The MSSM

as a HSNJL Model

— talk at PSROC LHC Sym (Jan 011)

OTTO C. W. KONG
— Nat'l Central U, Taiwan

The Nobel Prize in Physics 2008

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"

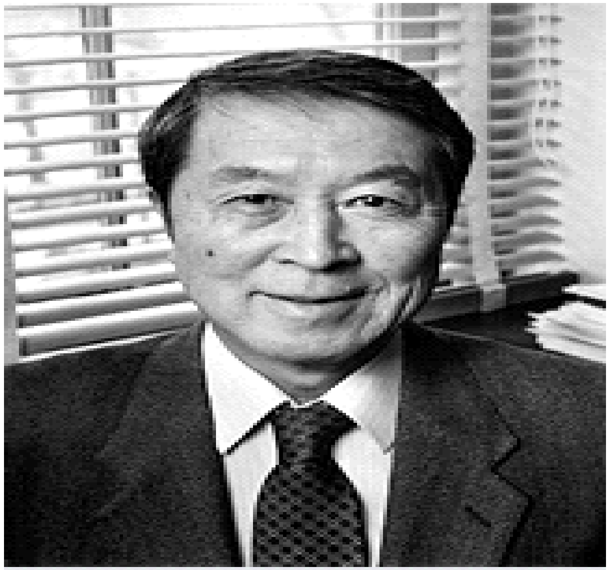

Photo: University of Chicago

Yoichiro Nambu

1/2 of the prize

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Makoto Kobayashi

1/4 of the prize

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Toshihide Maskawa

1/4 of the prize

Phys. Rev. **124**, 246
(issue of 1 October 1961)
Phys. Rev. **122**, 345
(issue of 1 April 1961)
Phys. Rev. Lett. **4**, 380
(issue of 1 April 1960)
Titles and Authors

14 October 2008

Nobel Focus: Particle Physics Gets a Break

A Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity II
Y. Nambu and G. Jona-Lasinio
Phys. Rev. **124**, 246
(issue of 1 October 1961)

A Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity I
Y. Nambu and G. Jona-Lasinio
Phys. Rev. **122**, 345
(issue of 1 April 1961)

Axial Vector Current Conservation in Weak Interactions
Yoichiro Nambu
Phys. Rev. Lett. **4**, 380
(issue of 1 April 1960)

References:

[1] Y. Nambu, "Quasi-Particles and Gauge Invariance in the Theory of Superconductivity," Phys. Rev. **117** 648 (1960).

Physics

Yoichiro Nambu

The Nobel Prize in Physics 2008

Nobel Lecture

Spontaneous Symmetry Breaking in Particle Physics: a Case of Cross Fertilization

The Nobel Lecture of Yoichiro Nambu was presented by Giovanni Jona-Lasinio, La Sapienza, University of Rome, Italy, 8 December 2008, at Aula Magna, Stockholm University. He was introduced by Professor Joseph Nordgren, Chairman of the Nobel Committee for Physics.

History repeats itself

1960 Midwest Conference in Theoretical Physics, Purdue University

A 'SUPERCONDUCTOR' MODEL OF ELEMENTARY PARTICLES AND ITS CONSEQUENCES by Y. Nambu (University of Chicago)†

(In absence of the author the paper was presented by G. Jona-Lasinio.)

The Nambu–Jona-Lasinio (NJL) model

Y. Nambu, G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961)

The Lagrangian of the model is

$$L = -\bar{\psi}\gamma_\mu \partial_\mu \psi + g\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right]$$

It is invariant under ordinary and γ_5 gauge transformations

$$\psi \to e^{i\alpha}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{-i\alpha}$$
$$\psi \to e^{i\alpha\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}$$

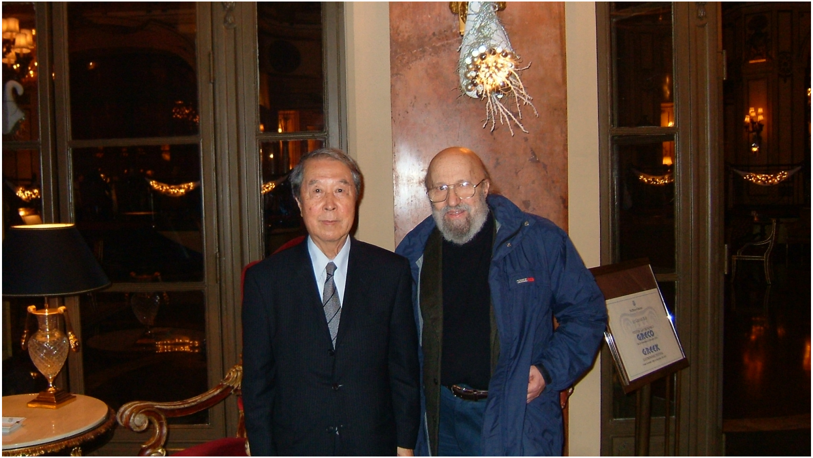
Other examples of BCS type SSB

- ^3He superfluidity
- Nuleon pairing in nuclei
- Fermion mass generation in the electro-weak sector of the standard model

