constrained from moment measurement in $B \to X_c \ell \nu$ (and $B \to X_s \gamma$)*

# Inclusive |Vub|: strategy

- *Fully reconstruct one B in hadronic decays*




*Requiring a lepton with P*>1GeV/c*




**- $p_{miss} = p_{\Upsilon(4S)}\quad p_{reco} - p_X - p_{lepton}$**

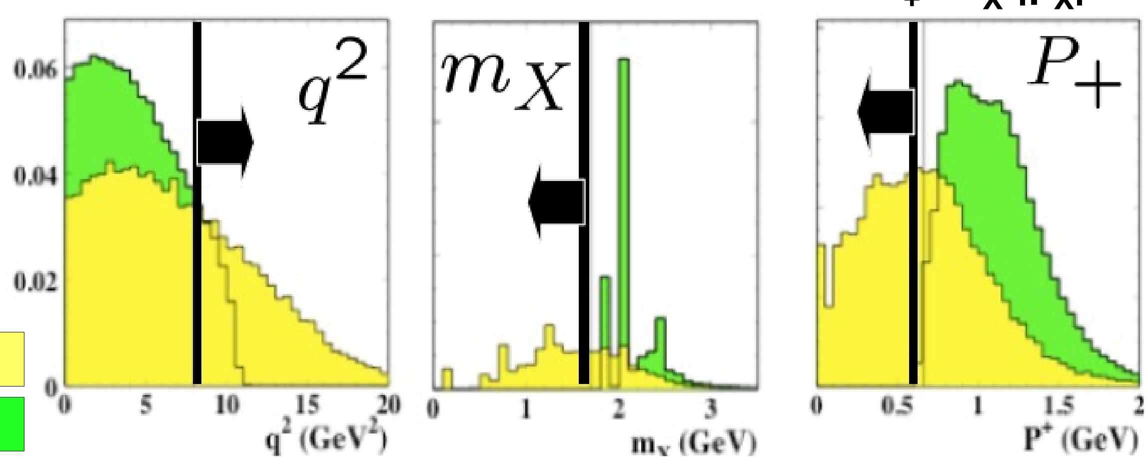
**- $m_X$: all remaining particles**

**$P_+ = m_X - |p_X|$**

Not to scale

$b \rightarrow u\ell\nu$

$b \rightarrow c\ell\nu$




*$m_X$ and $P_+$ require a sample of $B_{reco}$*

*Experimental Resolution leads to irreducible $b \rightarrow c\ell\nu$ contamination*

# Inclusive $|V_{ub}|$: results

ArXiv: 0708.3702[hep-ex]
accepted by PRL

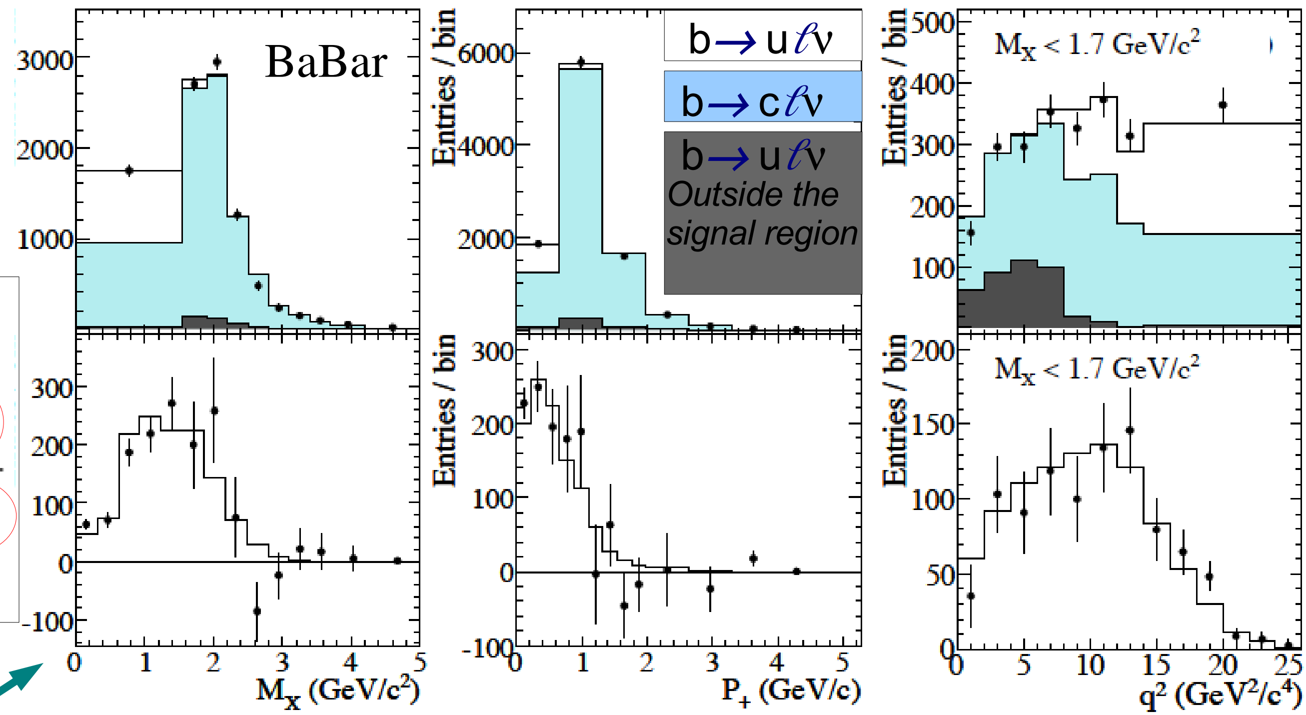
*Measure the partial branching ratio relative to the number of total semileptonic events*

