Dark matter constraint on SUSY parameter space

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1. Introduction

Cosmological observations

(i) Relic density

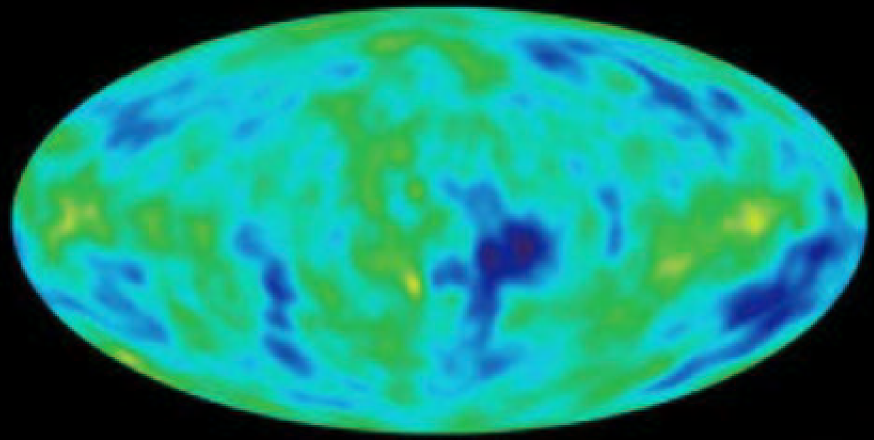
WMAP (2001–, NASA)

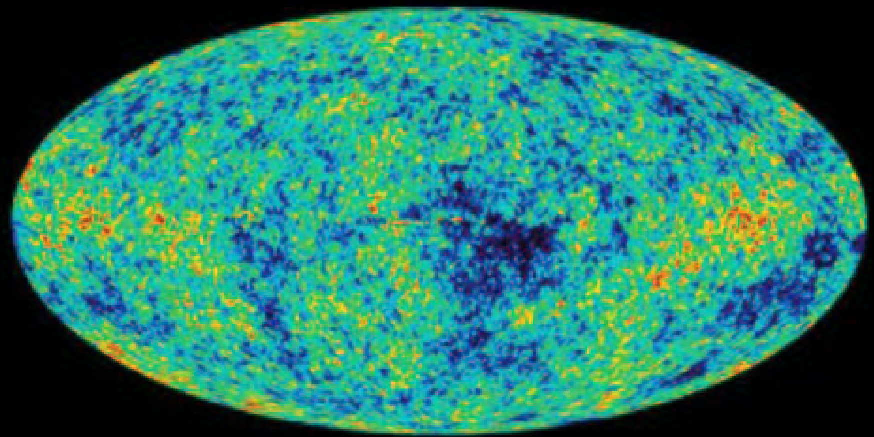
$$\Omega_\chi h^2 = 0.1126^{+0.008}_{-0.009}$$

(χ: CDM)

$\Omega_\chi \equiv \rho_\chi / \rho_c$

$h \approx 0.7$ ($H_0 = 100\, h$ km/s/Mpc)

