Lattice QCD:
A progress report

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Flavor Physics & CP Violation 2008

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Outline

1. Introduction: Lattice QCD
2. Decay constants: $P \to l\nu$
   - f_D and f_{D_s}: test of lattice QCD
   - f_B and f_{B_s}
3. Semileptonic decays
   - Exclusive $B \to D^* l\nu$: determination of $|V_{cb}|$
   - $B \to \pi l\nu$: determination of $|V_{ub}|$
4. Neutral meson mixing ($\Delta F = 2$)
   - Indirect CP violation in neutral kaons: $\hat{B}_K$
   - B^0 neutral mixing: $\Delta M_{d,s}$, $\Delta\Gamma_{d,s}$ and ξ
5. Conclusions and outlook

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`Goal: control systematic errors`

Quenched approximation: neglect vacuum polarization effects
→ uncontrolled and irreducible errors

Unquenched work with $N_f = 2$ or $\boxed{N_f = 2+1}$ flavours of sea quarks

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Light quark formalism: speed, discretization errors, chiral symmetry, technical issues.

Quark formulation	speed	chiral symmetry	Collaboration
Improved staggered	fast	$U(1) \times U(1)$ (ok)	HPQCD/MILC/FNAL
Overlap	very slow	exact	JLQCD
Domain Wall	slow	m_{res} (good)	RBC/UKQCD
Wilson	Moderately fast	bad	JLQCD/QCDSF/ ALPHA/CERN/PACS-CS
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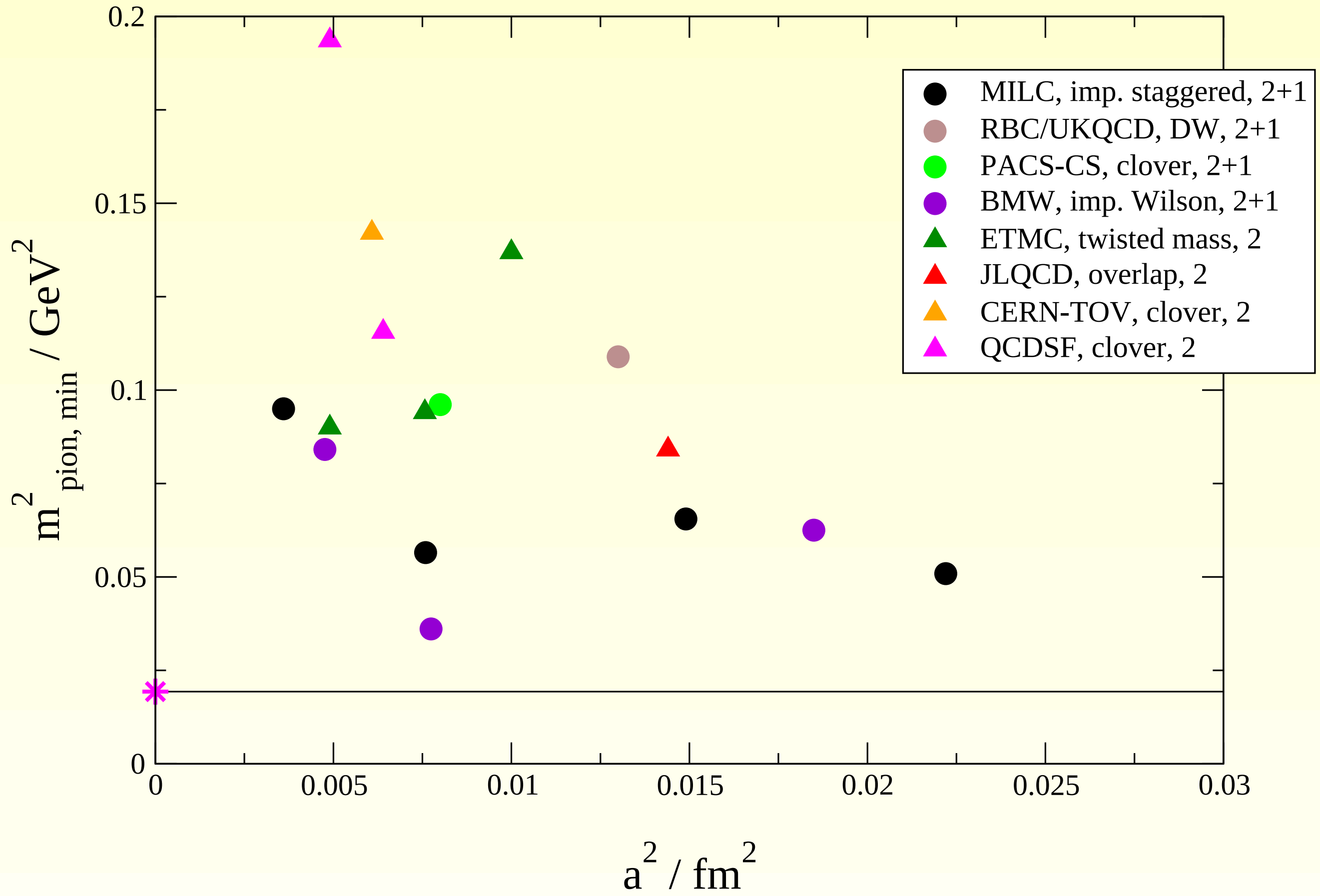
Mixed actions: Different formulations for sea and valence quarks.

$N_f = 2$ and $N_f = 2+1$ ensembles available



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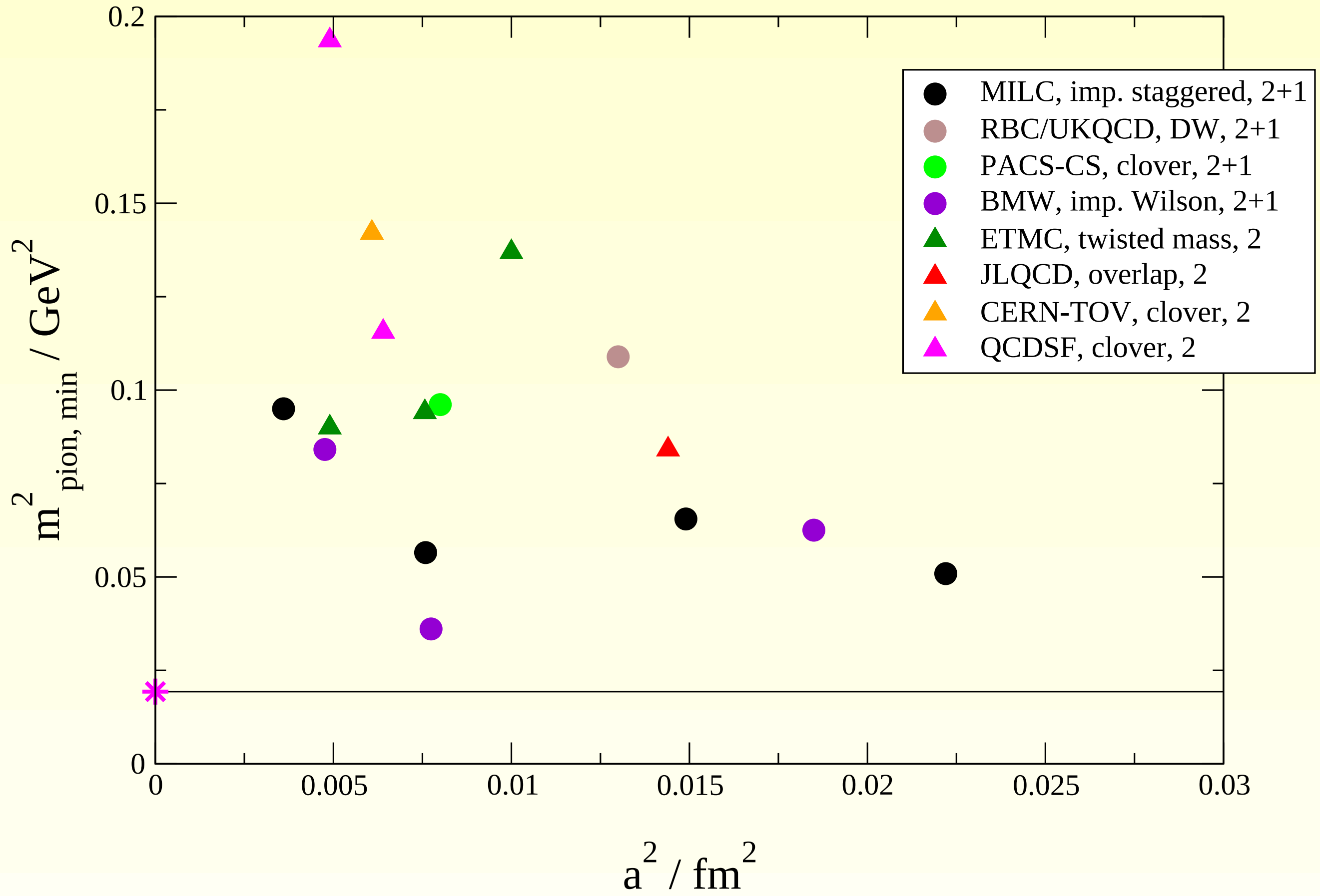

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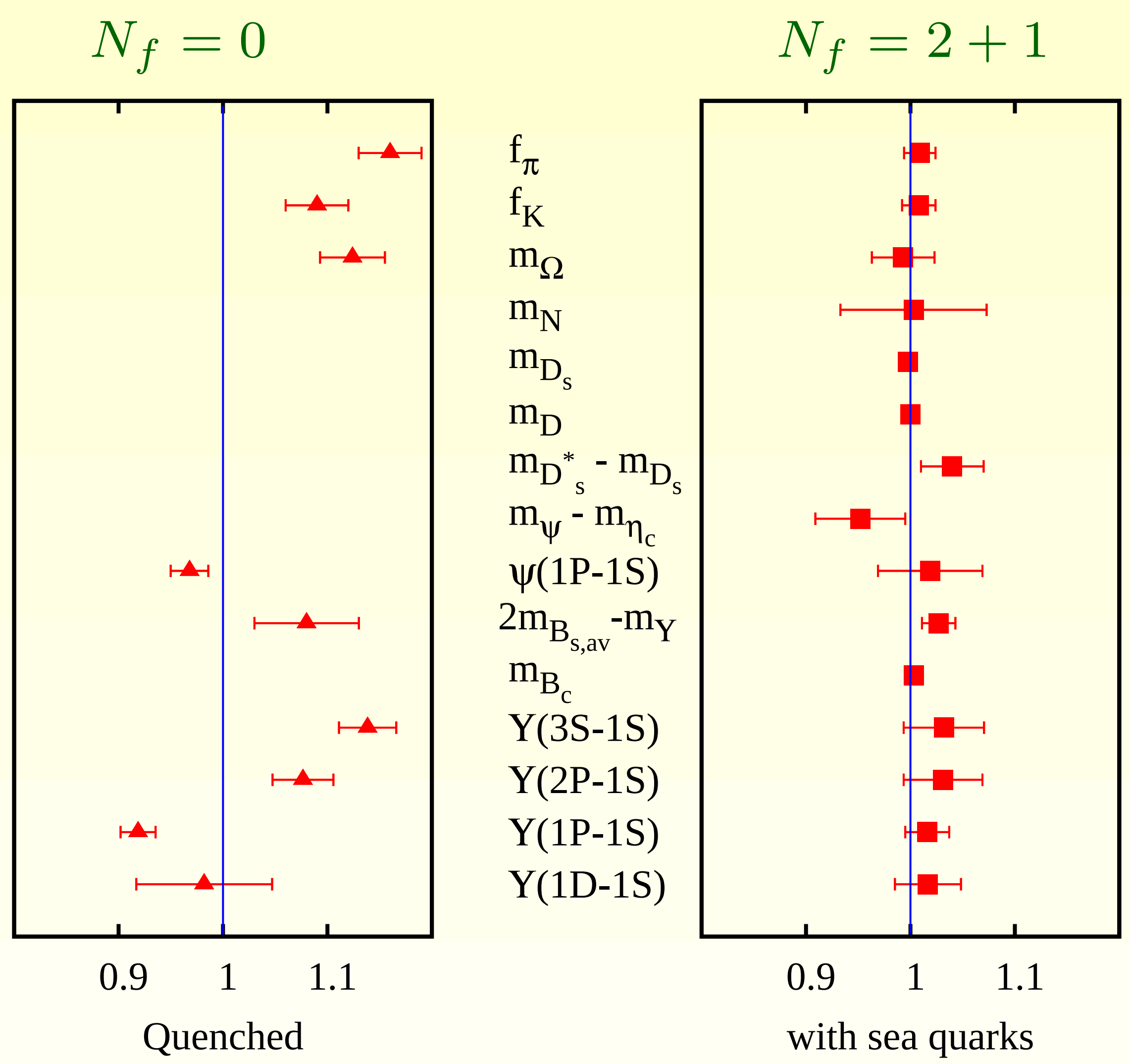
Use chiral perturbation theory to extrapolate to $m_{u,d}$