$$\frac{\Delta B(X_u\ell\nu)}{B(X\ell\nu)} = \frac{N_{b\to u}}{N_{X\ell\nu}} \cdot \frac{F}{\epsilon_{sel}}$$

Unfolding factor

Signal efficiency

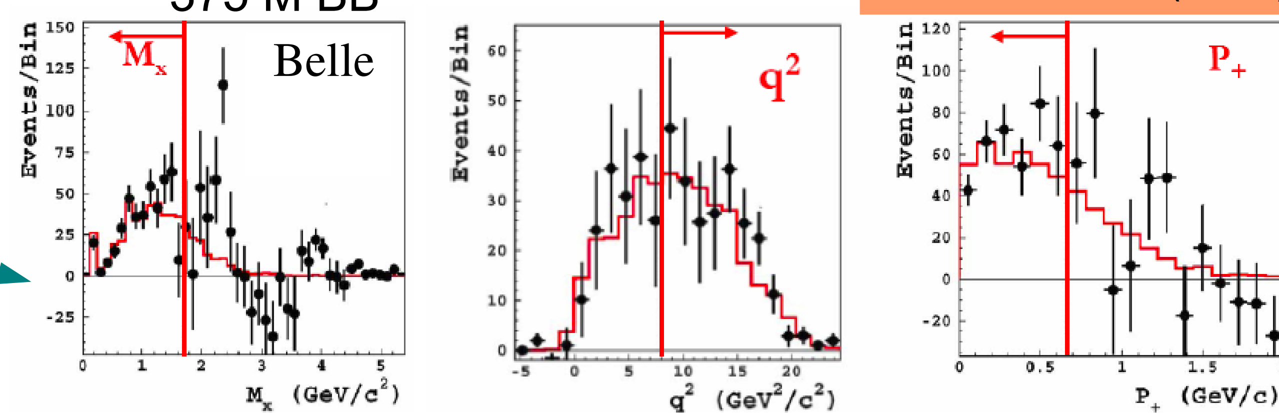
383 M BB




*Background subtracted distributions: not efficiency corrected*

PRL95,241801(2005)

375 M BB



# Inclusive $|V_{ub}|$: results

| Cut: | $N_u$ | $\lvert V_{ub}\rvert 10^3$ | stat. | syst. | theory | |
|---|---|---|---|---|---|---|
| $m_X$<1.55 GeV | 803 ±60 | 4.27 | ±0.16 | ±0.13 | ±0.30 | BLNP |
| | | 4.56 | ±0.17 | ±0.14 | ±0.32 | DGE |
| $P_+ = E_X - \lvert P_X\rvert < 0.66$ | 633 ±63 | 3.88 | ±0.19 | ±0.16 | ±0.28 | BLNP |
| | | 3.99 | ±0.20 | ±0.16 | ±0.24 | DGE |
| $m_X$<1.7 GeV & $q^2$ >8 GeV2 | 562 ±55 | 4.57 | ±0.22 | ±0.19 | ±0.30 | BLNP |
| | | 4.64 | ±0.23 | ±0.19 | ±0.25 | DGE |
| | | 4.93 | ±0.24 | ±0.20 | ±0.36 | BLL |

**Bosh,Lange,Neuber,Paz**
**PRL93,221801(2004)**
**PRD72,073006(2005)**

**Andersen,Gardi**
**JHEP0601,097(2006)**

**Bauer,Ligeti,Luke**
**PRD64,113004(2001)**

Single analysis
with 9% tot. uncertainty

*Compatibility taking into account the large correlations (stat. & syst.) between the different cuts:*