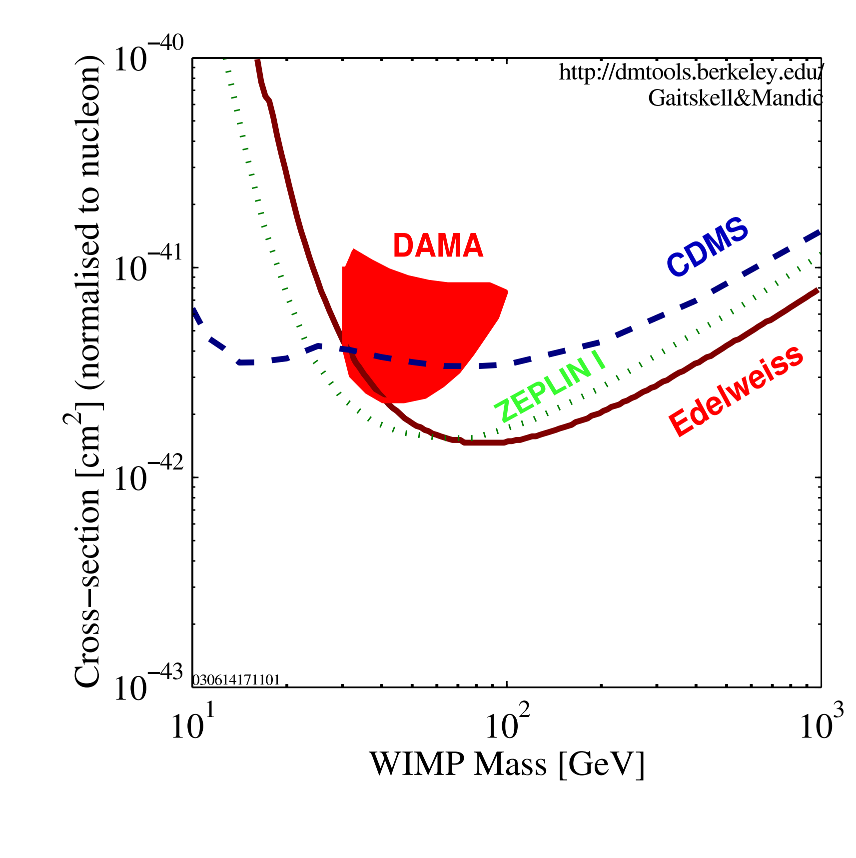

COBE


MAP

MSSM: $\chi =$ lightest neutralino

$\longrightarrow$ Strong constraint on the MSSM

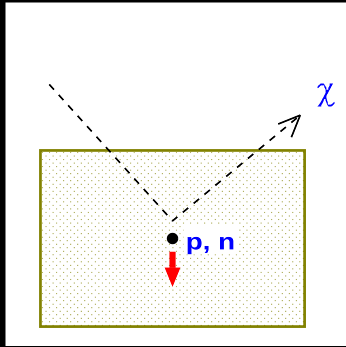
(ii) Direct detection



Weakly Interacting Massive Particle (WIMP)

$\chi p \to \chi p$

DAMA (NaI)
CDMS (Ge & Si)
ZEPLIN (Xe)
Edelweiss (Ge)
$\vdots$



Upper bounds on the cross sections

Spin-independent interaction: $\sigma_{\chi p}(\mathrm{SI}) \lesssim 10^{-6}\,\mathrm{pb}$

Spin-dependent interaction: $\sigma_{\chi p}(\mathrm{SD}) \lesssim 1\,\mathrm{pb}$

2. Theoretical framework

Minimal SUSY SM (MSSM)

- Relevant parameters

gaugino mass	$\frac{1}{2}(M_1\widetilde{B}\widetilde{B} + M_2\widetilde{W}^a\widetilde{W}^a + M_3\widetilde{g}^\alpha\widetilde{g}^\alpha)$
Higgsino mass	$\mu\widetilde{H}_1\widetilde{H}_2 + \mathrm{h.c.}$
Higgs mass	$\mu^2(\lvert H_1\rvert^2 + \lvert H_2\rvert^2) + \cdots$
	$\longrightarrow \tan\beta = \langle H_2^0\rangle/\langle H_1^0\rangle, \quad m_A^2\lvert A\rvert^2$
sfermion mass	$m_{\widetilde{q}_i}^2\lvert\widetilde{q}_i\rvert^2 + m_{\widetilde{\ell}_i}^2\lvert\widetilde{\ell}_i\rvert^2 + \cdots$
scalar trilinear	$A_t\widetilde{t}_R^\dagger\widetilde{q}_{3L}H_2 + A_b\widetilde{b}_R^\dagger\widetilde{q}_{3L}H_1 + A_\tau\widetilde{\tau}_R^\dagger\widetilde{\ell}_{3L}H_1$

Constrained MSSM (CMSSM)