Staggered χPT: remove leading $\mathcal{O}(a^2)$ errors in fits. Bernard, Sharpe and Aubin

Testing Lattice QCD



$m_D^{latt.} = 1.868(7)$

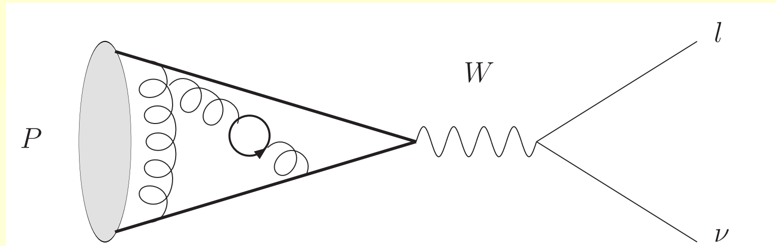
$m_D^{exp.} = 1.868$

$m_{D_s}^{latt.} = 1.962(6)$

$m_{D_s}^{exp.} = 1.968$

Experimental quantities are quite well reproduced by lattice when including realistic sea quark effects

2. Decay constants: $P \to l\nu$



Purely leptonic decays can be used to extract **CKM** matrix **elements**

$$\Gamma(P_{ab} \to l\nu) \propto f_P^2 |V_{ab}|^2$$

or testing **SM/lattice** predictions

f_D and f_{D_s}: test of lattice QCD

$$\underbrace{B(D_q \to l\nu)}_{\textbf{experiment}} \propto |V_{cq}|^2 \quad \underbrace{f_{D_q}^2}_{\textbf{lattice}}$$

Simple matrix element $\langle 0|\bar{q}\gamma_\mu\gamma_5 c|D_q(p)\rangle = i f_{D_q} p_\mu$ → precise calculations

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Results from two groups with $N_f = 2+1$

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Highly improved staggered quarks (**HISQ**): Reduction of $\mathcal{O}(a^2\alpha_s)$ and $\mathcal{O}((am_Q)^4)$ discretization errors $\rightarrow$ Very precise results for charm physics, charmonium and D, (m_c fixed by η_c). E. Follana et al (2007)HPQCD

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* **MILC** ensembles: 3 lattice spacings (0.09 fm ,0.12 fm, 0.15 fm)

* Renormalization partially non-pert. (FNAL/MILC, 1.5% error) and normalization via PCAC (HPQCD, no error)

* Simultaneous chiral and continuum extrapolation including all a, valence and sea quark masses:
  SChPT (FNAL/MILC) and continuum ChPT + $\mathcal{O}(a^2)$ terms (HPQCD).

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Latest Results (2007/08)

Sensitive to **BSM** physics: Starting to see evidence for nonstandard leptonic decays of D_s mesons? **Dobrescu and Kronfeld 2008**

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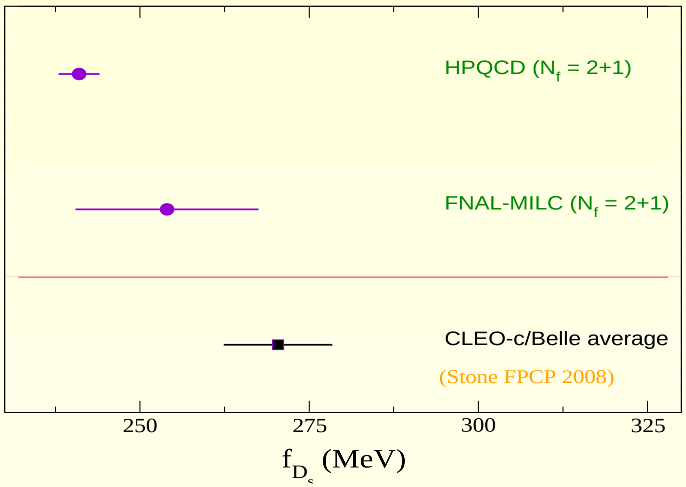

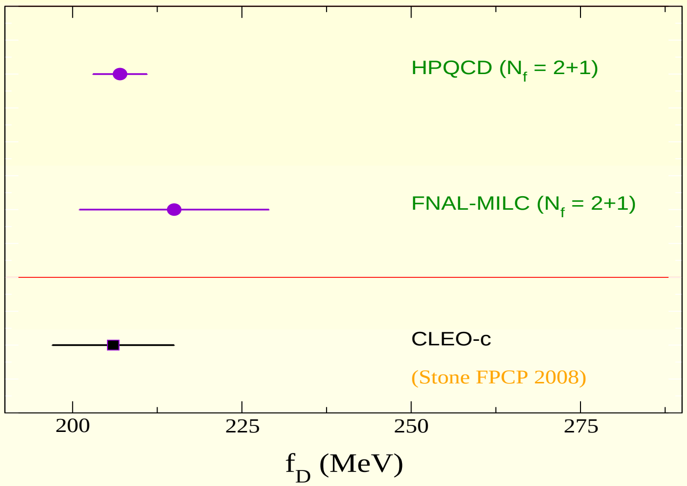

$$f_{D_s}^{HPQCD} = 241 \pm 3 \qquad f_D^{HPQCD} = 207 \pm 4$$

$> 3\sigma$ discrepancy between experiment and **HPQCD** lattice f_{D_s}.

Experiment-lattice agreement in f_K, f_π, f_D, m_D, m_{D_s}, $\frac{2m_{D_s} - m_{\eta_c}}{2m_D - m_{\eta_c}}$.

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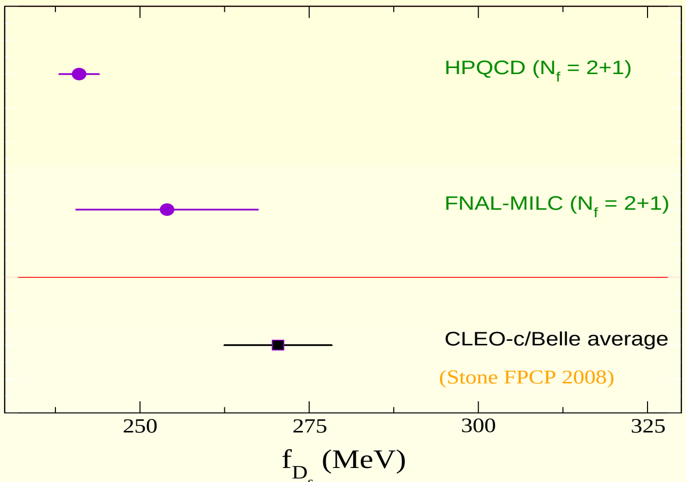

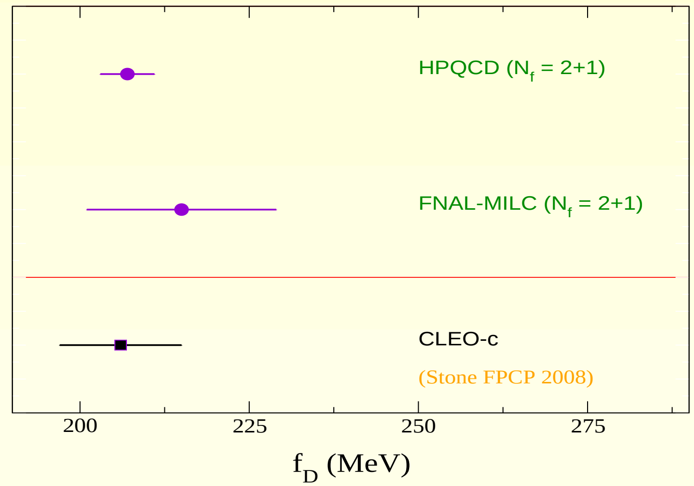

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Expected reduction of experimental errors

Experiment uses $V_{cs} = V_{ud}$.

Main sources of uncertainty in lattice f_{D,D_s}

FNAL/MILC plan to work on more (smaller) lattice spacings $\rightarrow$

* Smaller lattice spacing: $a = 0.06\ fm$ already exist, $a = 0.04\ fm$ in production.

Statistics: smearings, random wall sources, more ensembles, twice as many configurations.

Fixing m_c and improving the determination of the scale r_1 (Υ system).

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* expected reduction of the error by FNAL/MILC:

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Other fermion formulations.

f_B and f_{B_s}

Extraction of CKM matrix elements: $\underbrace{B(B^- \to \tau^- \bar{\nu}_\tau)}_{\textbf{experiment}} \propto |V_{ub}|^2 \underbrace{f_B^2}_{\textbf{lattice}}$

$(\langle 0|\bar{q}\gamma_\mu\gamma_5 b|B_q(p)\rangle = i f_{B_q} p_\mu)$

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Decay constants needed in the SM prediction for processes potentially very sensitive to BSM effects: for example, f_{B_S} for $B_s \to \mu^+\mu^-$

$B^- \to \tau^- \bar{\nu}_\tau$ is a sensitive probe of effects from charged Higgs bosons.