Nambu calls the last entry

> *my biased opinion, there being other interpretations as to the nature of the Higgs field*

Update of the NJL model



The (supersymmetric) Twist :-

- top mode SM Miransky, W. Bardeen, · · · '89/'90 (Nambu)
  — infared (quasi-)fixed point (IQFP) (Pendleton-Ross), Hill, Marciano, · · ·
  prediction : top mass $>$ 200 GeV ; VEV – top condensate

- supersymmetric NJL (formal – '82, SSM – '90)
  — $m_t = y_t \cdot v$, $m_b = y_b \cdot v'$; NJL predicts y not m ; $y_b < y_t$
  — other not very nice features as MSSM
  — lighter top fine, *but* . . . (172.1 GeV top, tan$\beta < 1.5$)

- our **holomorphic SNJL** (alternative supersymmetrization)
  — non-chiral symmetric 4-superfield interaction with t and b
  — superfield condensate : both scalar and fermion condensate
  — $y_t < y_b$; nice , experimentally viable (LHC)

NJL for Electroweak Symmetry Breaking :-

Nambu's Model Resurrected Again, with a Twist

[redacted]

PHYSICAL REVIEW D **81,** 031701(R) (2010)

Holomorphic supersymmetric Nambu—Jona-Lasinio model with application to dynamical electroweak symmetry breaking

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(Received 25 September 2009; published 10 February 2010)

Based on our idea of an alternative supersymmetrization of the Nambu–Jona-Lasinio model for dynamical symmetry breaking, we analyze the resulting new model with a holomorphic dimension-five operator in the superpotential. The approach provides a new direction for modeling dynamical symmetry breaking in a supersymmetric setting. In particular, we adopt the idea to formulate a model that gives rise to the minimal supersymmetric standard model as the low energy effective theory with both Higgs superfields as composites. A renormalization group analysis is performed to establish the phenomenological viability of the scenario, with admissible background scale that could go down to the TeV scale. We give the Higgs mass range predicted.

DOI: 10.1103/PhysRevD.81.031701 PACS numbers: 12.60.Jv, 11.30.Qc, 12.60.Rc

The Modern Perspectives :-

— or my perspectives (Nambu was before SM)

- only effective(quantum field) theories
  — layer after layer (scale)
- symmetry characterizes a theory (completely ?)
- no Dirac fermion / no chiral symmetry
  — nature has no parity symmetry (including space-time ?)
  — chiral (Weyl) fermions
- even scalar fields are chiral, with supersymmetry
  — gauge symmetry forbides (supersymmetric) mass,
  keeps symmetry breaking masses (naturally) ‘small’

⋆ **why NJL ?**

★ Gauge Symmetry fixes spin 1 sector

★ The Story of Fermions ···

— **3 families** of **15** spin $\frac{1}{2}$ quantum fields (**Weyl 2-spinors**)

under $SU(3)_C \times SU(2)_L \times U(1)_Y$

not 4 (Dirac) particles

- $(3, 2, 1)$: u_L u_L u_L d_L d_L d_L
- $(\bar{3}, 1, -4)$: u_R^c u_R^c u_R^c
- $(\bar{3}, 1, 2)$: d_R^c d_R^c d_R^c
- $(1, 2, -3)$: ν_L e_L^-
- $(1, 1, 6)$: e_R^+

— **minimal *chiral* set free from all gauge anomalies**

completely nontrivial cancellation (Vs vectorlike pairing)

SM spectrum :-

— one-family spectrum *very unique*

- taking $SU(3)_C \times SU(2)_L \times U(1)_Y$
- assuming a $(3,2,1)$ multiplet
- *obtained* as the minimal chiral set free from all gauge anomalies

O.K. Mod. Phys. Lett. A11, 2547
O.K. Phys. Rev. D55, 383

— $SU(3)$ requires $(\bar{3},1,a)$ and $(\bar{3},1,b)$

— $SU(2)$ requires an extra $(1,2,c)$

— $U(1)$ anomalies have no solution

$\longrightarrow$ adding a $(1,1,k)$ gives *the* unique solution

$\star$ idea extended to derive the 3-family spectrum

? alternative solution :

- $(3,2,0)$: u_L u_L u_L d_L d_L d_L
- $(\bar{3},1,-c)$: u^c_R u^c_R u^c_R
- $(\bar{3},1,c)$: d^c_R d^c_R d^c_R
- $(1,2,0)$: ν_L e^-_L
- $(1,1,0)$: e^+_R

— some lectures on EW (from famous theorist)

"What would be the simplest fully chiral representation of ...... give an anomaly free theory? ...