*- $m_x$ and ($m_x$,$q^2$) agree at 1$\sigma$*

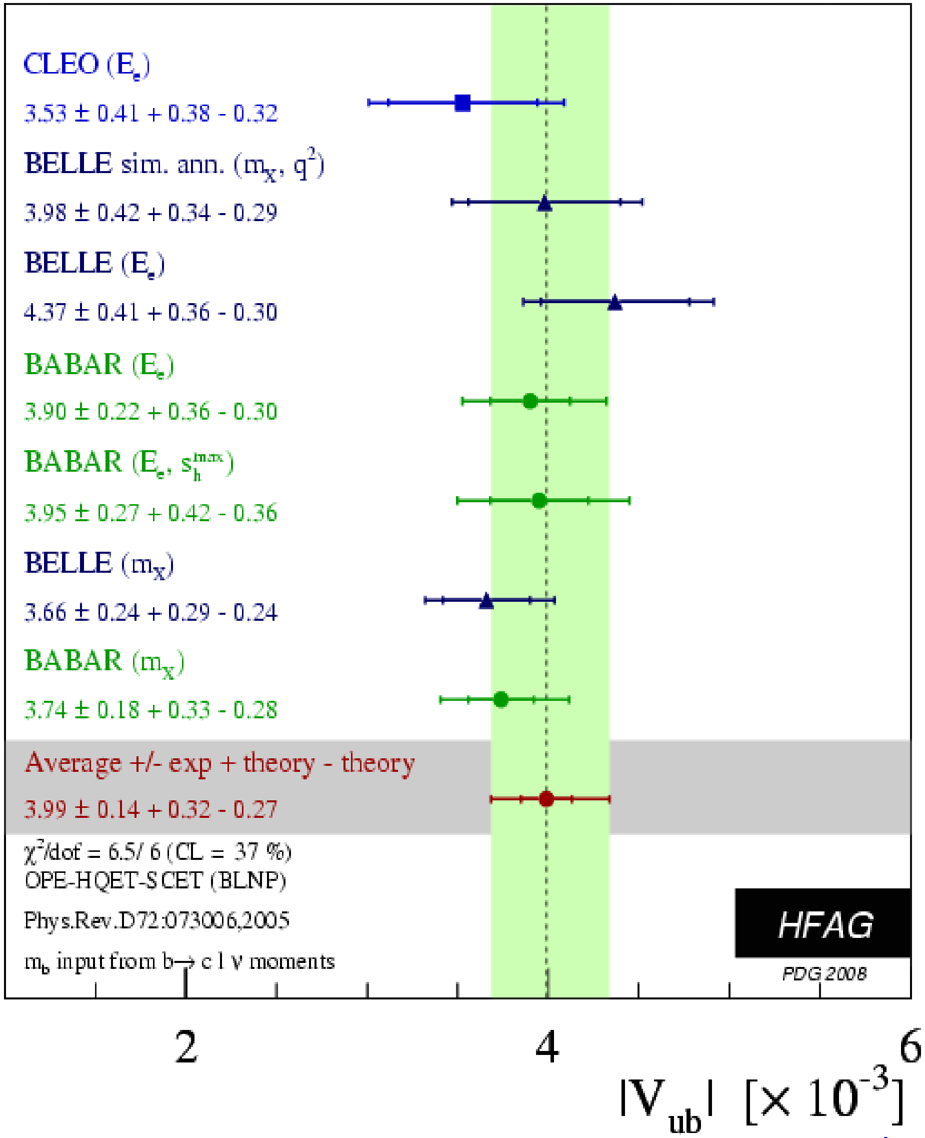
*- $P_+$ differs by 2.5$\sigma$*

Experimental systematics on $\Delta\mathcal{B}(B\to X_u\ell\nu)$ expressed in %

| *Method* | Detector effects | Shape function | $\mathcal{B}(\bar{B}\to X_u\ell\bar{\nu})$ $X_u=\pi,\rho,\ldots$ | Gluon splitting | $\mathcal{B}(\bar{B}\to X_c\ell\bar{\nu})$ | $B\to D^*\ell^-\bar{\nu}$ form factors | $\mathcal{B}(D)$ | $m_{ES}$ fit | Monte Carlo statistics | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| $M_X$ | 1.92 | 0.90 | 2.08 | 1.62 | 0.87 | 0.21 | 0.44 | 3.71 | 3.22 | 6.07 |
| $P_+$ | 3.88 | 1.31 | 2.22 | 1.47 | 2.80 | 0.39 | 0.73 | 3.98 | 4.62 | 8.38 |
| $M_X, q^2$ | 3.83 | 2.43 | 2.71 | 1.02 | 1.17 | 0.55 | 0.79 | 5.17 | 4.29 | 8.81 |