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$N_f = 2+1$ determinations

heavy valence quarks HPQCD NRQCD, FNAL/MILC Fermilab action

	FNAL-MILC (LAT2007)	HPQCD (2005)		
f_B (MeV)	197 ± 13	216 ± 22		
f_{B_s} (MeV)	240 ± 12	260 ± 26		
f_{B_s}/f_B	1.22 ± 0.03	1.20 ± 0.03		

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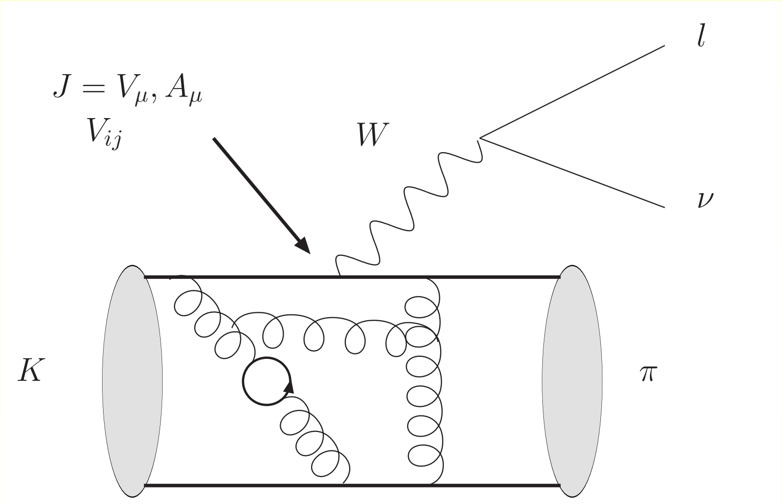
	FNAL-MILC (LAT2007)	HPQCD (2005)	errors % current	errors % in 2-5 years
f_B (MeV)	197 ± 13	216 ± 22	6.8-10.3	4.0
f_{B_s} (MeV)	240 ± 12	260 ± 26	5.1-10.1	3.5
f_{B_s}/f_B	1.22 ± 0.03	1.20 ± 0.03	2.7-2.6	2.0

Extraction of f_{B_s}/f_B from **double ratios**: e.g. $[f_{B_s}/f_B]/[f_K/f_\pi]$

3. Semileptonic decays



Exclusive $B \to D^* l\nu$: determination of $|V_{cb}|$

$B \to D^* l\nu$ rate at zero recoil $\propto |V_{cb} h_A(1)|$

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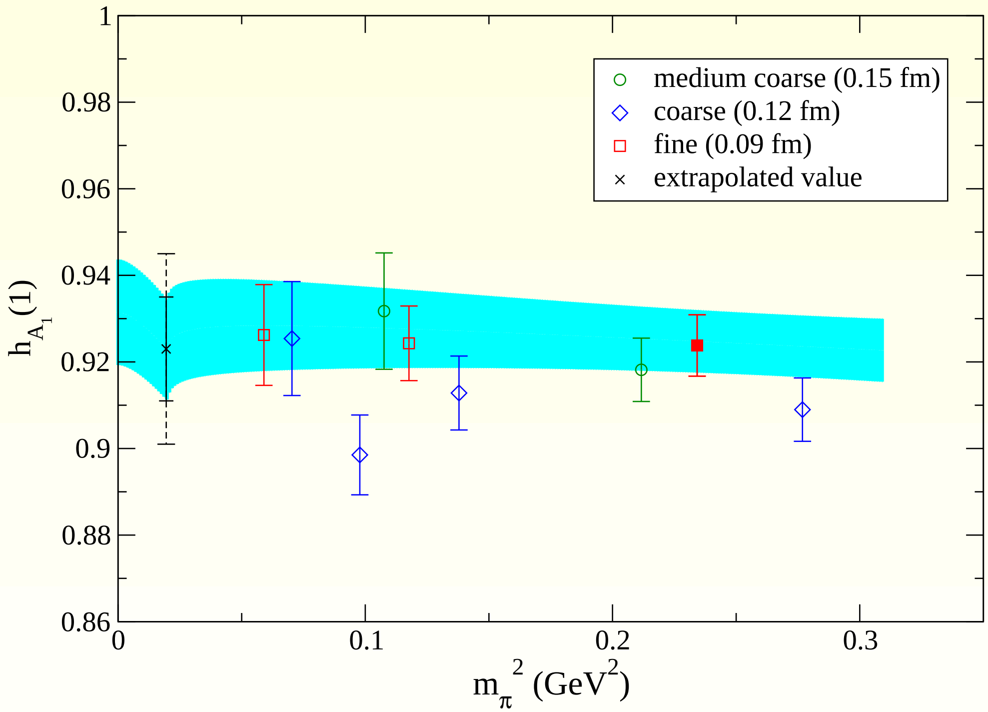

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HFAG

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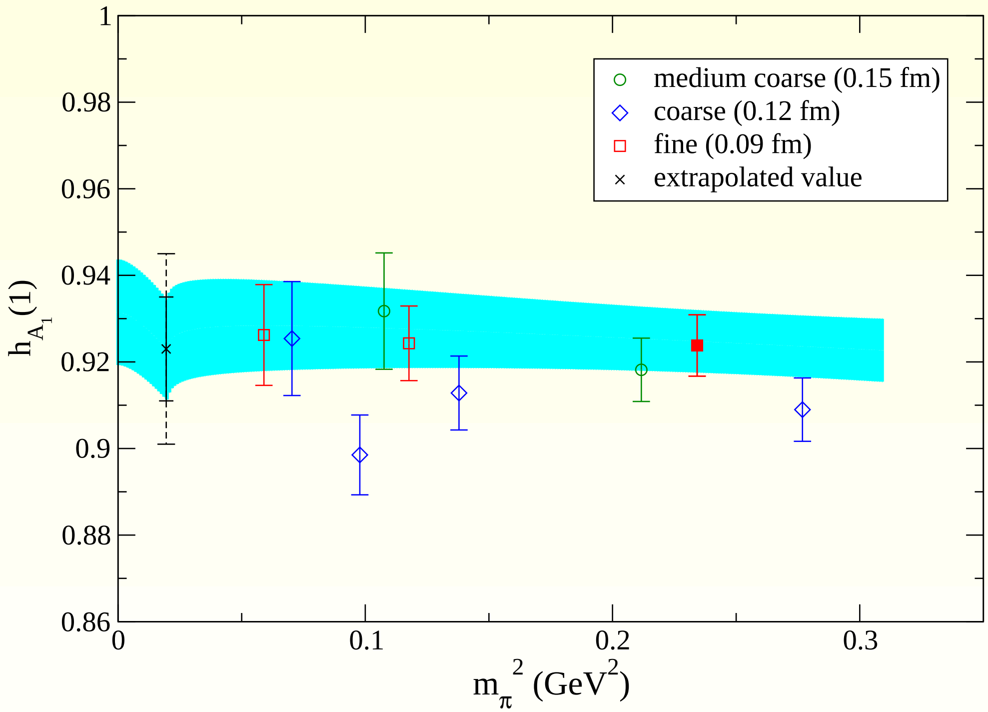

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↓ stat. + disc. errors

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(one year)

$B \to \pi l\nu$: determination of $|V_{ub}|$

Only $N_f = 2+1$ calculation so far: staggered **HPQCD PRD73/75 (2006/07)**

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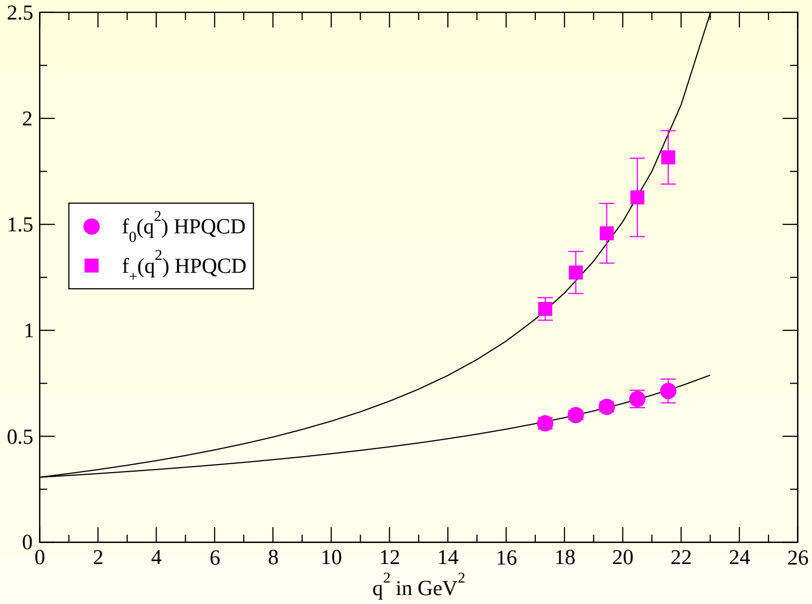

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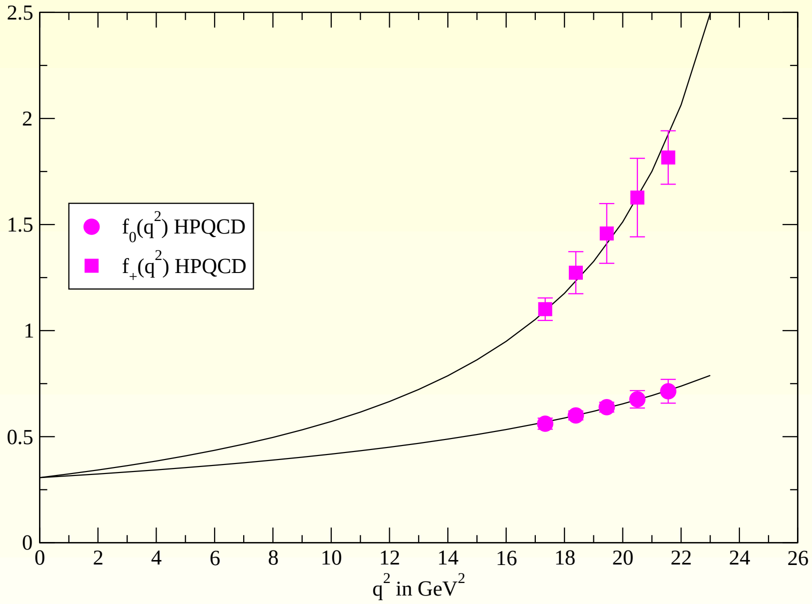

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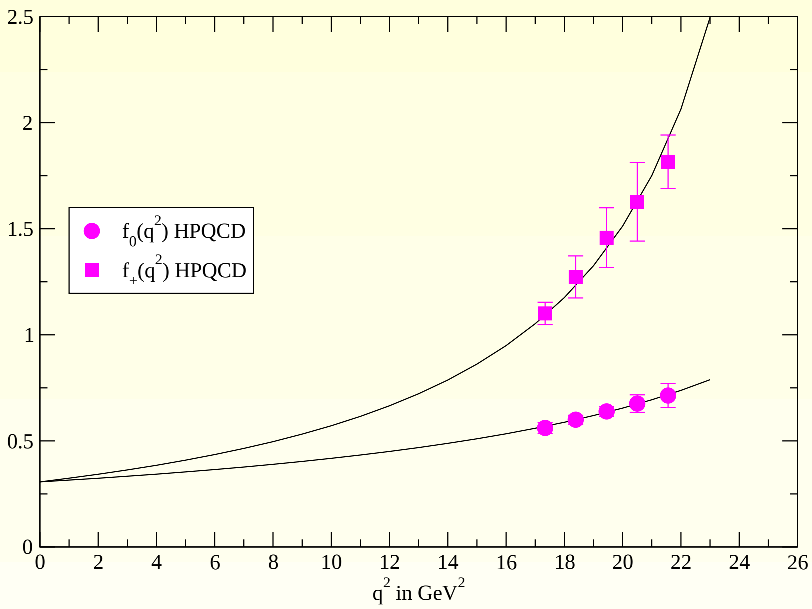

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Poor overlap in q^2 between lattice and experiment
$\rightarrow$ increases the total error

Work in progress to reduce total error

* **Moving NRQCD**: Generate data at low q^2 + keeping statical errors under control **K. Wong Lattice2007**.