... one generation of the Standard Model,... fails by little ...

The simplest representation is in fact a charge 1/2 quark, ..."

- **Witten's anomaly requires at least $(1,2,0)$, ...**

- against vectorlike pair – Georgi's survival hypothesis

  invariant mass at cutoff scale

- SM $\rightarrow$ BSM — hierarchy/fine-tuning problem

  scalar field is somewhat sick

- scalar field content — only part arbitrary (*cf.* gauge symmetry)

- SUSY — **technically** natural hierarchy

  scalar as (part of) **chiral** superfield (constrained as fermions)

  Vs

BUT μ-problem — vectorlike pair of Higgs superfields

- **SNJL models solve our problem**

— and avoid fine-tuning of "four-quark" coupling(s)

Nambu–Jona-Lasinio Model :-

- dynamical symmetry breaking
- four-fermion interaction

$$\mathcal{L}_\psi = i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- + g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_-$$

$$\longrightarrow \mathcal{L}_\psi - (\mu\phi^\dagger + g\psi_+\psi_-)(\mu\phi + g\bar{\psi}_+\bar{\psi}_-)$$

$$= i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- - \mu^2\phi^\dagger\phi - \mu g(\phi^\dagger\bar{\psi}_+\bar{\psi}_- + \phi\psi_+\psi_-)$$

- auxiliary scalar field ϕ (no kinetic term)
- EL-eq for $\phi^\dagger$ gives ϕ as composite

$$\phi = -g/\mu\,\bar{\psi}_+\bar{\psi}_-$$

- $\langle\phi\rangle \neq 0 \Longrightarrow$ symmetry breaking and fermion mass

→ low energy effective field theory :-

- 1-loop effective potential for ϕ gives gap equation

$$\langle\phi\rangle \neq 0 \text{ solution for } \frac{g^2\Lambda^2}{8\pi^2} > 1$$

- Dirac fermion mass $m = \mu g \langle\phi\rangle$ for $\psi_+ - \psi_-$

$$g^2 \bar{\psi}_+ \bar{\psi}_- \psi_+ \psi_- \qquad \Longrightarrow \qquad g^2 \langle \bar{\psi}_+ \bar{\psi}_- \rangle \; \psi_+ \psi_-$$

- kinetic term for ϕ through wave-function renormalization

  fermion-loop propagator with Yukawa vertices ($m \ll \Lambda$)

$$Z = \frac{N_c \mu^2 g^2}{16\pi^2} \left[\ln \frac{\Lambda^2}{M^2} + O(1) \right]$$

- Higgs with mass $2m$ and a Goldstone boson

$\phi \longrightarrow \phi/\sqrt{Z}$:-

- $$\mathcal{L}_\psi = i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- + \partial_\mu\phi^\dagger\partial^\mu\phi$$
$$-\tilde{\mu}^2\phi^*\phi - \frac{\tilde{\lambda}}{2}(\phi^\dagger\phi)^2 - \tilde{y}\phi\psi_+\psi_- + h.c.$$

— $\tilde{y} = \frac{\mu g}{\sqrt{Z}} = \frac{4\pi}{\sqrt{N_c}}\frac{1}{\sqrt{\ln(\Lambda^2/M^2)}}$

— $\tilde{\mu}^2 = \left[\frac{8\pi^2}{N_c g^2} - (\Lambda^2 - M^2)\right]\frac{2}{\ln(\Lambda^2/M^2)}$

— $\tilde{\lambda} = \frac{32\pi^2}{N_c \ln(\Lambda^2/M^2)}$

- condition for $\langle\phi\rangle \neq 0$ gives gap equation result

Supersymmetrizing the NJL Model (Naively):-

- $i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ \quad \longrightarrow \quad \int d^4\theta \;\; \bar{\Phi}_+\Phi_+$
- $g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_- \quad \longrightarrow \quad \int d^4\theta \;\; g^2\bar{\Phi}_+\bar{\Phi}_-\Phi_+\Phi_-$
- $-\mu g\,\phi\psi_+\psi_- \quad \longrightarrow \quad \int d^2\theta \;\; \mu g\,\Phi\Psi_+\Psi_-$
- $-\mu^2\phi^*\phi \quad \longrightarrow \quad \int d^2\theta \;\; \frac{\mu}{2}\Phi\Phi$

BUT :-

- $\phi = -g/\mu\,\bar{\psi}_+\bar{\psi}_-$ implies

$$\mu^2\phi^*\phi = -\mu g\,\phi\psi_+\psi_- = g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_- \qquad \text{(no SUSY !)}$$