# $|V_{ub}|$ results (HFAG average, BLNP)

- *Many different theoretical approach ⇒many $|V_{ub}|$ values*



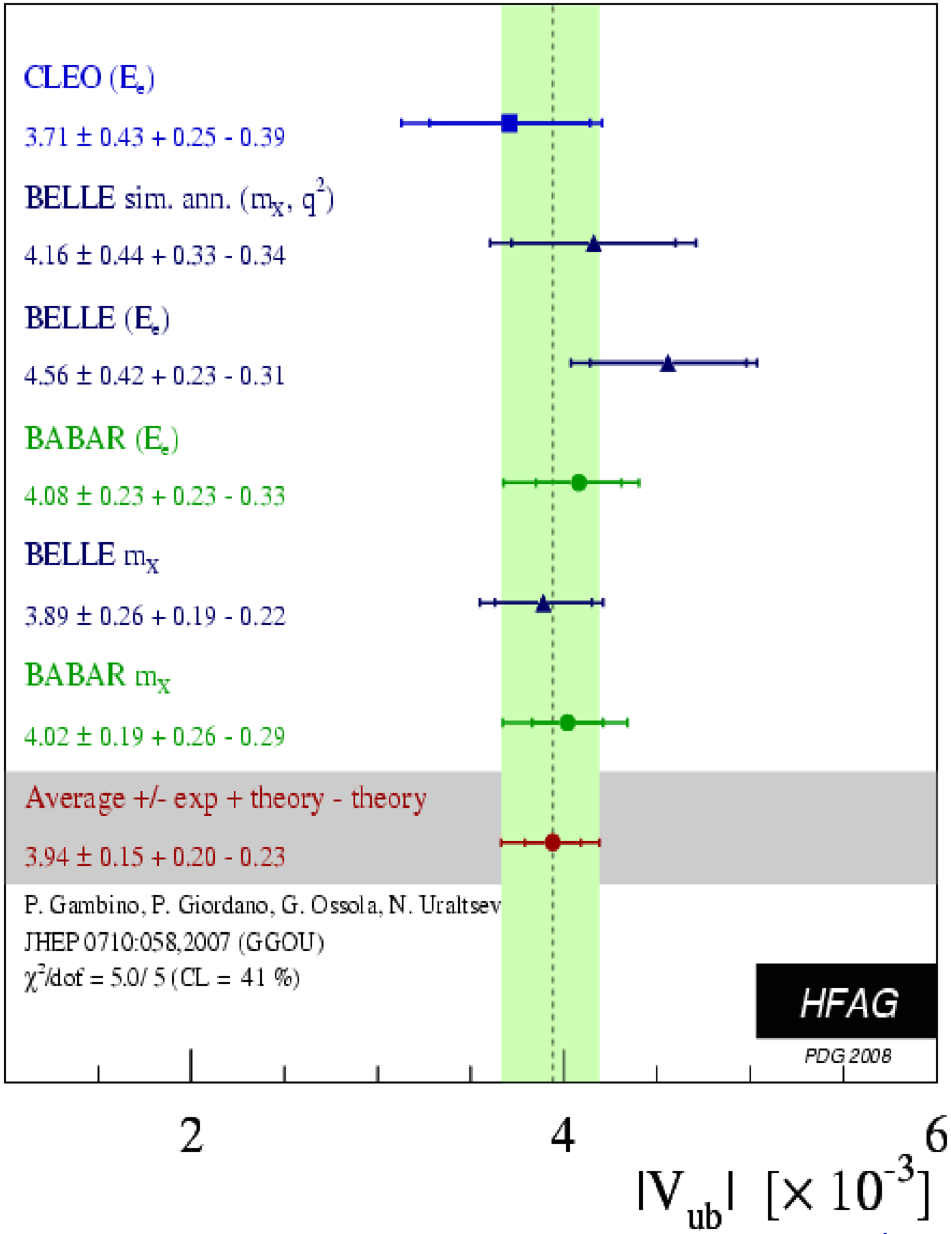

- *Here only BLNP, with $m_b$ from $B\rightarrow Xc\ell\nu$ Global Fit (Kinetic Scheme), including also uncertainty on the KS⇔SF Scheme translation*
  - *$B\rightarrow Xs\gamma$ not included: theory not under control (Neubert @ LP2007)*

$$|V_{ub}|=(3.99 \pm 0.14 \pm 0.30)10^{-3}$$

| δ\|Vub\| | +8.8% -7.7% |
|---|---|
| Statistical | 2.0% |
| Exp.systematics | 2.3% |
| $b\rightarrow c\ell\nu$ model | 1.3% |
| $b\rightarrow u\ell\nu$ model | 1.4% |
| HQ parameters | 7.0% |
| SF + Sub. SF | 0.6% |
| matching | 3.6% |
| Weak Annihilation | 1.3% |

*For $B_{reco}$ analysis use only $m_x$ cut:*
*- large unpublished correlations with P+ and q2*

# $|V_{ub}|$ results (HFAG average, GGOU)




**Gambino, Giordano, Ossola, Uraltsev**
**JHEP0710:058(2007)**

- *$m_b$ from Global Fit (Kinetic Scheme), including also $B \to Xs\gamma$*

$$|V_{ub}|=(3.94 \pm 0.15 \pm 0.23)10^{-3}$$

| δ\|Vub\| | +6.3% -7.0% |
|---|---|
| Statistical | 2.2% |
| Exp.systematics | 2.2% |
| $b \to c \ell \nu$ model | 1.3% |
| $b \to u \ell \nu$ model | 1.5% |
| Non pert.- | 3.9% |
| Higher order par. | 1.8% |
| q2 tail model | 2.6% |
| Weak Annihilation | -3.1% |