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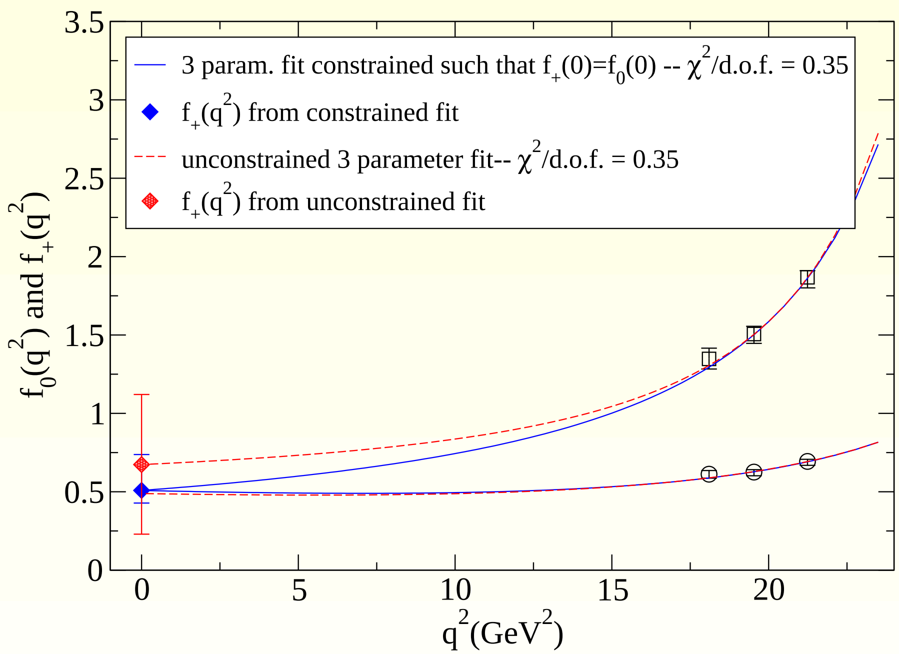

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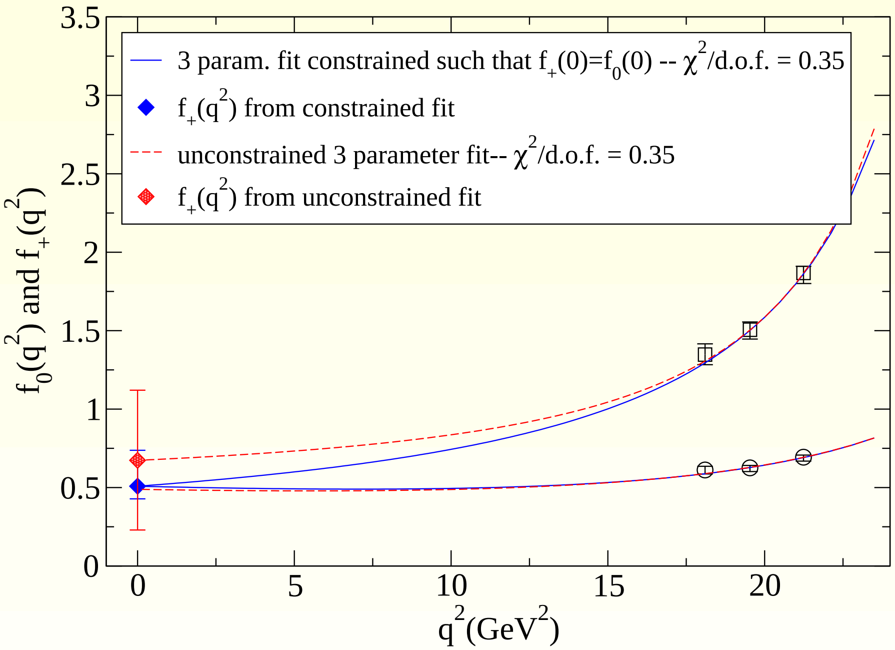


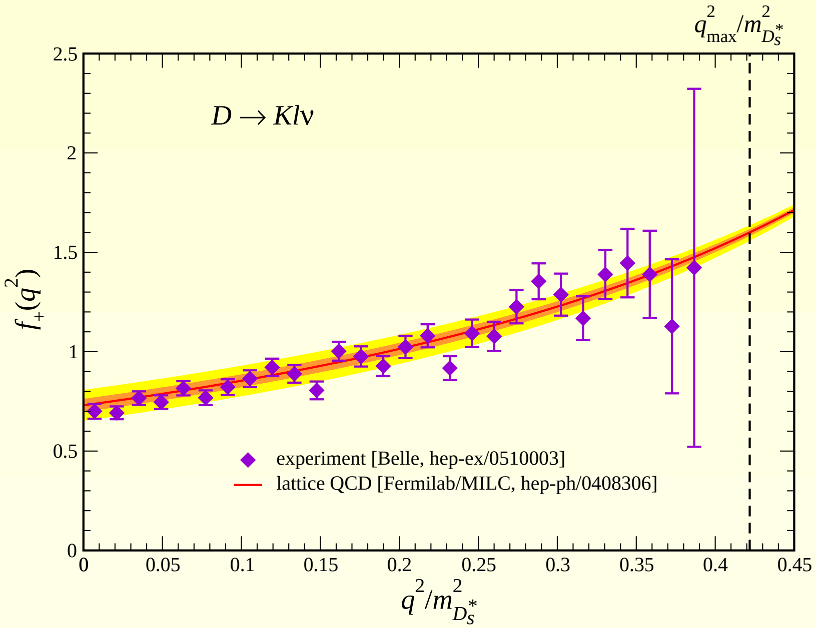
P. Mackenzie and **R. Van de Water** (2007)

Work underway to extend lattice results → **FNAL-MILC (Mackenzie, LAT07)**

`total error after finishing current analysis` $\sim 10\%$.

Semileptonic decays: Improvements in progress

$D \to \pi l \nu$ and $D \to K l \nu$:



Andreas Kronfeld (2005)

* FNAL-MILC 2005 results are consistent with experiment for $f_+(0) \leftrightarrow$ predict $V_{cd}(V_{cs})$ with $14\%(11\%)$

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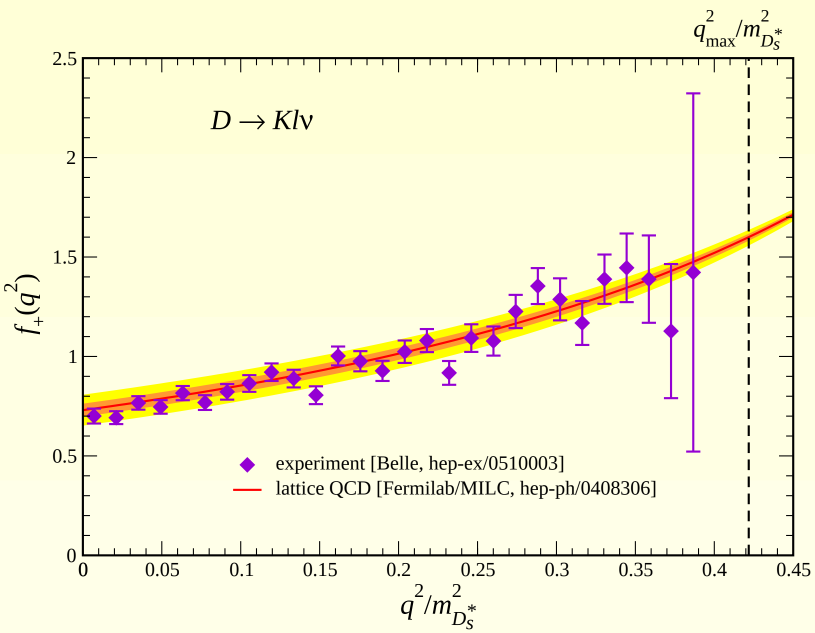

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FNAL-MILC working on $N_f = 2 + 1$ calculation of the form factors $f_+^{D\to\pi}(0)$ and $f_+^{D\to K}(0)$ (reduction of discr. errors) $\to$ V_{cd} and V_{cs}.

$\boxed{D \to \pi l \nu \text{ and } D \to K l \nu\text{:}}$

* $\frac{\Gamma(D \to l\nu)}{\Gamma(D \to \pi l \nu)}$ independent of $|V_{cq}|$ $\to$ consistency check
* $\frac{\Gamma(D_s \to l\nu)}{\Gamma(D \to K l \nu)}$ CKM independent test of lattice (QCD)

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* **Becirevic, Haas and Mescia**: Testing systematic errors reduction for several double ratios with $N_f = 2$ `Wilson fermions.`

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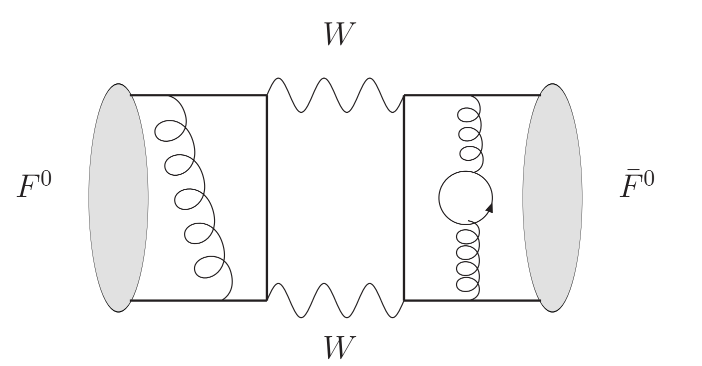
$B \to D l \nu$ (**alternative determination of V_{cb}**):

de Divitiis et al 2007 Quenched analysis in the framework of HQET

* Including the case of `non-vanishing lepton mass.`

* Demonstrate feasibility of using methodology in the unquenched theory

4. Neutral meson mixing ($\Delta F = 2$)



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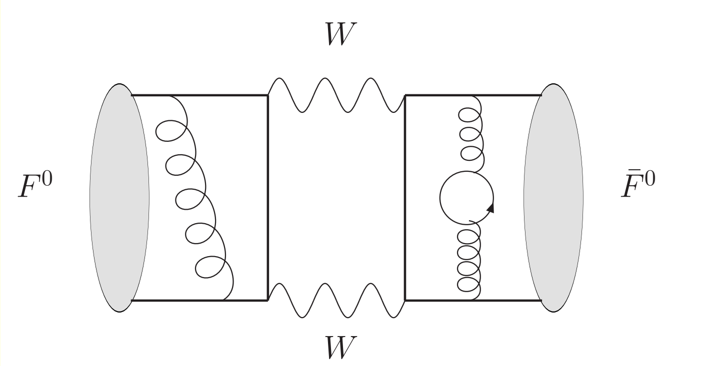

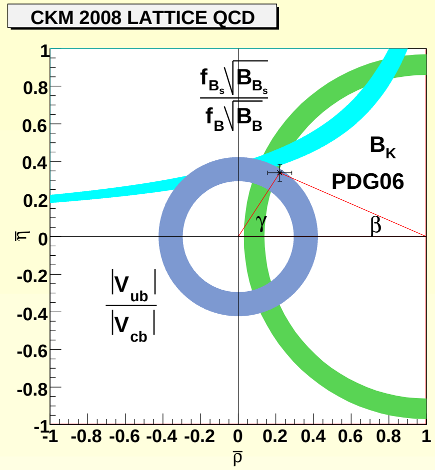

Indirect CP violation in neutral kaons: $\hat{B}_K$

$$\underbrace{|\epsilon_K|}_{\text{experimental}} = \left| \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} \right|$$

$$\underbrace{B_K(\mu)}_{\text{lattice}} = \frac{\langle \bar{K}^0 | Q_{\Delta S=2}(\mu) | K^0 \rangle}{\frac{8}{3} \langle \bar{K}^0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K^0 \rangle}$$

} hyperbole in the $\rho - \eta$ plane of the **UT** $(\mathrm{Im}\,(V_{ts}V_{td}^*) \sim \bar{\eta}[(1-\bar{\rho}) + const.])$.