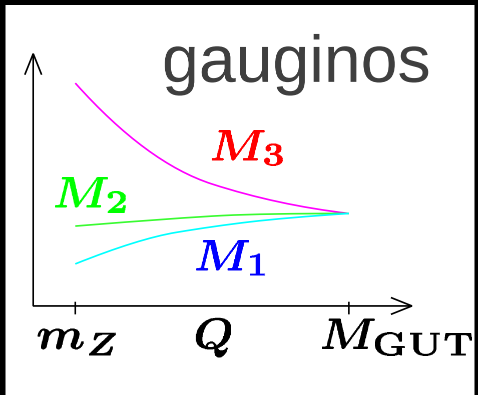
- Soft ~~SUSY~~ parameters at $Q = M_{\rm GUT}$ ($\approx 2 \times 10^{16}$ GeV)

  scalars $\quad m^2_{\tilde{q}_i} = m^2_{\tilde{\ell}_i} = m^2_{H_1} = m^2_{H_2} = m_0^2$

  gauginos $\quad M_1 = M_2 = M_3 = m_{1/2}$

  trilinear $\quad A_{U,D,E} = A_0 Y_{U,D,E}$

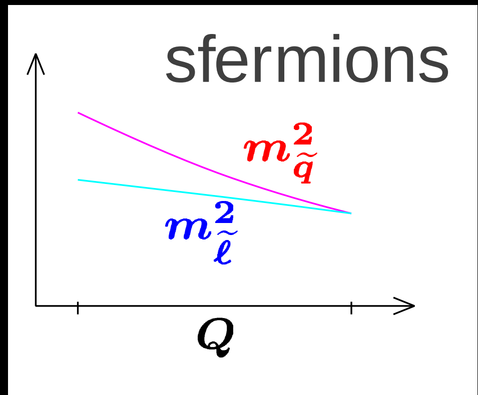
- RGE: $M_{\rm GUT} \longrightarrow m_Z$