- no nontrivial vacuum without SUSY breaking

The Supersymmetric NJL Model :-

- $i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ \longrightarrow \int d^4\theta\ \bar{\Phi}_+\Phi_+\ (1 - m^2\theta^2\bar{\theta}^2)$
- $g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_- \longrightarrow \int d^4\theta\ g^2\bar{\Phi}_+\bar{\Phi}_-\Phi_+\Phi_-$
- $-\mu g\,\phi\psi_+\psi_- \longrightarrow \int d^2\theta\ \mu g\,\Phi_2\Psi_+\Psi_-$
- $-\mu^2\phi^*\phi \longrightarrow \int d^2\theta\ \mu\,\Phi_1\Phi_2 + \int d^4\theta\ \bar{\Phi}_1\Phi_1$

BUT :-

- EL-eq for Φ_2 gives $\Phi_1 = -g\,\Phi_+\Phi_-$ implies

$$\int d^4\theta\ \bar{\Phi}_1\Phi_1 = \int d^4\theta\ g^2\bar{\Phi}_+\bar{\Phi}_-\Phi_+\Phi_-$$

- Φ_2 not the composite Φ_1 plays the Higgs superfield $\langle\Phi_1\rangle = 0$

An Alternative Supersymmetrization ?

- $i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ \qquad \longrightarrow \qquad \int d^4\theta\;\; \bar{\Phi}_+\Phi_+\;\; (1 - m^2\theta^2\bar{\theta}^2)$
- $-\mu g\,\phi\psi_+\psi_- \qquad \longrightarrow \qquad \int d^2\theta\;\; \mu g\,\Phi_0\Psi_+\Psi_-$
- $-\mu^2\phi^*\phi \qquad \longrightarrow \qquad \int d^2\theta\;\; \frac{\mu}{2}\Phi_0\Phi_0$

$$\Longrightarrow \qquad \mathcal{L} = \int d^4\theta\,\left[(\bar{\Phi}_+\Phi_+ + \bar{\Phi}_-\Phi_-)(1 - m^2\theta^2\bar{\theta}^2)\right]$$
$$+\int d^2\theta\,\left[\frac{\mu}{2}\Phi_0^2 + \sqrt{\mu G}\Phi_0\Phi_+\Phi_-\right] + h.c.$$

- consider superpotential $\quad W = \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-$

$$\longrightarrow W - \frac{1}{2}(\sqrt{\mu}\Phi_0 + \sqrt{G}\Phi_+\Phi_-)(\sqrt{\mu}\Phi_0 + \sqrt{G}\Phi_+\Phi_-)$$

With Holomorphic Four-Chiral Superfield Interaction :-

- $W = \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-$ contains no $g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_-$

- EL-eq for auxiliary superfield Φ_0 gives $\Phi_0 = -\sqrt{G/\mu}\,\Phi_+\Phi_-$

  implies $\quad \frac{\mu}{2}\Phi_0^2 = -\frac{\sqrt{\mu G}}{2}\Phi_0\Phi_+\Phi_- = \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-$

- $\langle\Phi_0\rangle \quad \Longrightarrow \quad \frac{G}{2}\langle\Phi_+\Phi_-\rangle\ \Phi_+\Phi_-$ $\qquad$ Dirac mass for Φ_+–Φ_-

- kinetic term for Φ_0 from wave-function renormalization

  through Φ_+–Φ_- loop with Yukawa vertices

→ low energy effective field theory :-

- (gauged-)kinetic term $\int d^4\theta \;\; Z_0 \, \bar{\Phi}_0 \, e^{2V_\Phi} \, \Phi_0 \;\; \left[1 + (2m^2 + A^2)\theta^2\bar{\theta}^2\right]$

  where $Z_0 = \frac{N_c \mu G}{16\pi^2}\left[\ln\frac{\Lambda^2}{M^2} + O(1)\right]$

— $\tilde{m}_0^2 = -(2m^2 + A^2)$, tachyonic soft mass (*cf.* radiative EWSB)

— $\tilde{y} = \frac{\sqrt{\mu G}}{\sqrt{Z_0}} = \frac{4\pi}{\sqrt{N_c}} \frac{1}{\sqrt{\ln\left(\Lambda^2/M^2\right)}}$

— $\tilde{\mu} = \frac{\mu}{Z_0} = \frac{16\pi^2}{N_c G} \frac{1}{\ln\left(\Lambda^2/M^2\right)}$

- $\frac{\mu}{2}\Phi^2$ term $\Longrightarrow$ Φ in real representation of symmetry