*For $B_{reco}$ analysis use only $m_x$ cut:*
*- large unpublished correlations with P+ and q2*

# $|V_{ub}|$ using SF independent analyses

- *QCD interaction affecting $b \to s\gamma$ and $b \to u\ell\nu$ are the same*
- *Take ratio of weigheted rates*

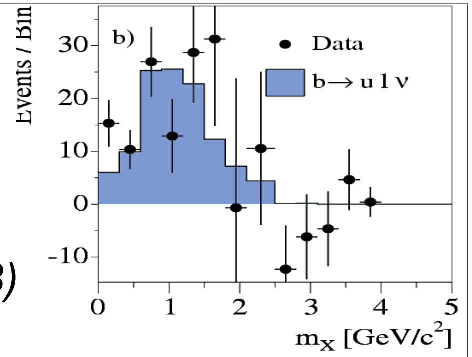
**Lange,Neuber,Paz JHEP0510:084 (2005)**

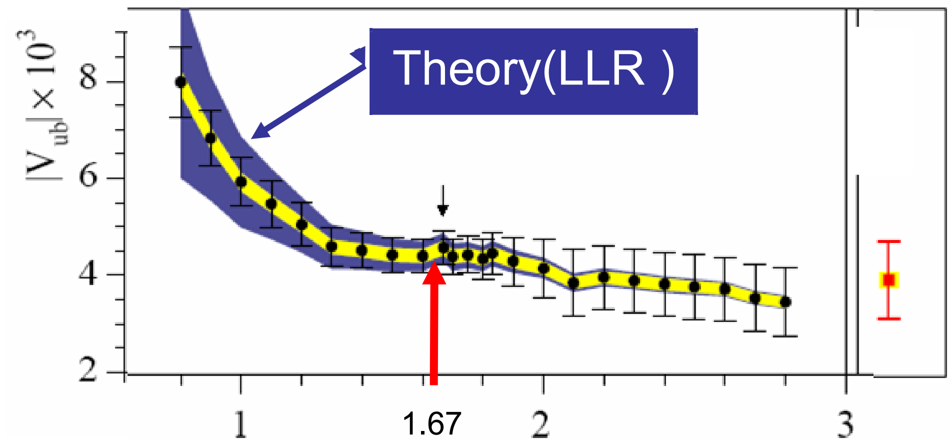
**Leibovich,Low,Rothstein PRD61,053006(2000) PLB513,83 (2001)**

$$\int_0^{m_{max}} \frac{d\Gamma(b\to u)}{dm_X} dm_X \quad \Longleftrightarrow \quad \int_{E_{min}}^{m_B/2} \frac{d\Gamma(b\to s\gamma)}{dE_\gamma} W(E_\gamma, E_{min}) dE_\gamma$$

PRL96,221801(2006)