* B_K largest source of error to extract CKM combination from experimental ε_K

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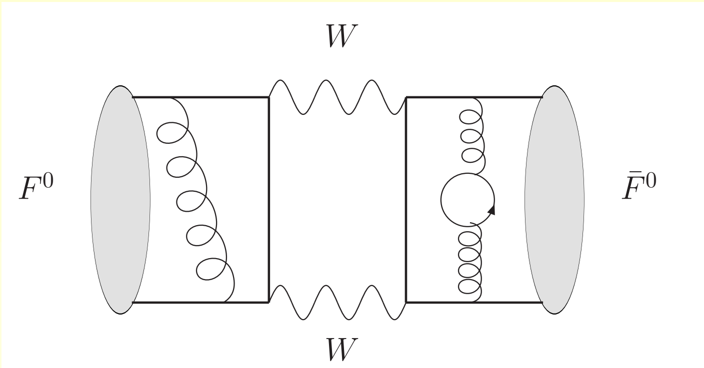

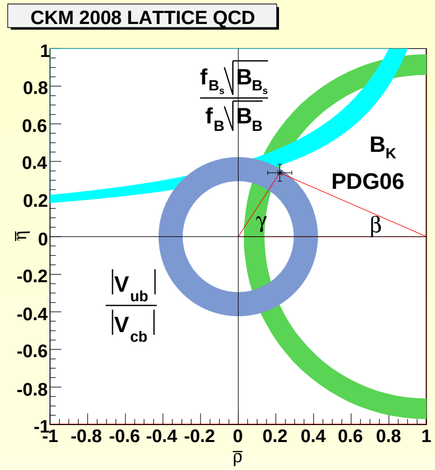

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* B_K largest source of error to extract CKM combination from experimental ε_K

** Expected `reduction` in B_K error to a `few per-cent` level $\rightarrow V_{cb}$ error becomes significant.

Indirect CP violation in neutral kaons: $\hat{B}_K$

Calculate box in lattice QCD needs good chiral symmetry $\rightarrow$ control the allowed operator mixing $\rightarrow$ simplify renormalization (smaller errors)

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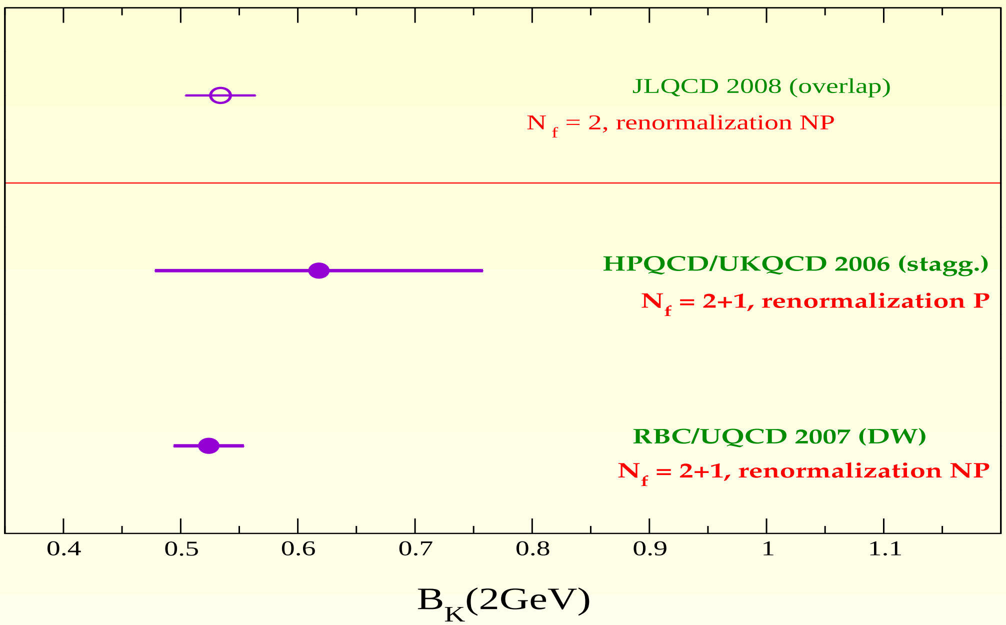

Recent unquenched results

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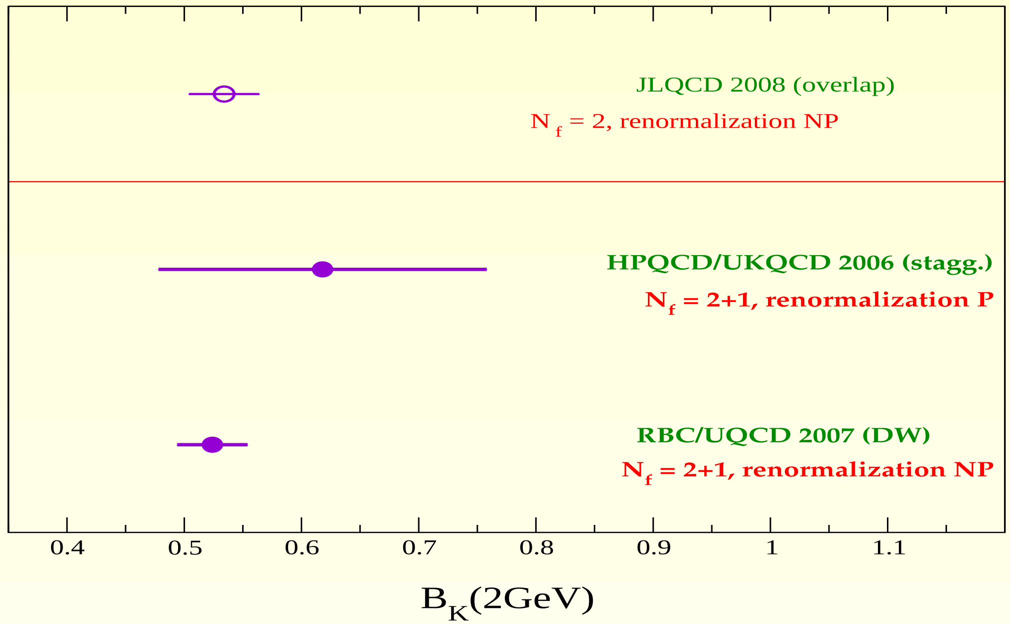

Recent unquenched results

Current most accurate result by RBC-UKQCD:

$$B_k^{\overline{MS}}(2\ GeV) = 0.524(10)_{stat.}(25)_{sys.}(13)_{ren.}$$

(6% error)

* NLO $SU(2) \times SU(2)$ χPT (2% error).

* Only one lattice spacing (4% error) → need check discret. errors.

(work in progress)

JLQCD generating $N_f = 2 + 1$ ensembles (two different volumes)

$\longrightarrow$ remove dominant uncertainties.

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Underway `overlap valence + twisted mass sea` $N_f = 2$ calculation

ALPHA, Scorzato LAT2007

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ALPHA, Scorzato LAT2007

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* twisted mass: computationally more efficient than overlap or DW

Underway `DW valence + staggered sea` $N_f = 2+1$ calculation:

Aubin, Laiho, Van de Water

* Non-perturbative renormalization

* Large number of staggered sea quark configurations **MILC**

* $\simeq 5-6\%$ error expected.

B^0 neutral mixing: $\Delta M_{d,s}$, $\Delta\Gamma_{d,s}$ and ξ

Experimental measurements:

CDF

$$\Delta M_s|_{exp.} = 17.77 \pm 0.12\, ps^{-1}$$

PDG07 average

$$\Delta M_d|_{exp.} = 0.507 \pm 0.005\, ps^{-1}$$

$$\Delta\Gamma_s|_{exp.}^{\mathbf{D\emptyset}} = 0.17 \pm 0.09 \pm 0.02\, ps^{-1}$$

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- theoretically: In the Standard Model

$$\Delta M_q|_{theor.} \propto |V_{tq}^* V_{tb}|^2 \quad f_{B_q}^2 \hat{B}_{B_q}$$

$\Longrightarrow$ **Need accurate theoretical calculation of** $f_{B_q}^2 \hat{B}_{B_q}$

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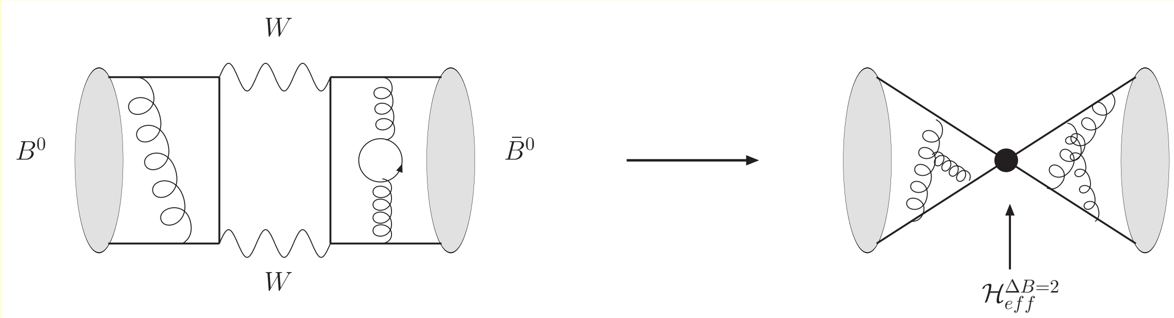
Precise determination of CKM matrix elements

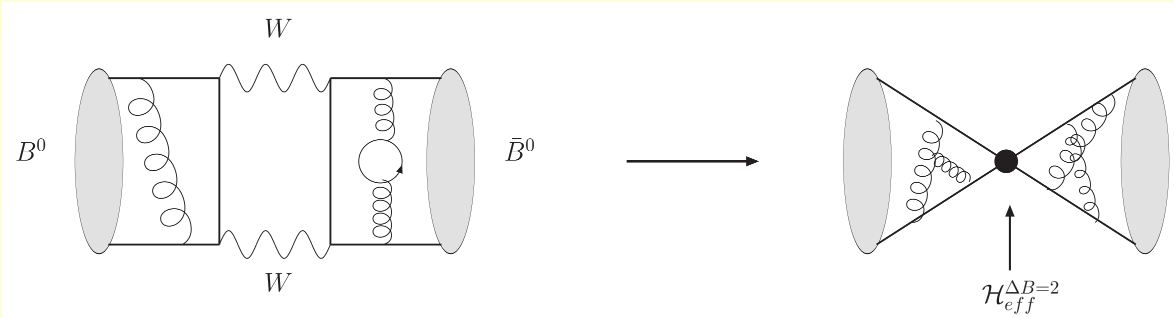
$$\left|\frac{V_{td}}{V_{ts}}\right| = \underbrace{\frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}}_{\xi} \sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}}$$

* Many uncertainties in the theoretical (lattice) determination cancel totally or partially in the ratio



NP could enter through new particles in box diagrams.