Condensate/Mass Generation — A Comparison :-

- NJL : $g^2\langle\bar{\psi}_+\bar{\psi}_-\rangle\ \psi_+\psi_- \quad\longrightarrow\quad -\mu g\,\langle\phi\rangle\ \psi_+\psi_-$

— symmetry breaking with bi-fermion condensate $\langle\phi\rangle$

- SNJL : $\int d^4\theta\ \ g^2\langle\bar{\Phi}_+\bar{\Phi}_-\rangle\ \Phi_+\Phi_-$

$\longrightarrow\quad -g\left\langle F_1^\dagger\right\rangle\ [A_+F_- + A_-F_+ - \psi_+\psi_-] \qquad\qquad \left(F_1^\dagger = -\mu A_2\right)$

— $\langle F_1\rangle = -g\,\langle A_+F_- + A_-F_+ - \psi_+\psi_-\rangle$, sbi-fermion condensate

- HSNJL : $\int d^2\theta\ \ -G\langle\Phi_+\Phi_-\rangle\ \Phi_+\Phi_-$

$\longrightarrow\quad \sqrt{\mu G}\ \langle A_0\rangle\ [A_+F_- + A_-F_+ - \psi_+\psi_-]\ \ + \sqrt{\mu G}\,\langle F_0\rangle\ A_+A_-$

— $\langle A_0\rangle = -\sqrt{G/\mu}\ \langle A_+A_-\rangle$, a bi-scalar condensate

Towards

EW Symmetry Breaking

NJL Model → SM :-

- four-fermion interaction $g^2\bar{Q}\bar{t^c}Qt^c$
- Higgs doublet as top-composite

$$\phi = -g/\mu(\bar{Q}\bar{t^c})$$

- top condensate breaks EW symmetry → fermion masses

— gives top quark mass at to infared quasi-fixed point

- high $m_t \sim 218$ GeV ($\Lambda \sim 10^{19}$ GeV) Bardeen, Hill, Lindner 90

  $m_t \sim 214 - 202$ GeV ($\Lambda \sim 10^{15} - 10^{19}$ GeV) Marciano 89,90

  $m_t \sim 253$ GeV Miransky, Tanabashi, Yamawaki 89; King & Mannan 90,91
- extensions, e.g. two-Higgs-doublet model

Towards the MSSM :-

- consider $W = G\, \varepsilon_{\alpha\beta}\, \hat{Q}^{\alpha} \hat{U}^{c} \hat{Q}'^{\beta} \hat{D}^{c}\ (1 + B\theta^2)$

$$
\begin{aligned}
W \;\longrightarrow\; & W - \mu\,(\hat{H}_d - \lambda_u \hat{Q}\hat{U}^c)(\hat{H}_u - \lambda_d \hat{Q}'\hat{D}^c)(1 + B\theta^2) \\
&= (-\mu \hat{H}_d \hat{H}_u + y_u \hat{Q}\hat{H}_u \hat{U}^c + y_d \hat{H}_d \hat{Q}'\hat{D}^c)(1 + B\theta^2)
\end{aligned}
$$

- two composites — $\hat{H}_u = \frac{y_d}{\mu}\, \hat{Q}'\hat{D}^c$ and $\hat{H}_d = \frac{y_u}{\mu}\, \hat{Q}\hat{U}^c$

- low energy effective theory looks like MSSM ($A = B$)
- symmetric role for $\hat{H}_u$ and $\hat{H}_d$ $\left(\mathrm{also:}\ \mu\lambda_{\mathrm{u}}\lambda_{\mathrm{d}} = \frac{\mathrm{y_u y_d}}{\mu} = \mathrm{G}\right)$

— numerical lifted through non-universal soft masses

— expect $\langle h_u \rangle \gtrsim \langle h_d \rangle$ (Vs UBB in D-flat)

