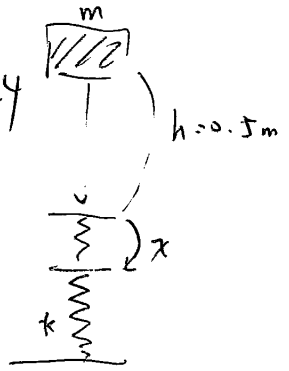
H.W (II)

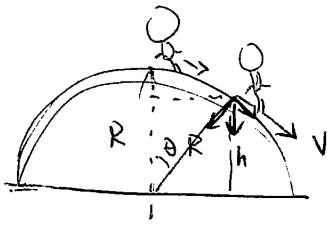
8-24



$$mg(0.5+x)=\frac{1}{2}kx^2$$

$$\Rightarrow x \doteq 11\ cm \quad \#$$

8-34,



$$\begin{cases} mg(R-h)=\frac{1}{2}mv^2 \\ mg\cos\theta = m\frac{v^2}{R} \end{cases}$$

$$\Rightarrow g\cos\theta = \frac{2g}{R}(R-h),\quad h=R\cos\theta$$

$$\Rightarrow \cos\theta = 2(1-\cos\theta)$$

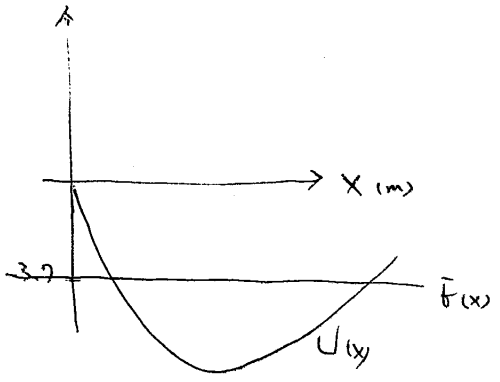
$$\Rightarrow \cos\theta = \frac{2}{3}$$

$$\therefore h = R\cos\theta = \frac{2}{3}R \doteq 8.33\ m \quad \#$$

8-41, (a) 保守力 → 力學能守恒

$$\therefore E = U+K = -4.5\cdot e^{-\frac{5}{4}} + 2 = -3.7\,J \quad \#$$

(b)



8.41,

∵ K (動能耗盡) = 0

⇒ $-4x e^{\frac{-x}{4}} = -3.7$

⇒ $x = 1.3\,m$ or $9.1\,m$

(c) 1.3 m #

(d) 9.1 m #

(e) ∵ U 最小 ⇒ K 最大

$\frac{d(-4x e^{\frac{-x}{4}})}{dx} = 0 \Rightarrow -4e^{\frac{-x}{4}} + x e^{\frac{-x}{4}} = 0$

⇒ $x = 4$ 時，U 有極小值.

∴ $U(4) \doteq -5.9$

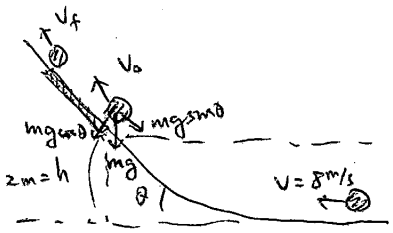
⇒ $K = (-3.7) - (-5.9) = 2.2\,J$ #

(f) $x = 4\,m$ #

(g) $F(x) = -\frac{dU(x)}{dx} = (4-x)e^{\frac{-x}{4}}$ #

(h) $x = 4\,m$ #

8.62, $f_k = \mu_k N = 0.4\,mg\cos\theta$



$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + mgh$

⇒ $v_0 \doteq 4.9\,m/s$

∴ $F = mg\sin\theta + f_k = ma$

⇒ $mg \cdot \frac{1}{2} + 0.4 \cdot mg \cdot \frac{\sqrt{3}}{2} = ma$

⇒ $a \doteq 8.5\,m/s^2$

∴ $v_f^2 = v_0^2 + 2ad$

⇒ $v_f^2 = 4.9^2 - 2 \cdot 8.5 \cdot 0.75$

⇒ $v_f \doteq 3.6\,m/s$ #

8.77,

(參考 Fig. 8-61)