- *Measuring $m_X$ using the $B_{reco}$ sample*
- *Statistics limited (89 M BB)*

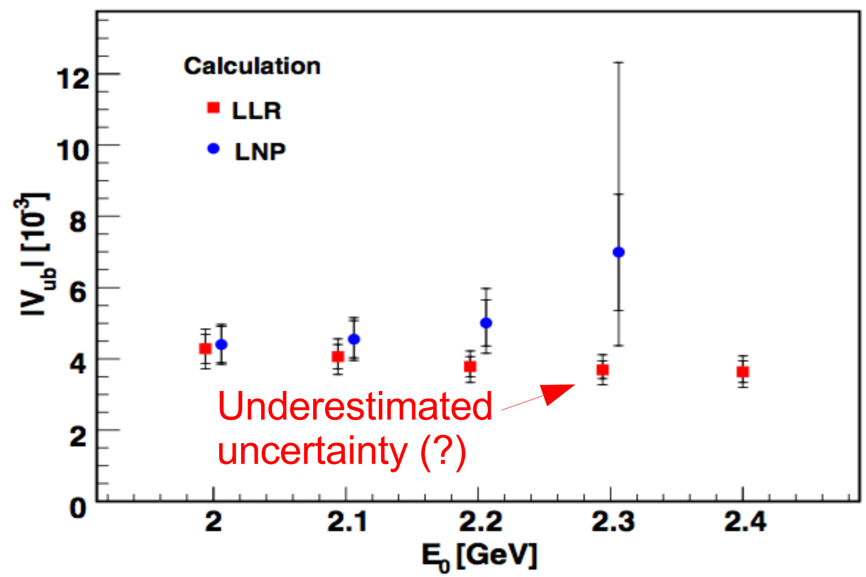


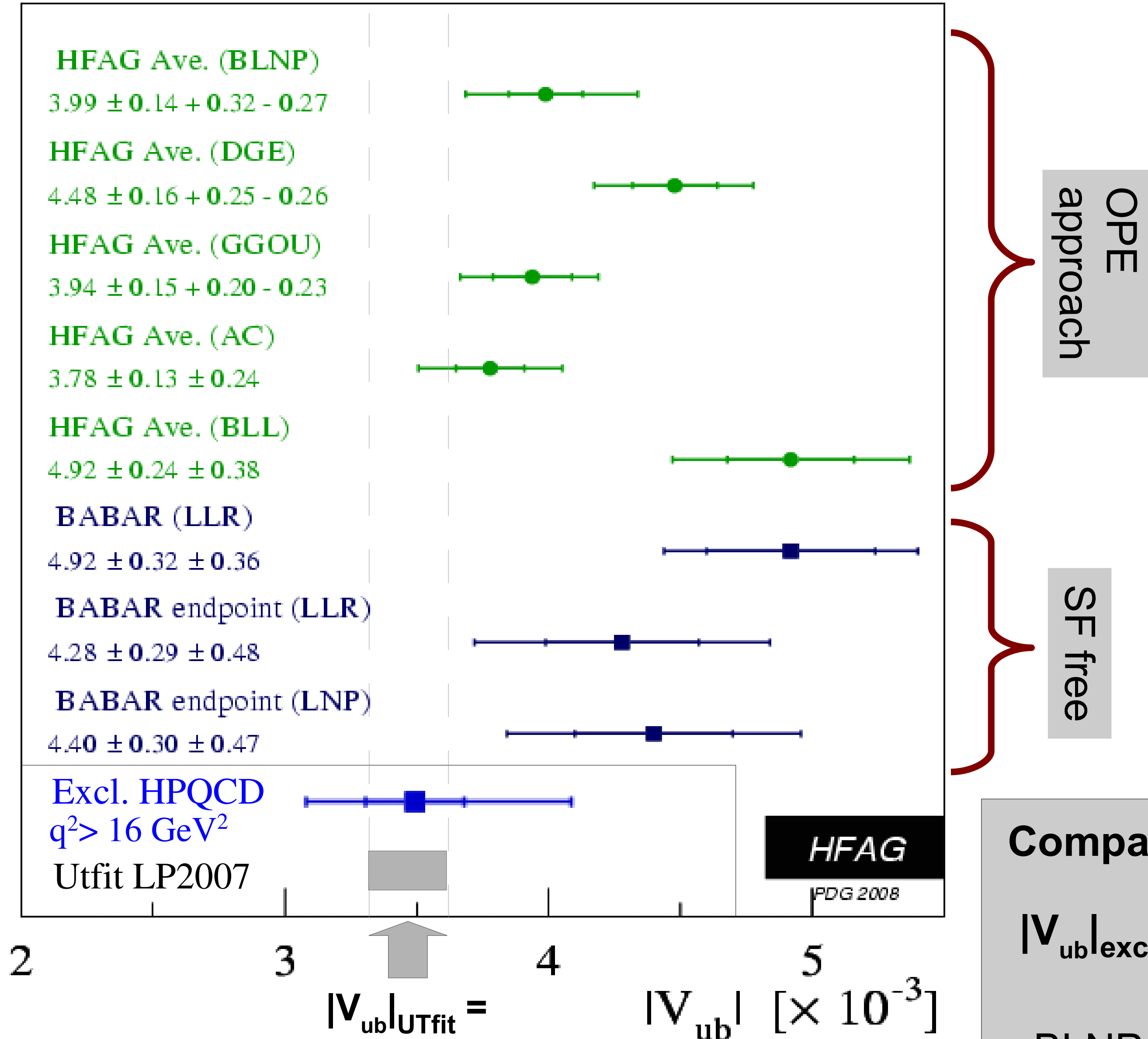

$|V_{ub}|=(4.43 \pm 0.30_{stat} \pm 0.25_{syst} \pm 0.29_{theo})\times 10^{-3}$

Golubev,Skovpen,Luth PRD76,114003(2007) using the BaBar data

- Using the $E_\ell$ endpoint spectrum (BaBar data PRD73,012006(2006)
- cosistent results (except at high $E_\ell$)