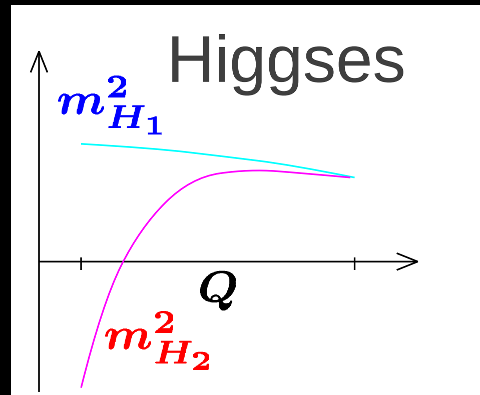


$@\ Q = m_Z$

$M_1 \approx M_2/2$

$M_3 \approx 3M_2$





Radiative EW symmetry breaking

$m^2_{\rm Higgs} = m_0^2 > 0$ at $M_{\rm GUT}$

$\longrightarrow m^2_{\rm Higgs} < 0$ at m_Z

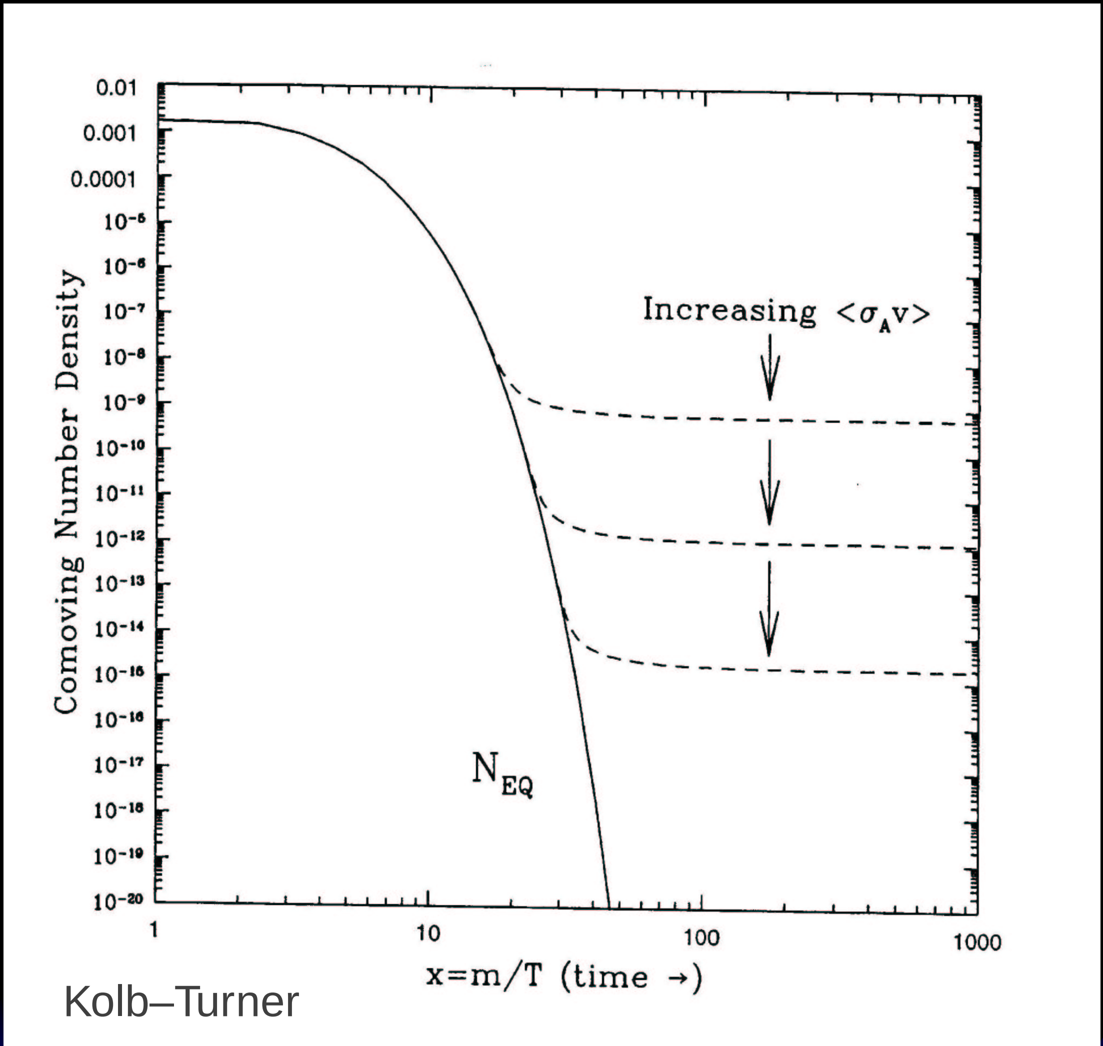
$\left[\frac{m_Z^2}{2} = \frac{m^2_{H_1} - m^2_{H_2}\tan^2\beta}{\tan^2\beta - 1} - \mu^2\right]$

$\Longrightarrow$ $4 + 1$ parameters

$m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ {\rm sgn}(\mu)$

3. Relic density of the neutralino

Boltzmann eq. : $\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma v_{\rm rel}\rangle(n_\chi^2 - n_\chi^{\rm eq2})$



Interaction rate

$\Gamma \sim n\sigma v$

$n_{\rm eq} \sim e^{-m/T}$ $(T \ll m)$

Expansion rate

$H \sim T^2/M_{\rm Pl}$ (RD)

$\Gamma > H$: thermal

$\Downarrow \longleftarrow T_f \approx m_\chi/20$

$\Gamma < H$: decoupled (freeze out)

Accurate calculation

1. All the contributions to $\sigma = \sigma(\chi\chi \to \mathrm{all})$
   "all" $= f\bar{f}, W^+W^-, ZZ, H_r^0 H_s^0, H^+H^-, ZH_r^0, W^\pm H^\mp$

2. Exact formula for $\langle \sigma v \rangle$ Gondolo–Gelmini (1991)
   $\longleftrightarrow$ expansion $\langle \sigma v \rangle = a + bx, \;\; x = T/m_\chi$