Recent claims of NP effects in $B_s^0 - \bar{B}_s^0$ mixing (**Bona et al. (UTfit Col.)**) and $B_d^0 - \bar{B}_d^0$ mixing (**Lunghi and Soni**)



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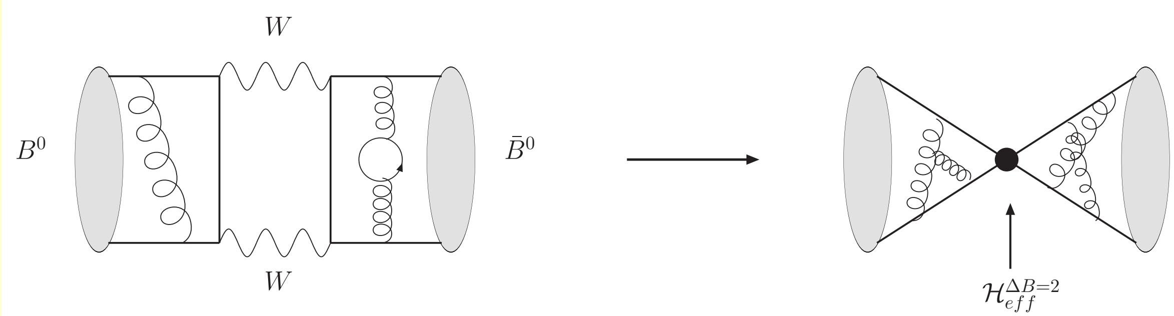
Two unquenched $N_f = 2 + 1$ calculations underway: HPQCD and MILC/FNAL

* Improved staggered (Asqtad) for light quarks and NRQCD (HPQCD) Fermilab action (MILC/FNAL)

* Calculation of all the matrix elements needed to determine $\Delta M_{d,s}$, $\Delta\Gamma_{d,s}$ and ξ.



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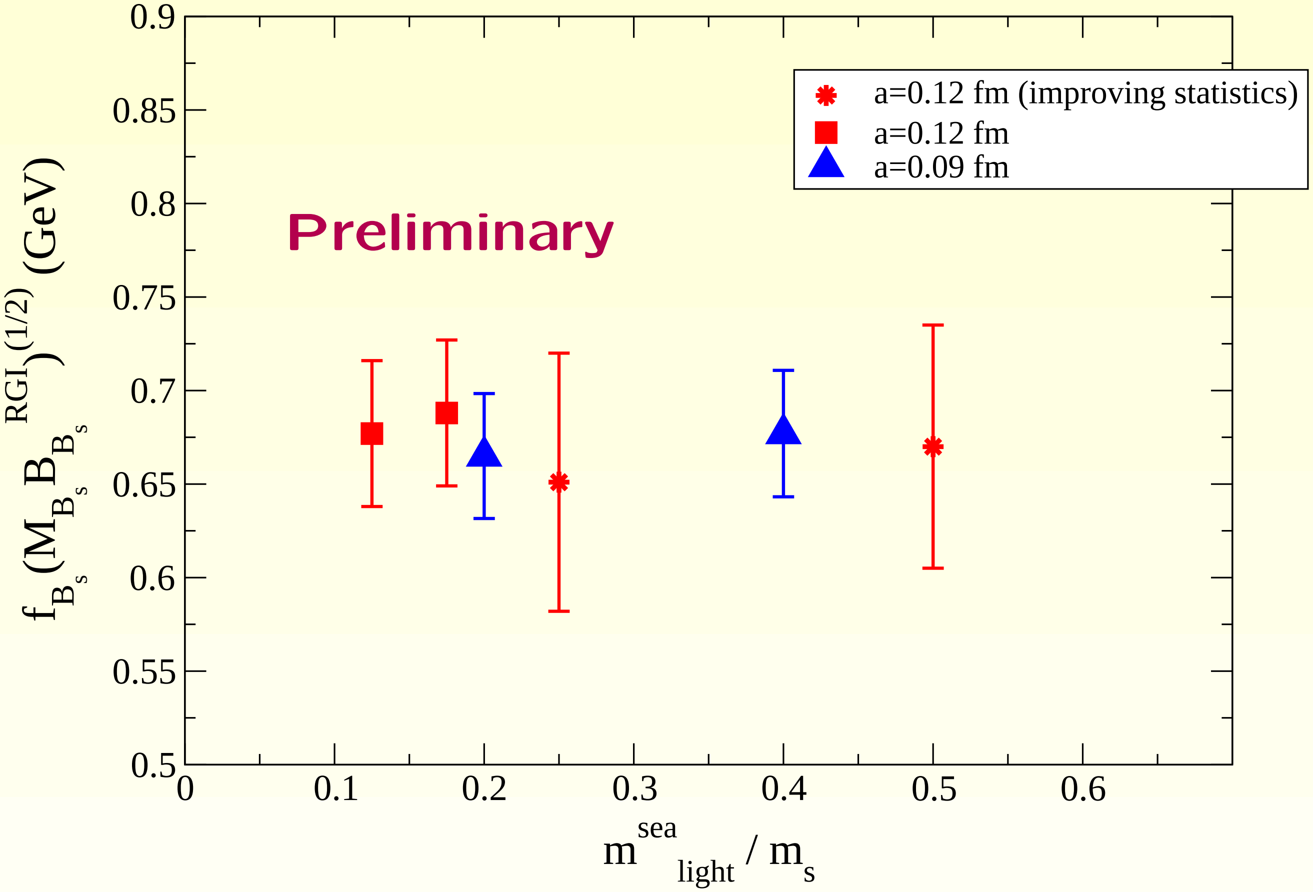
- * Calculation of all the matrix elements needed to determine $\Delta M_{d,s}$, $\Delta\Gamma_{d,s}$ and ξ.

Current status: working on the chiral extrapolation
(NLO+analytic NNLO SχPT)

Preliminary results for $f_{B_q}\sqrt{M_{B_q}B_{B_q}}$

$f_{B_s}\sqrt{M_{B_s}\hat{B}_{B_s}}(GeV^{3/2})$

HPQCD



with $m_s^{valence}$ fixed to its physical value and m_s^{sea} very close to it.

statistics+fitting errors $\sim 1-2\%$

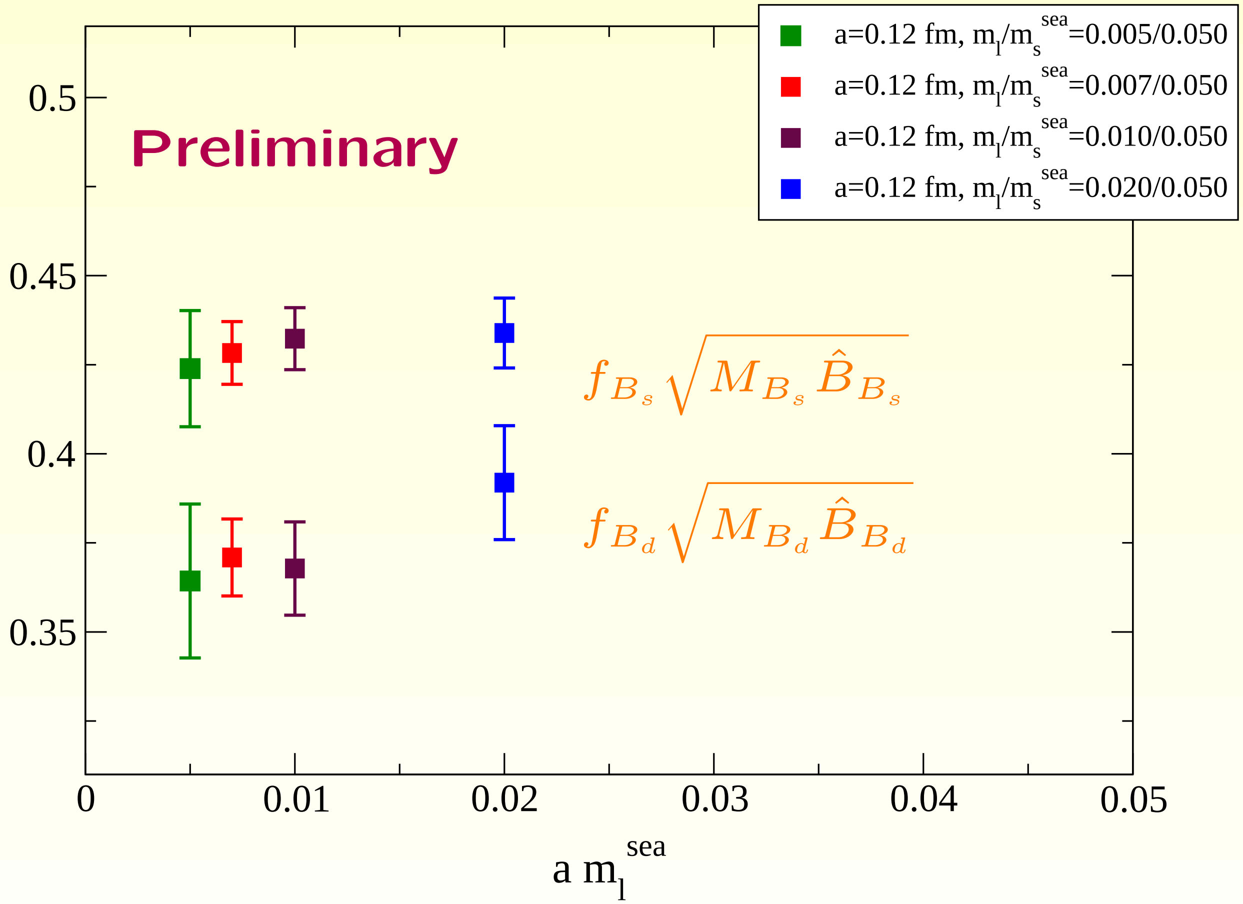
Statistics and systematic errors included