Holomorphic Vs Old Model (for MSSM) :-

• bottom together with (vs only) top mass at quasi-fixed point

⋆ both (vs one) Higgs superfields as composites

• large (vs small) $\tan\beta$

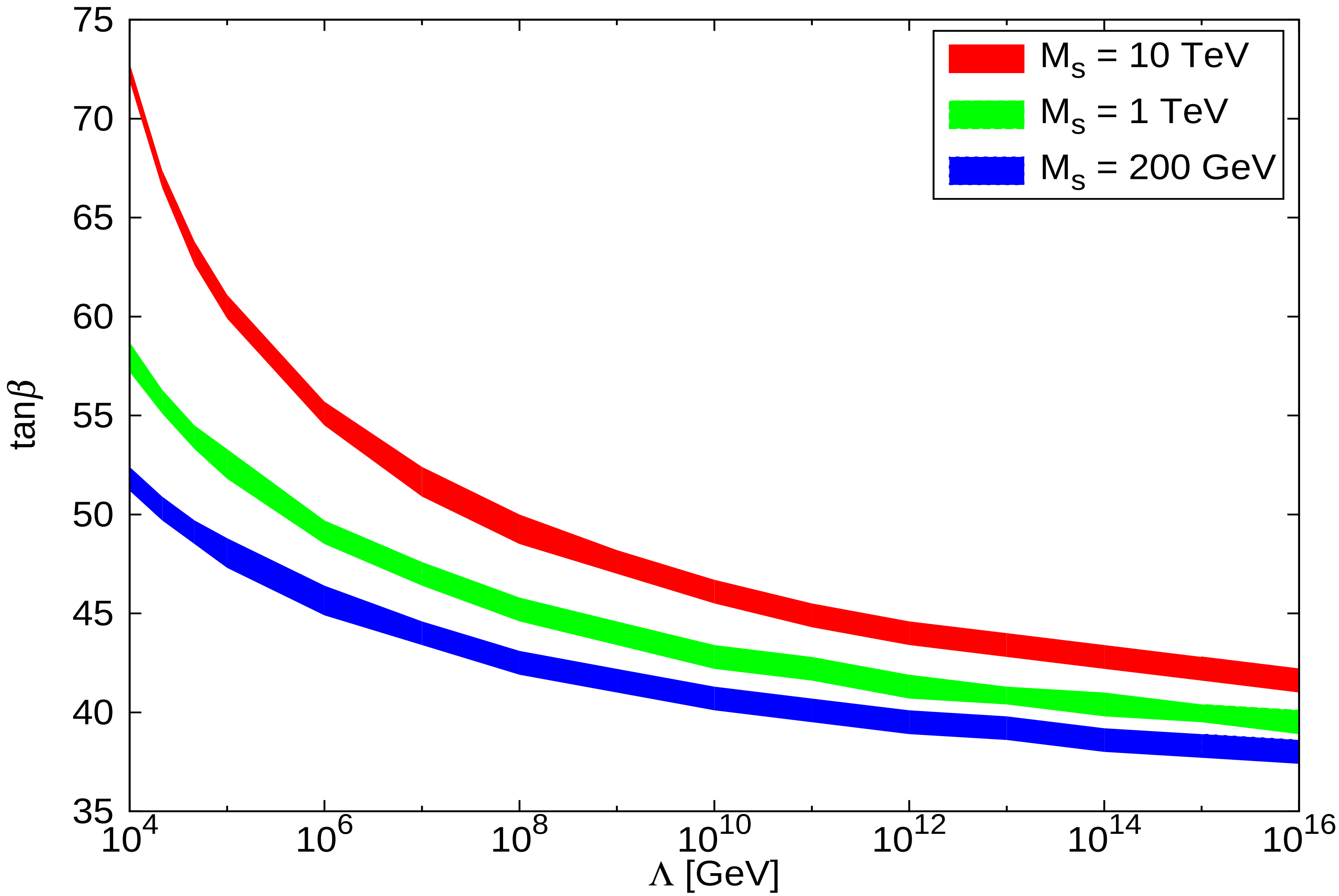
• $A_t \simeq A_b \simeq B \quad$ (vs $A_t \simeq 0$)

• $m^2_{H_d} \simeq -(m^2_Q + m^2_b + |A_b|^2)$
plus (vs only) $m^2_{H_u} \simeq -(m^2_Q + m^2_t + |A_t|^2)$