3. Accurate treatment of Boltzmann eq.
   $\longleftrightarrow$ approximate solution
   $\rho_\chi \propto 1/\int_0^{x_f} dx \langle \sigma v \rangle, \;\; x_f = T_f/m_\chi \sim 1/20$

4. Coannihilation $(\chi\chi' \to \cdots, \chi'\chi' \to \cdots, \cdots)$ Griest–Seckel (1991) Mizuta–Yamaguchi (1993)
   χ': NLSP $(\tilde{\tau}_1, \chi_1^\pm, \chi_2^0, \cdots)$
   relevant for $m_{\chi'} \lesssim 1.1 m_\chi$

$\sigma \to \sigma_{\mathrm{eff}}$
enhanced

Cross section $\sigma(\chi\chi \to \text{all})$

Process	s-channel	t & u-channel
$\chi\chi \to f\bar{f}$	h, H, A, Z	$\tilde{f}_{1-6}$
$\chi\chi \to hh, hH, HH$	h, H	χ^0_{1-4}
hA, HA	A, Z	χ^0_{1-4}
AA	h, H	χ^0_{1-4}
H^+H^-	h, H, Z	$\chi^\pm_{1,2}$
$\chi\chi \to W^+W^-$	h, H, Z	$\chi^\pm_{1,2}$
ZZ	h, H	χ^0_{1-4}
$\chi\chi \to Zh, ZH$	A, Z	χ^0_{1-4}
ZA	h, H	χ^0_{1-4}
$W^\pm H^\mp$	h, H, A	$\chi^\pm_{1,2}$

Thermal average $\langle \sigma v \rangle$

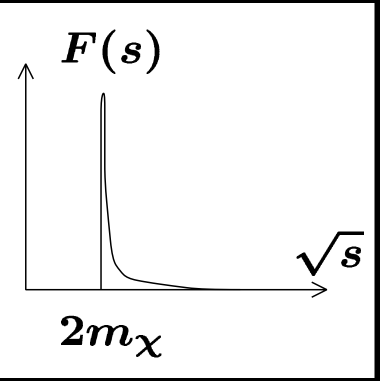
[1] **Exact formula**

Gondolo–Germini (1991)

$$\langle \sigma v_{\rm rel} \rangle = \frac{\int d^3p_1 d^3p_2\, \sigma v_{\rm rel}\, e^{-E_1/T} e^{-E_2/T}}{\int d^3p_1 d^3p_2\, e^{-E_1/T} e^{-E_2/T}}$$

$$= \frac{1}{8 m_\chi^4 T K_2^2(\frac{m_\chi}{T})} \int_{4m_\chi^2}^{\infty} ds\, \sigma \sqrt{s}(s-4m_\chi^2) K_1\left(\frac{\sqrt{s}}{T}\right)$$

$$\sim \int_{4m_\chi^2}^{4m_\chi^2(1+\epsilon)} ds\, \sigma \cdot F(s)$$



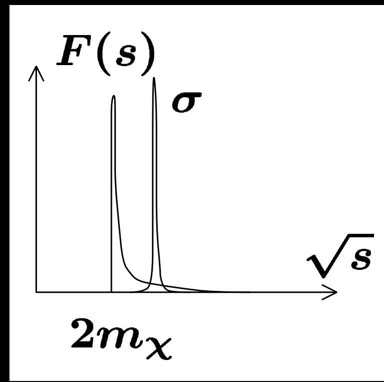
[2] **Expansion**

$$\langle \sigma v_{\rm rel} \rangle = a + bx, \quad x = T/m_\chi$$

$$a = w(4m_\chi^2), \; b = -\frac{3}{2}(2w - w')|_{s=4m_\chi^2}, \; w(s) = \frac{1}{2m_\chi^2}\sqrt{s(s-4m_\chi^2)}\sigma(s)$$