# $|V_{ub}|$ results (different calculations)




8 May 2008

*Different theoretical calculation*

*Results vary from $3.78 \times 10^{-3}$ (AC) to $4.92 \times 10^{-3}$ (BLL)*

*very different b-quark masses dependences*

*Goal for the future: quote one value of $|V_{ub}|$*

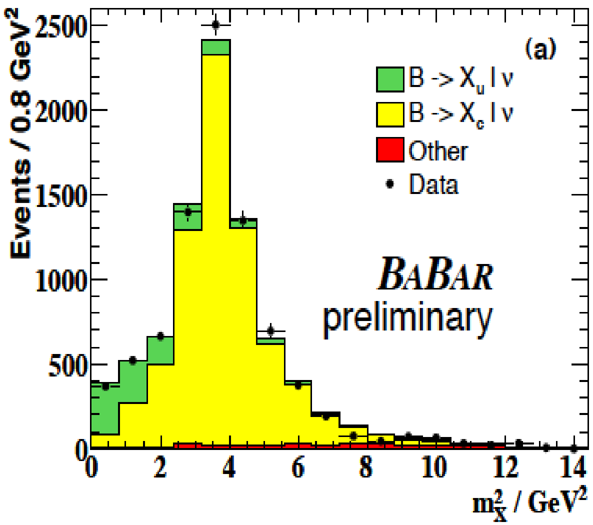
**Comparison with Excl. (HPQCD)**

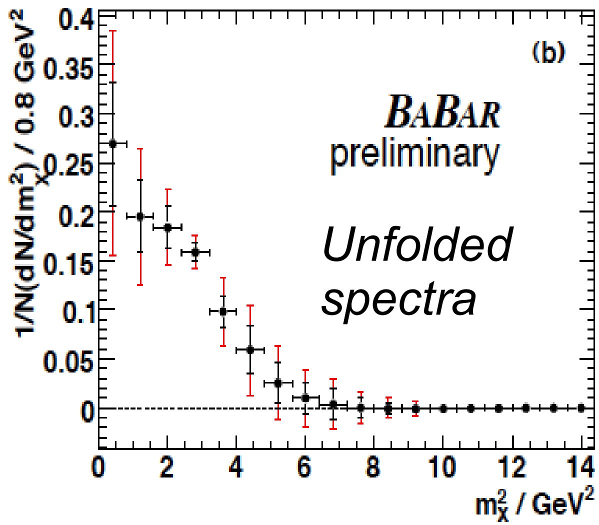
$$|V_{ub}|_{excl.} = (3.47 \pm 0.20^{+0.60}_{-0.39\,FF})10^{-3}$$

BLNP-HPQCD=0.5 +/- 0.7

# Hadronic Moments in $B \to X_u \ell \nu$ decays

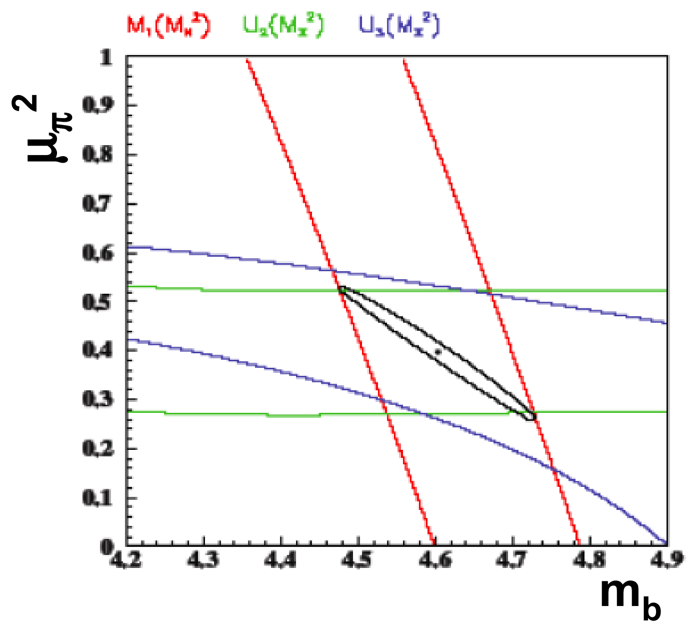
- *Measure hadronic mass spectrum over full $m_X$ range (same strategy/datasample used to extract $|V_{ub}|$)*
- *Mass moments related to $m_b$: extract moments with upper cut $m_x^2 < 6.4$ GeV$^2$*




ArXiv: 0801.2985[hep-ex]

| Mom | | Stat. | Syst. | |
|---|---|---|---|---|
| M1 | 1.96 | ±0.34 | ±0.53 | GeV$^2$ |
| U2 | 1.92 | ±0.59 | ±0.87 | GeV$^4$ |
| U3 | 1.79 | ±0.62 | ±0.78 | GeV$^6$ |




**Calculations of Gambino, Ossola, Uraltsev JHEP09(2005)010**

*First measurement of $m_b$ in $B \to X_u \ell \nu$ decays (in the Kinetic scheme)*

$$m_b = 4.604 \pm 0.250 \text{ GeV}$$

*compatible with Global Fit*

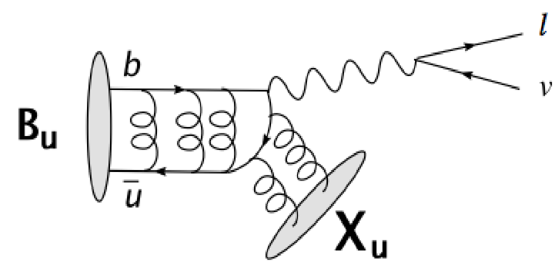
# Weak annihilation in $B \to X_u \ell \nu$