Same for $f_{B_d}\sqrt{B_{B_d}}$

Preliminary results for $f_{B_q}\sqrt{M_{B_q}B_{B_q}}$

$a^{3/2} f_{B_q}\sqrt{M_{B_q}B_{B_q}}$

Fermilab/MILC

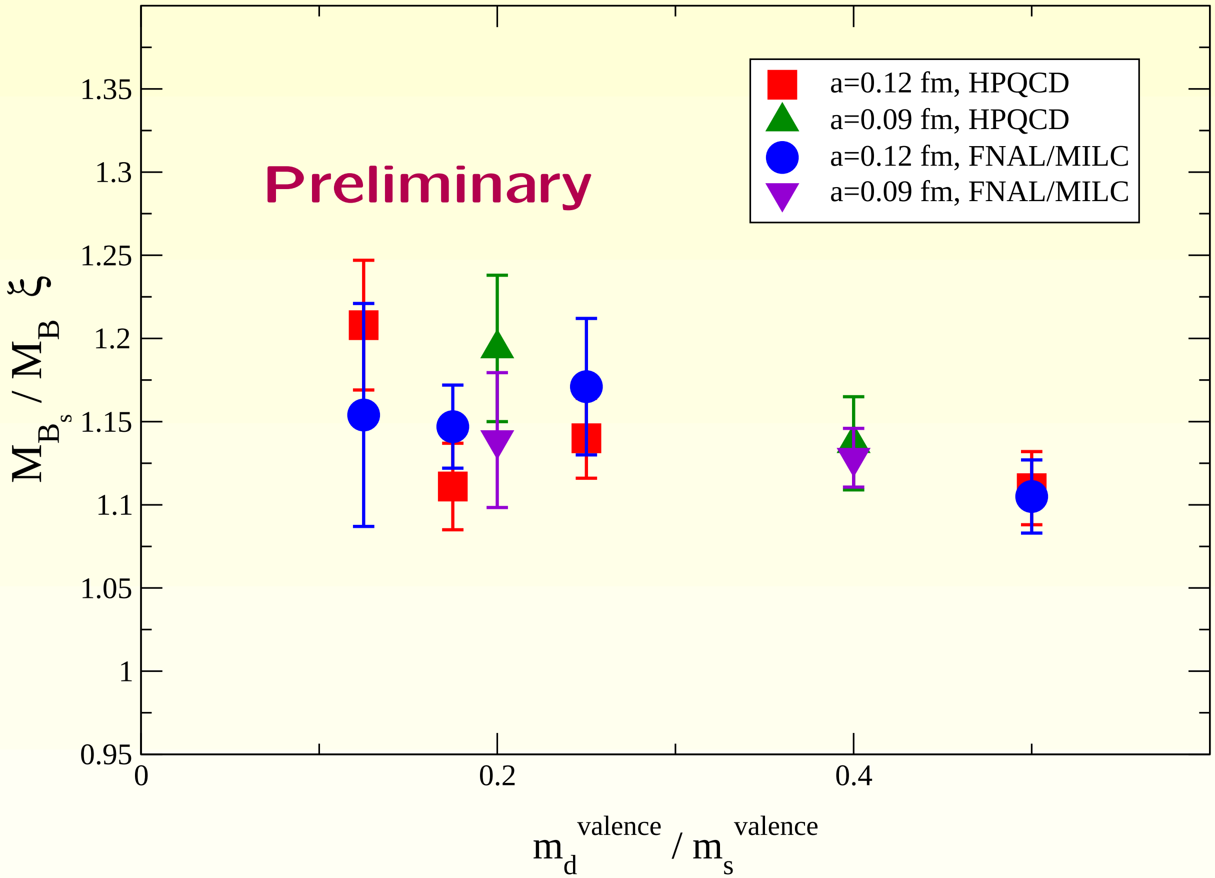


Example: Ensembles with $a = 0.12\ fm$.

Full QCD: only statistical errors included

Preliminary results for ξ: Full QCD

$$\xi\, M_{B_s}/M_{B_d} = (f_{B_s}\sqrt{M_{B_s} B_{B_s}})/(f_{B_d}\sqrt{M_{B_d} B_{B_d}})$$



Only statistical errors included.

Only full QCD points included.

Discussion of errors

	$f_{B_q}\sqrt{B_{B_q}}$	ξ
Total (estimate)	$5-7\%$	$2-3\%$

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Expected improvements in 2 years: `smaller lattice spacings`, `better statistics`, development of `non-perturbative` or partially non-perturbative matching, more accurate `inputs` (am_b, a, ...).

Reduction of errors by a factor of $1.5-2$

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Underway RBC/UKQCD: **C. Albertus** *et al.*

* In an early stage: static limit, $m_{pion} \geq 400 MeV$,...

B_0 mixing beyond the SM

Effects of heavy new particles seen in the form of effective operators built with **SM** degrees of freedom

$$\mathcal{H}_{eff}^{\Delta F=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \widetilde{C}_i \widetilde{Q}_i$$

** With Q_i and $\widetilde{Q}_i$ four-fermion operators

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- $\langle \bar{F^0} | Q_i | F^0 \rangle$ calculated on the lattice

SM predictions + BSM contributions + experiment

$\rightarrow$ constraints on BSM physics

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Complete $N_f = 2+1$ analysis of $\Delta B = 2$ matrix elements expected from both **Fermilab lattice-MILC** and **HPQCD** collaborations in 1-2 years with errors$< 10\%$.

D^0 neutral mixing

See **Eugene Golowich's talk**

What can lattice calculate?

Long-distance:

Current lattice techniques are inefficient
for calculating non-local operators

* Straightforward approach requires a unreasonable increase of computing time to account for non-locality.

$\Downarrow$

* Need to develop new techniques to have accurate ($\sim 10\%$ errors) results.

What can lattice calculate?

Short-distance: We can calculate the matrix elements involved in the the SM and general BSM analysis on the lattice.

* Same techniques and effective hamiltonian as for B^0 mixing.

* This kind of studies can `exclude large regions of parameters` in many models, `constraining BSM building`.

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* A consistent unquenched determination of all matrix elements involved, free of the uncontrolled uncertainties associated to quenching is needed

* **FNAL/MILC** col. plans to calculate these matrix elements in the next 2 years with at least a 10% precission.

5. Conclusions and outlook

Important progress in lattice calculations including **sea quarks** $(N_f = 2 + 1)$

* Precise new results (few percent errors) in `Kaon` and `D` sectors.

* **Expected for this year**: precise results in b physics: B^0 mixing parameters, decay constants.

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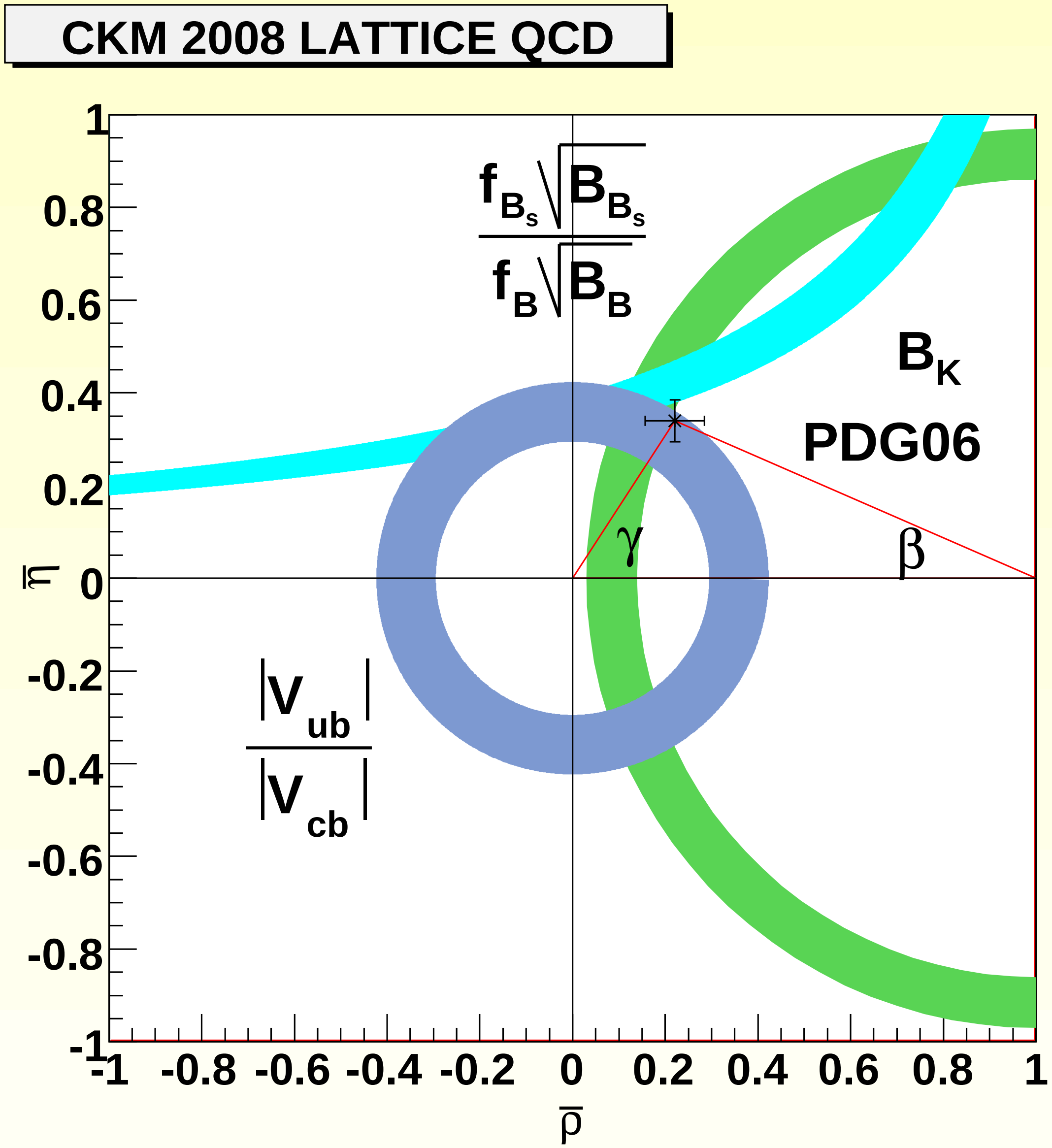
Prospects for next two years

* Reduction in uncertainties of quantities relevant for CKM physics by a factor of around 2.

* Consistency checks of lattice QCD methods by ...

** more comparison against experiment.

** comparing lattice calculations using different fermion formulations.



- * $\hat{B}_K$ from RBC/UKQCD
- * $\frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}}$ preliminary result from FNAL/MILC
- * $|V_{ub}|$ from Flynn and Nieves, 0705.3553
- * $|V_{cb}|$ from Jack Laiho, LAT2007
- * $|V_{us}|$ from $K_{l2}^{exp.} + \underbrace{\frac{f_K}{f_\pi}}_{\text{HPQCD}}$

C. Davies & C. McNeile

Other Heavy-light semileptonic decays

	Flavour neutral	Unstable	affordable now	in 5 years?
$B \to \eta l \nu$	$\surd$		possible but expensive	
$B \to \eta' l \nu$	$\surd$	$\surd$		$\surd$
$B \to \rho l \nu$		$\surd$		$\surd$
$B \to \omega l \nu$	$\surd$	$\surd$		$\surd$
$B \to K l l$			$\surd$	
$B \to K^* l l$		$\surd$		$\surd$
$B \to \phi l l$	$\surd$	$\surd$		$\surd$
$B \to K^* \gamma$		$\surd$		$\surd$

R. Van de Water

HISQ action

E. Follana et al, HPQCD coll.

- Highly improved staggered action.
- Much improved control of discretization errors.
  - * Highly reduce $\mathcal{O}(a^2\alpha_s)$ errors (an order of magnitude)
  - * Substantially reduce taste-changing with respect to Asqtad
  - * No tree-level $\mathcal{O}((am)^4)$ at first order in the quark velocity v/c