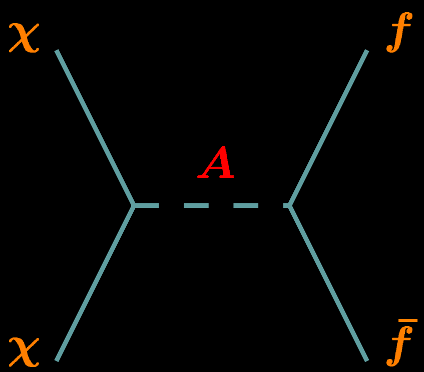
Exact vs Expansion

Naively, “expansion” should converge quickly,
because $F(s)$ decays quickly.
However, this is not necessarily true
when σ varies rapidly with s.



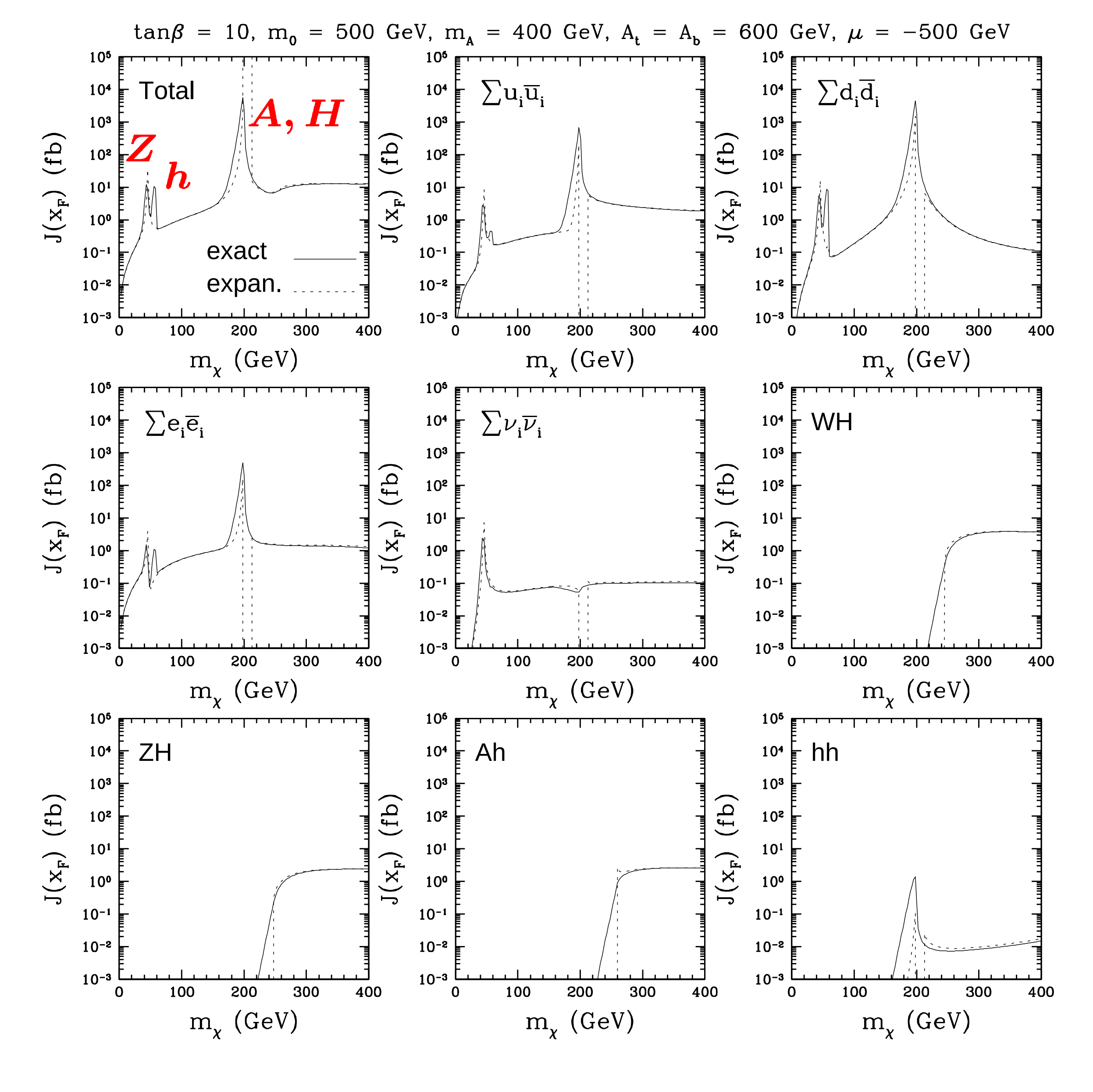
[e.g. near resonance,
threshold of new channel]