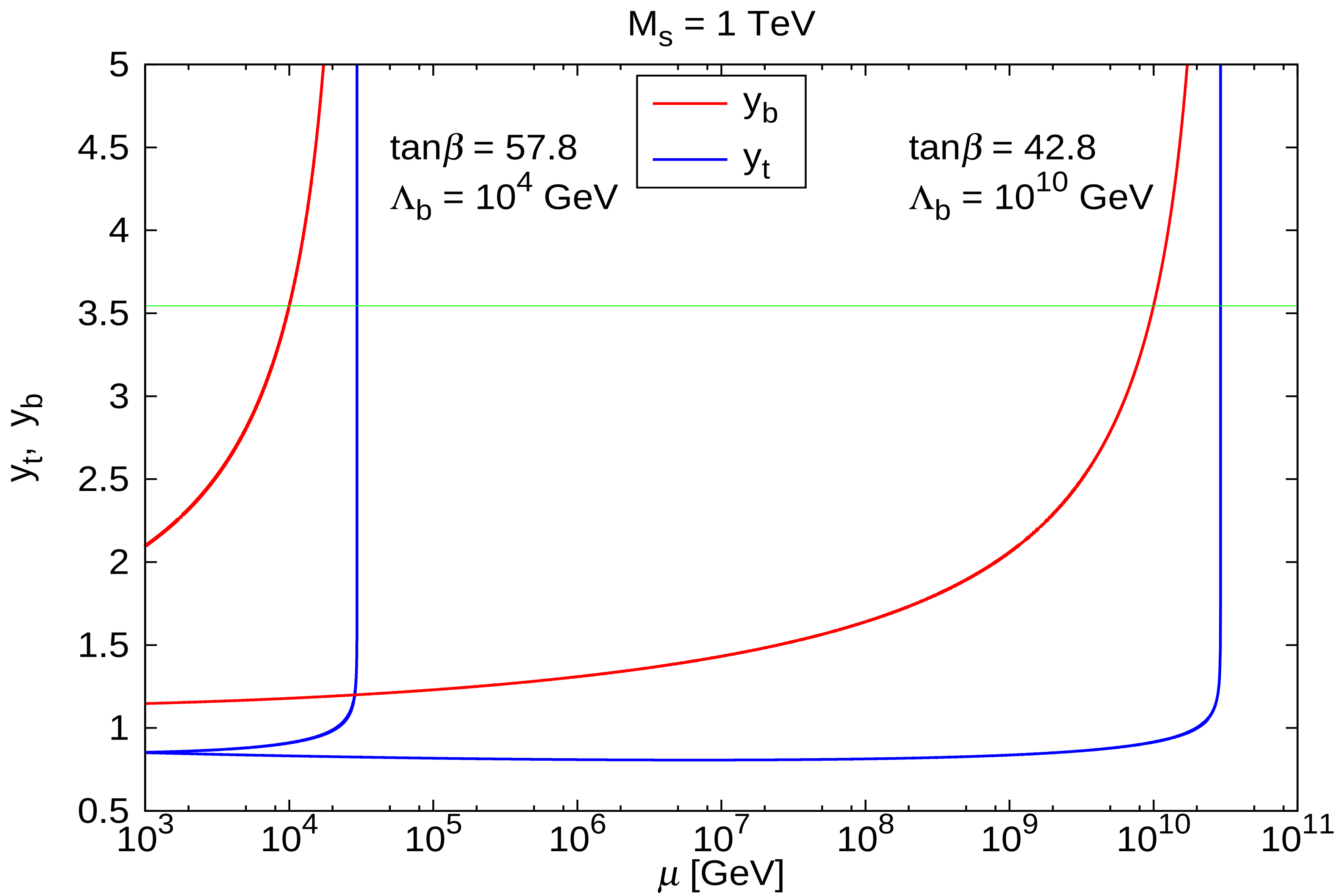
⋆ full W $[= G_{ijkh}\, Q_i U^c_j Q_k D^c_h (1 + A\theta^2) + G^e_{ij}\, Q_3 U^c_3 L_i E^c_j (1 + A\theta^2)]$
— non-holomorphic case needs similar holomorphic terms
for Yukawa couplings of down-type quarks and charged leptons

• sbottom and stop condensates for u_i and $d_i + \ell_i$ masses
(vs top condensate and stop condensates for u_i and $d_i + \ell_i$ masses)