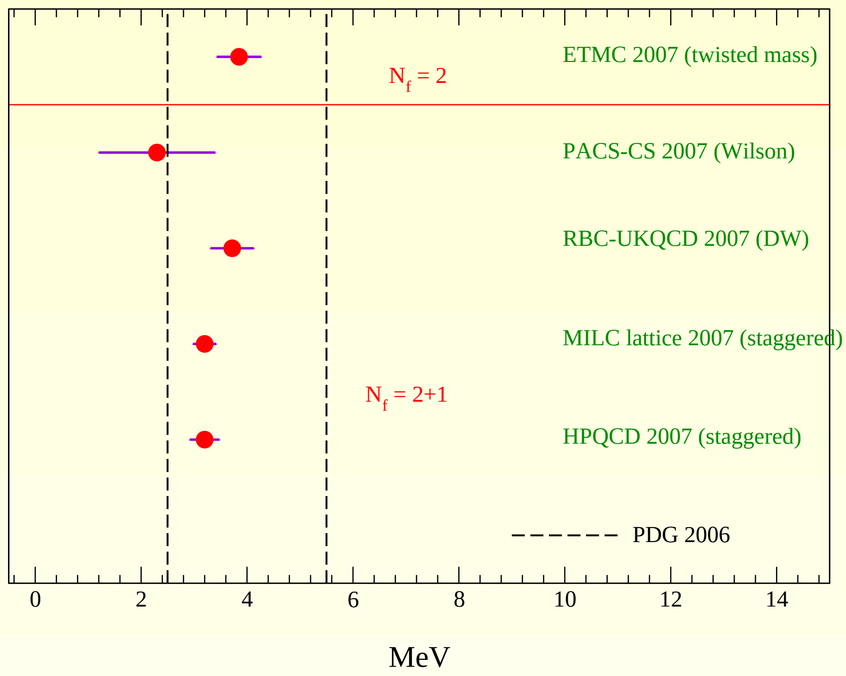
    $\longrightarrow$ accurate results for charm quarks

Error budget for decay constants

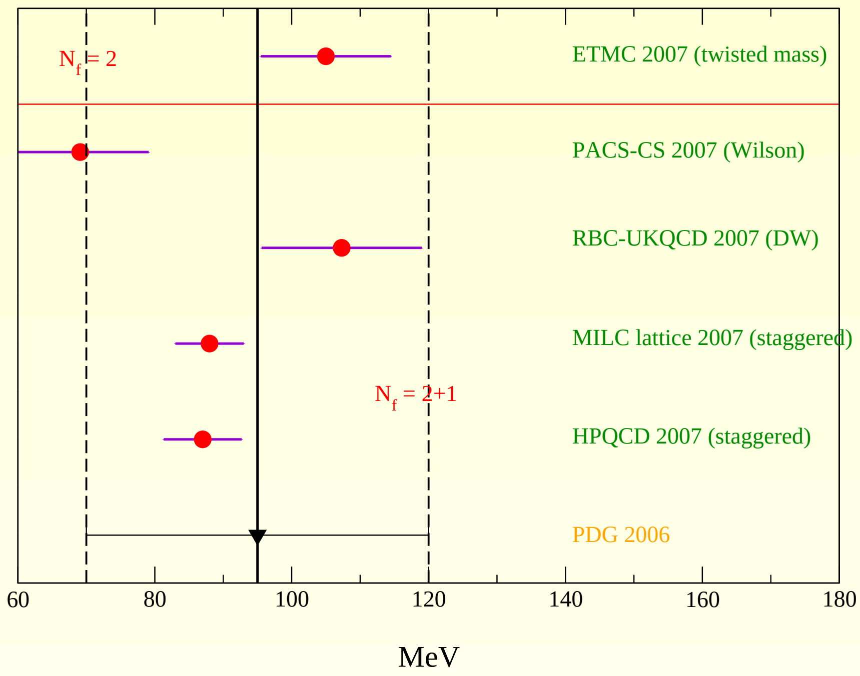
	f_π	f_K	f_K/f_π	f_D	f_{D_s}	f_{D_s}/f_D
r_1 uncert.	1.4	1.1	0.3	1.4	1.0	0.4
a^2 extrap.	0.2	0.2	0.2	0.6	0.5	0.4
finite volume	0.8	0.4	0.4	0.3	0.1	0.3
$m_{u/s}$ extrap.	0.4	0.3	0.2	0.4	0.3	0.2
statistical	0.5	0.4	0.2	0.7	0.6	0.5
m_s evol.	0.1	0.1	0.1	0.3	0.3	0.3
m_d, QED, etc	0.0	0.0	0.0	0.1	0.0	0.1
Total(%)	1.7	1.3	0.6	1.8	1.3	0.9

Quark masses

$$\hat{m}^{\overline{MS}}(2GeV) = \frac{(m_u+m_d)}{2}$$



$$m_s^{\overline{MS}}(2GeV)$$



New: Determination of the charm quark mass

HPQCD coll., Chetyrkin, Kühn, Steinhauser & Sturm

m_c extracted from current-current correlators.

* **HISQ** action used to determine moments G_n of charm-quark pseudoscalar, vector and axial-vector correlators.

$$G_n \equiv \sum_t (t/a)^n G(t)$$

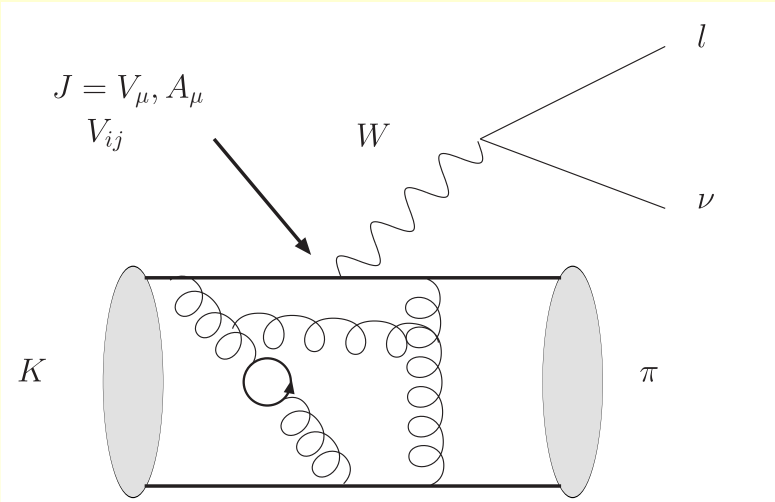
with

$$G(t) \equiv a^6 \sum_{\vec{x}} (am_{0c})^2 \langle 0|J(\vec{x},t)J(0,0)|0\rangle$$

* Four-loop results from continuum perturbation theory for the moments.

$m_c(m_c) = 1.266(16) GeV$ or equivalently $m_c(3GeV) = 0.983(13) GeV$

$K \to \pi l \nu$: $|V_{us}|$ from Kaon semileptonic decays (K_{l3})



See Federico Mescia's talk

Latest result [RBC/UKQCD (2007)]: $f_+^{K\pi}(0) = 0.9644(33)_{stat.}(34)_{q^2,\chi}(14)_a$

* Only one lattice spacing $\to$ need check disc. errors

$\to$ calculation in a `second lattice spacing` in progress.

* Further technical improvements to reduce systematic and statistical errors in progress (twisted boundary conditions, stochastic volume averages)

Calculations with other fermion formulations:

ETMC (twisted mass), **FNAL-MILC** (staggered)

Same programme can be applied for extra operators

$$\langle \bar{B^0}_{d(s)} | Q_{i=1-5} | B^0_{d(s)} \rangle$$

- Chiral perturbation theory more complicated (extra free parameters):

$$\langle \overline{B^0_{d(s)}} | Q_{i=1-5} | B^0_{d(s)} \rangle \rightarrow_{chiral} \Gamma_i (1 + L) + \underbrace{\Gamma_i' L'}_{i \neq 1} + \text{analytic terms}$$

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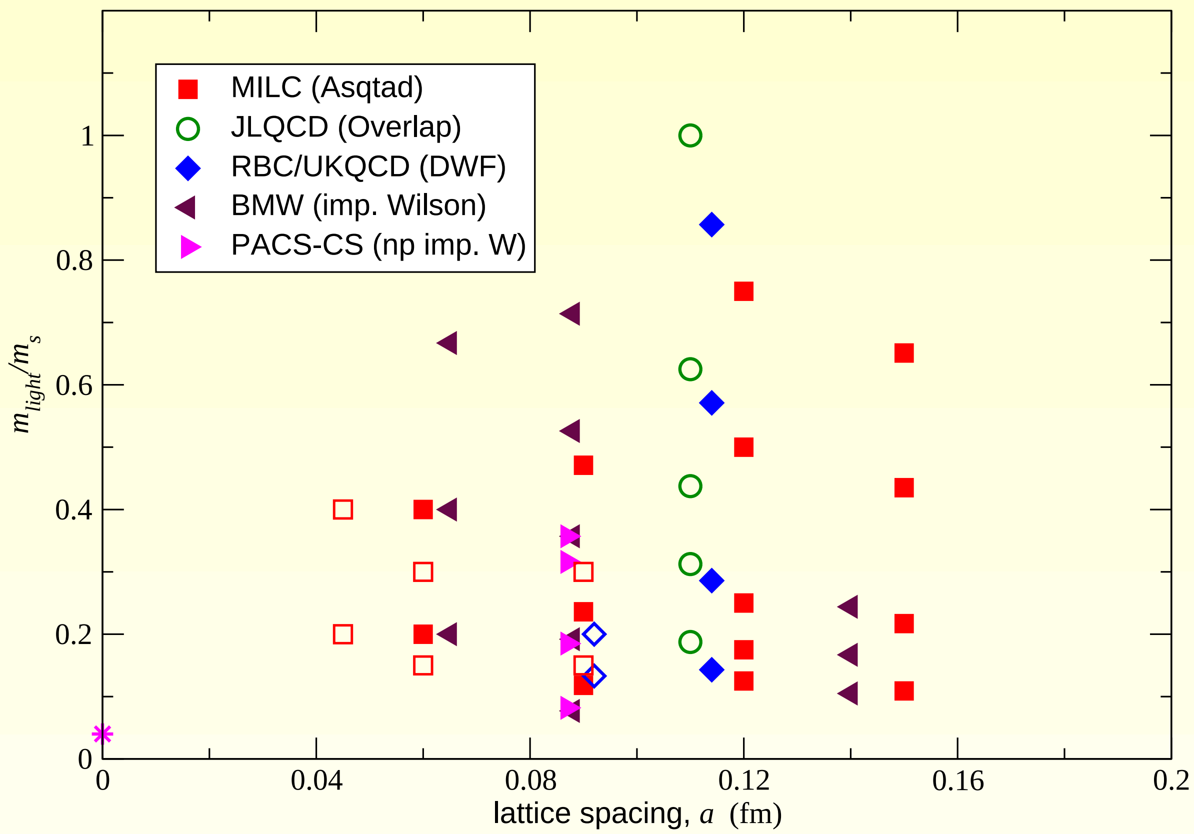
Complete N_{f+1} analysis of $\Delta B = 2$ matrix elements expected from both **Fermilab lattice-MILC** and **HPQCD** collaborations in 1-2 years with `errors` $< 10\%$.

* First results: One-loop renormalization for **HPQCD** study

(**E.G,Shigemitsu,Trottier**)

$N_f = 2 + 1$ ensembles available or in production



A. El-Khadra, 2007