ArXiv: 0708.1753
383 M BB

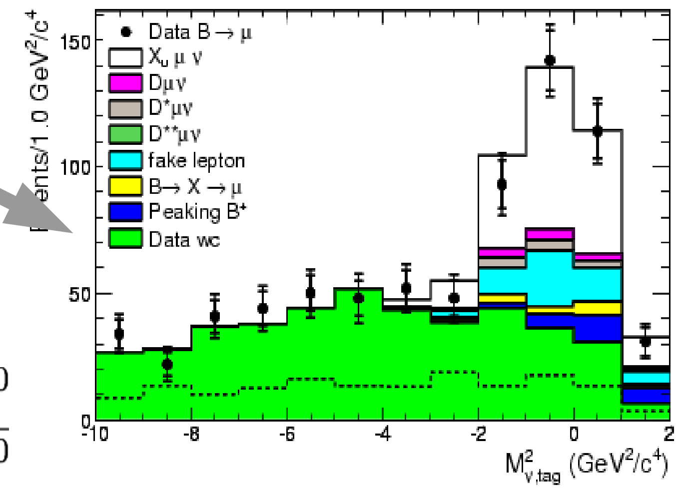
- *Small contribution to $B \to X_u \ell \nu$ decays:*

$$\frac{|\Gamma_{WA}|}{.\Gamma_u} < 7.4\% \ at \ 90\% C.L.$$

*CLEO, studing the $q^2$ spectra PRL96,121801 (2006)*




- *Introduce difference between $B^0$ and $B^+$ decays*
- *Tag with partial reconstructed $B^0 \to D^{*+} \ell \nu$*
- *Neutrino mass from kinematics: $m_\nu^2 = (P_B - P_{D^*} - P_\ell)^2$*
  - *Compare $B^0$ partial rate to charge averaged $B \to X_u \ell \nu$ rate in the large $p_\ell$ region (to enhance the WA contribution) PRD73,012006(2006)*




Charge asymmetry: $A^{+/0} = \frac{\Delta\Gamma^+ - \Delta\Gamma^0}{\Delta\Gamma^+ + \Delta\Gamma^0}$

| $\Delta p$ | $\Delta\mathcal{B}(B) \cdot 10^4$[8] | $\Delta\mathcal{B}(B^0) \cdot 10^4$ | $A^{+/0}$ |
|---|---|---|---|
| $2.2 - 2.6\,\mathrm{GeV}/c$ | 2.31±0.10±0.18 | 2.62±0.33±0.16 | -0.17±0.15±0.11 |
| $2.3 - 2.6\,\mathrm{GeV}/c$ | 1.46±0.06±0.10 | 1.30±0.21±0.07 | 0.08±0.15±0.08 |
| $2.4 - 2.6\,\mathrm{GeV}/c$ | 0.75±0.04±0.06 | 0.76±0.15±0.05 | -0.05±0.20±0.10 |

*For $\Delta p$=2.3-2.6 GeV*
*~300 $X_u$ events (e+μ)*

$$\frac{|\Gamma_{WA}|}{\Gamma_u} < \frac{3.8\%}{f_{WA}(2.3 - 2.6)} \ at \ 90\% C.L.$$

*- $f_{WA}(\Delta p)$ is the fraction of WA in a $\Delta p$ interval*

# Conclusion

- *Determination of* $|V_{ub}/V_{cb}|$ *complements* **sin2β** ∩ $|V_{td}/V_{ts}|$ *to test the SM*
- *Significant improvemets of understanding semileptonic decays in the last years:*
  - *Thanks to the continuous* ***theory*** ⇔ ***experiment*** *interactions*
- *Inclusive* $B \to X_c \ell \nu$ *decays precisely determined* $|V_{cb}|$, $m_b$ *etc.*
- *Inclusive* $B \to X_u \ell \nu$ *achieved <~9% error on* $|V_{ub}|$ *(crucial the role of* $m_b$*)*
  - *room for improvements on both systematics and statistics*
  - *challenge the theory: WA and HQE parameters from decays* $b \to u \ell \nu$

# Backup Slides

# *Theory: references*

- *Approaches based on the OPE, require the SF*
  - *3-scale OPE based on HQET, SCET:  Bosch, Lange, Neubert, Paz PRD72:073006(2005)*
  - *Kinetic Scheme: Gambino, Giordano, Ossola, Uraltsev JHEP10(2007)058*
  - *Relate b uln to b sg with weight functions: Lange, Neubert, Paz JHEP 0510:084,2005; Leibovith,Low,Rothstein PLB486:86*
  - *Select a region with reduced SF dependece: Bauer, Ligeti, Luke PRD64:113004(2001)*
- *Parton level approach, no SF is needed but require a model of non-perturbative QCD effects (can be tested on other measurements: b quark fragmentations data)*
  - *Dressed Gluon Exponentiation: Andersen, Gardi JHEP0601:097(2006)*
  - *Analytuc Coupilg: Aglietti, Ferrera, Ricciardi PR74 (2006) 034006, PRD74(2006) 034005, PRD74(2006)034004*