(a) $F(x) = -\frac{dU(x)}{dx} \doteq +4.7$ N

(b) +x - direction,

$E(2) = -7 + \frac{1}{2} \cdot 2 \cdot (4.5)^2 = -4.75$

$K = 0 \Rightarrow U = -4.75 \Rightarrow x = 1.7$ m or 13.8 m

(c) 1.5 m

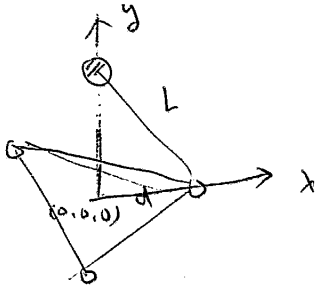
(d) 13.8 m

(e) $K = E - U = -4.75 - (-7)$

$\Rightarrow \frac{1}{2} \times 2 \cdot v^2 = 12.25$

$\Rightarrow v = 3.5$ m/s #

9-7,

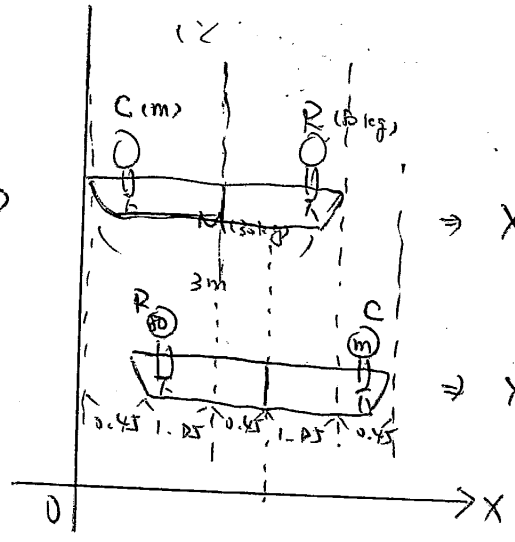


(a) $X = \frac{13.9 \cdot 0 + 1 \times 3 \times 0}{13.9 + 1 \times 3} = 0$ #

(b) $y = \frac{13.9 \cdot (L^2 - d^2)^2 + 1 \times 3 \times 0}{13.9 + 1 \times 3} \doteq 3.13 \times 10^{-11}$ m #

9-16,

無外力 ⇒ 質心不動!!



① $\Rightarrow X_{CM} = m \cdot 0 + 30 \cdot 1.5 + 80 \cdot 3$

② $\Rightarrow X'_{CM} = 80 \times 0.45 + 30 \times 1.95 + m \times 3.45$

$\Rightarrow X_{CM} = X'_{CM} \Rightarrow m \doteq 55.2$ kg #

9.32, (a) $F = \frac{dP}{dt}$, $\Rightarrow \int_{t_i}^{t_f} d\vec{P} = \int_{t_i}^{t_f} \vec{F}\cdot dt = \vec{J}$

$V(0) = 0 \Rightarrow \vec{P}(0) = 0$

$\therefore \vec{P}(4) - \vec{P}_{(0)} = \vec{P}(4) = \int_0^4 \vec{F}\,dt = \int_0^2 5t\hat{i}\,dt + \int_2^4 10\hat{i}\,dt$

$= \left(\frac{5}{2}t^2\Big|_0^2 + 10t\Big|_2^4\right)\hat{i}$

$= (10 + 40 - 20)\hat{i}$

$= 30\,\hat{i}\,(kg\cdot m/s)$ #

(b) $\vec{P}(7) = \int_0^7 \vec{F}\,dt = \int_0^2 5t\,\hat{i}\,dt + \int_2^4 10\,\hat{i}\,dt + \int_4^7 (-5t+30)\,dt$

$= \left(\frac{5}{2}t^2\Big|_0^2 + 10t\Big|_2^4 + \left(\frac{-5}{2}t^2 + 30t\right)\Big|_4^7\right)\hat{i}$

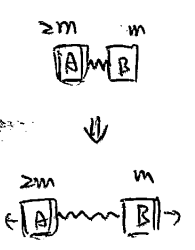
$= (10 + 40 - 20 - 122.5 + 210 + 40 - 120)\,\hat{i}$

$= 37.5\,(kg\cdot m/s)$ #

(c) $\vec{P}(9) = \int_0^9 \vec{F}(t)\,dt = 32.5\,\hat{i}\,(kg\,m/s)$

$\therefore m\vec{v}_9 = 32.5\,\hat{i} \Rightarrow \vec{v}_9 = 13\hat{i}\,(m/s)$ #

9.48, 水箭量守恒, 動量守恒
↓
且無外力作用, 系統質心不變!!

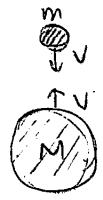


$$\begin{cases} 80 = \frac{1}{2}(2m)V_A^2 + \frac{1}{2}(m)V_B^2 \\ 2mV_A = mV_B \end{cases}$$

$\Rightarrow$ (a) $K_A = \frac{1}{2}(2m)V_A^2 = 26.7\,J$ #

(b) $K_B = \frac{1}{2}mV_B^2 = 53.3\,J$ #

9.69,

(a) 

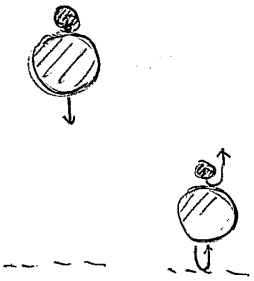
撞前，$V_{cm} = \frac{MV - mV}{m+M}$

撞後

動量守恆 $MV - mV = M \cdot 0 + mV_m'$

$\Rightarrow V_m' = \frac{(M-m)}{m}V$

$V_{cm}' = \frac{M \cdot 0 + m\frac{(M-m)}{m}V}{M+m} = \frac{MV-mV}{m+M} = V_{cm}$，(彈性碰撞之必然結果)