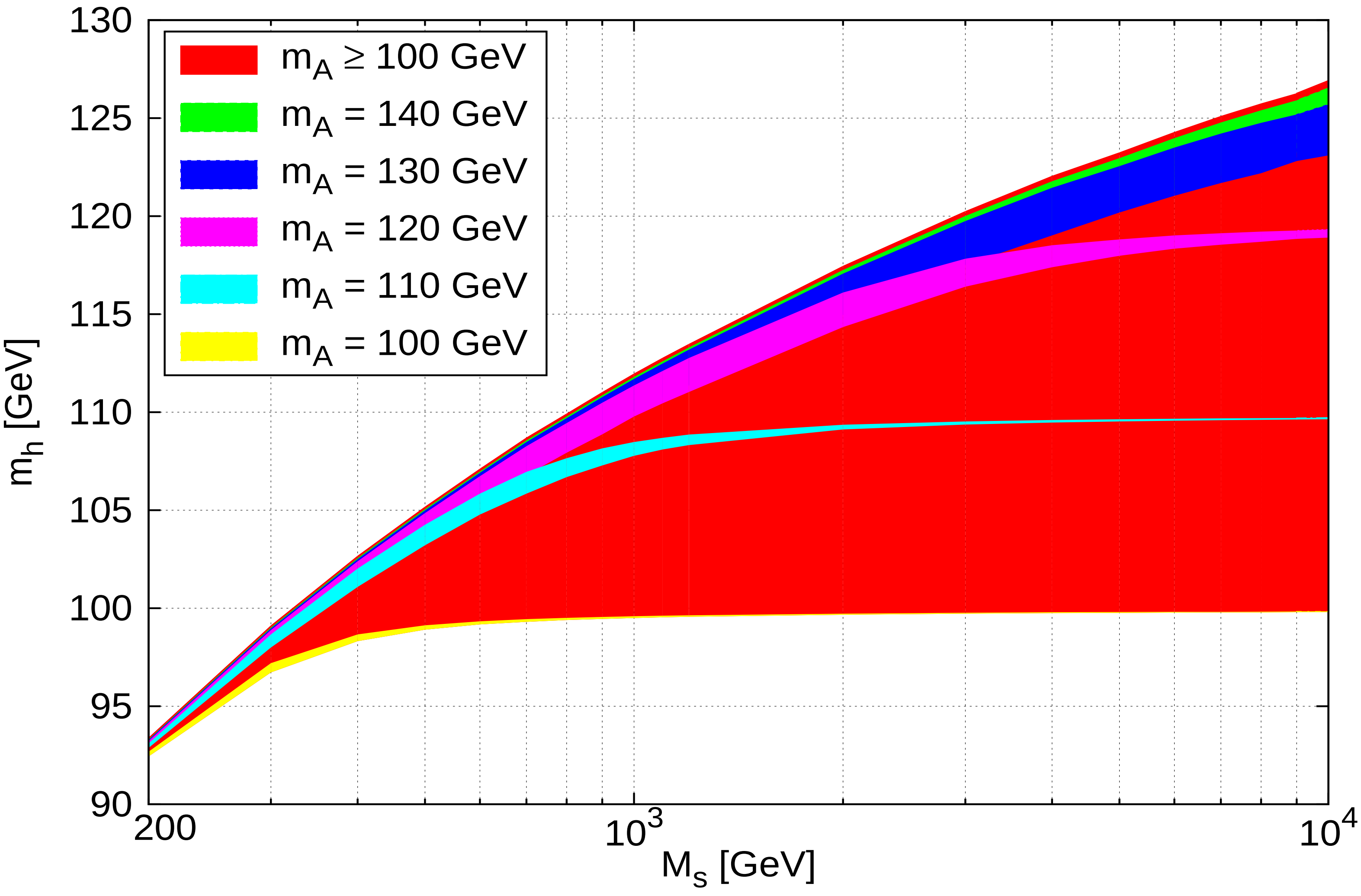
Our Solution :-



Illustrative y_t and y_b :-



Mass of the lightest Higgs boson :-



Final Remarks :-

- SNJL model with holomorphic term works
- may provide more interesting version of MSSM

— SUSY : scalar $\longrightarrow$ chiral superfield

— *problematic* MSSM superfield spectrum — vectorlike Higgs superfields, turn up as composites

— four-superfield (G) term from integrated out *heavy* Higgs superfields ?

— more natural B (and A) term, and all Yukawa coupling

- chiral symmetry explicitly broken

THANK YOU !

e.g. $SU(4)_A \times SU(3)_C \times SU(2)_L \times U(1)_X$

multiplets	X	Gauge anomalies					$U(1)_Y$ states	
		t-1	441	331	221	1^3		
$(\mathbf{4},\mathbf{3},\mathbf{2})$	**1**	24	6	8	12	24	3 $\mathbf{1}(Q)$	$\mathbf{-5}(Q')$
$(\bar{\mathbf{4}},\bar{\mathbf{3}},\mathbf{1})$	**5**	60	15	20		1500	3 $\mathbf{-4}(\bar{u})$	$\mathbf{2}(\bar{d})$
$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	**3**	24	6		12	216	3 $\mathbf{-3}(L)$	$\mathbf{3}(\bar{L})$
$(\bar{\mathbf{4}},\mathbf{1},\mathbf{1})$	**9**	36	9			2916	3 $\mathbf{-6}(\bar{E})$	$\mathbf{0}(N)$
$(\mathbf{6},\mathbf{1},\mathbf{1})$	**-18**	-108	-36			-34992	3 $\mathbf{6}(E)$	3 $\mathbf{12}(S)$
$(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$	**-10**	-60		-20	-30	-6000	$\mathbf{5}(\bar{Q}')$	
$(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$	**-4**	-12		-4		-192	$\mathbf{2}(\bar{d})$	
$(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$	**-4**	-12		-4		-192	$\mathbf{2}(\bar{d})$	
$(\mathbf{1},\mathbf{1},\mathbf{2})$	**6**	12			6	432	$\mathbf{-3}(L)$	
3 $(\mathbf{1},\mathbf{1},\mathbf{1})$	**24**	72				41472	3 $\mathbf{-12}(\bar{S})$	
3 $(\mathbf{1},\mathbf{1},\mathbf{1})$	**-12**	-36				-5184	3 $\mathbf{6}(E)$	
Total		0	0	0	0	0		