動能守恆，$\frac{1}{2}MV^2 + \frac{1}{2}mV^2 = 0 + \frac{1}{2}m\left(\frac{(M-m)}{m}V\right)^2$

$\Rightarrow m = \frac{M}{3} = 0.21\ kg$ #

(b) $Mgh = \frac{1}{2}MV^2 \Rightarrow V \doteq 5.94\ m/s$

$\therefore \frac{1}{2}m\left(\frac{M-m}{m}\right)^2 V^2 = mgh'$

$\Rightarrow h' \doteq 7.2\ m$ #

10.7,

(a) $\theta = \frac{2\pi}{8} = \frac{\pi}{4}$ between adjacent spokes

1秒，2.5圈

$\Rightarrow$ 0.05秒，$\frac{1}{8}$ 圈

$V = \frac{d}{t} = \frac{0.2}{0.05} = 4\ m/s$ #

(b) NO!! (角速度都一樣)

10.32,

(a) $\omega = \omega_0 + \alpha t$，$\omega = \frac{2\pi}{T}$

$\Rightarrow \frac{2\pi}{0.033 + 1.26\times10^{-5}} = \frac{2\pi}{0.033} + \alpha(31536000)$

$\Rightarrow \alpha \doteq -2.304\times10^{-9}\ rad/s^2$ #

(b) $0 = \frac{2\pi}{0.033} + \alpha t \Rightarrow t \doteq 2620\ years$ #

(c) Year 2012, Now

α is fixed

$\therefore \omega = \omega_0 + \alpha(2012 - 1054)^{year}$

$= \frac{2\pi}{0.033} = \frac{2\pi}{T_0} + \alpha(958\ year)$

$\Rightarrow T_0 \doteq 0.024\ s$ #

10-40 / (a) $I = 15 \times \left(\frac{1}{2}MR^2\right) + 2 \times \frac{M}{15}\left(2R^2 + 4R^2 + \cdots 4R^2\right)$

$\left(R = \frac{1}{30}\,m\right)$ $= 15 \times \frac{1}{2} \frac{100}{15} \frac{1}{1000}\left(\frac{1}{30}\right)^2 + \frac{2}{15} \times 0.1 \left(2\left(\frac{1}{30}\right)^2 + 4\left(\frac{1}{30}\right)^2 + \cdots + 14\left(\frac{1}{30}\right)^2\right)$

$= 5.56 \times 10^{-5}\ kg \cdot m^2 +$

$= 0.00833\ kg \cdot m^2$

$= \underline{8.332 \times 10^{-3}\ kg \cdot m^2}$ #

(b) $I = \frac{1}{12} M L^2 = \frac{1}{12} \times 0.1 \times 1^2 = 8.333 \times 10^{-3}\ kg \cdot m^2$

(c) $\frac{(8.332 \times 10^{-3} - 8.333 \times 10^{-3})}{8.332 \times 10^{-3}} \doteq 0.22\ \%$

#

10.51) ,

(a) $\tau = I\alpha$,

$\Rightarrow (0.5t + 0.3t^2) \cdot 0.1 = 10^3 \times \alpha$

$\Rightarrow \alpha(t) = 50t + 30t^2$

$\alpha(3) \doteq \underline{4.2 \times 10^2\ rad/s^2}$ #

(b)

$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$

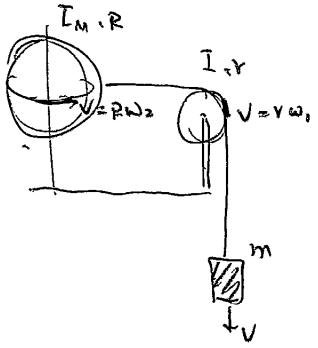
$\Rightarrow d\omega = \alpha\, dt$

$\Rightarrow \int_{t=0}^{t=3} d\omega = \int_0^3 \alpha\, dt$

$\Rightarrow \omega(3) = 25t^2 + 10t^3 \Big|_0^3 = 495\ rad/s$

#

10.66, $mgh = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_M \omega_2^2 + \frac{1}{2} m v^2$



$\Rightarrow 0.6 \times 9.8 \times 0.82 = \frac{1}{2}\left(3 \cdot 10^{-3}\right)\left(\frac{V}{0.05}\right)^2 + \frac{1}{2}\left(\frac{2}{3} \cdot 4.5 \cdot (0.085)^2\right)\left(\frac{V}{0.085}\right)^2 + \frac{1}{2} \cdot 0.6 \times V^2$

$\Rightarrow V \doteq 1.42\ m/s$ #