Griest–Seckel (1991)



resonance at
$2m_\chi \approx m_A$

$$J(x_f) = \int_0^{x_f} dx \langle \sigma v \rangle$$



T.N.–Roszkowski–Ruiz (JHEP0108)

Expansion causes huge errors around the poles and thresholds

VV, VH can be dominant for large m_χ.

$\Omega_{\rm expansion}/\Omega_{\rm exact}$

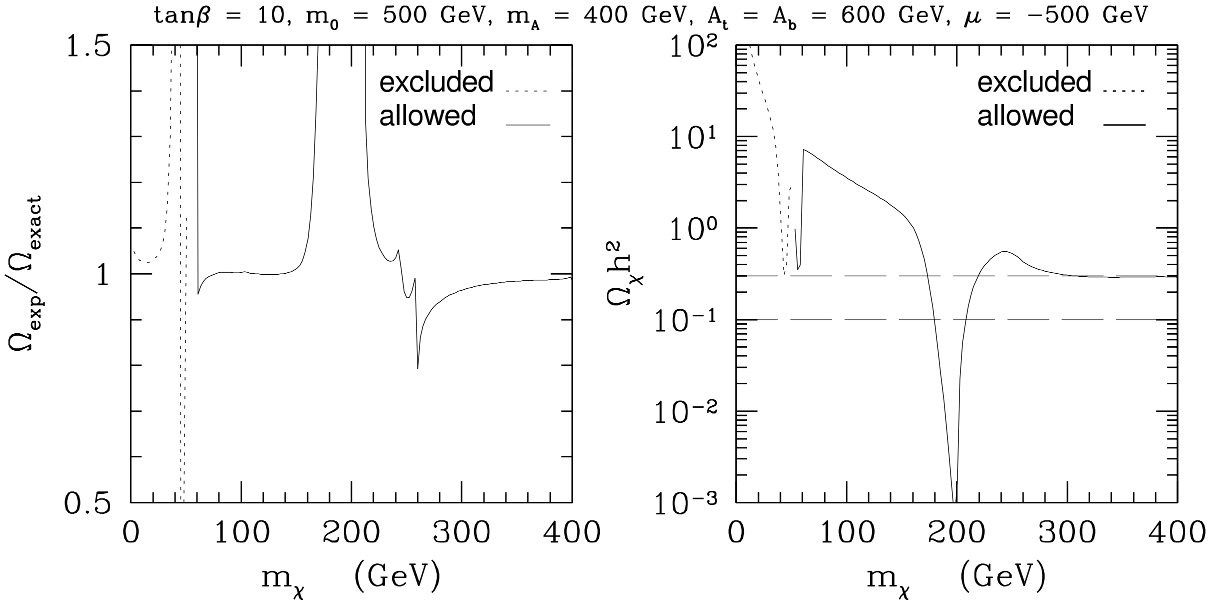


Figure 4: The ratio $\Omega_{\rm exp}/\Omega_{\rm exact}$ (a) and the relic density $\Omega_\chi h^2$ (b) for the same choice of parameters as in Fig. 1. The solid (dotted) curves are allowed (excluded) by current experimental constraints. In window (a) the relic aboundance in both cases is computed by solving Eq. (3) iteratively and using Eq. (4). In window (b) we show $\Omega_\chi h^2$ is computed using our numerical code. The band between the two horizontal dashed lines corresponds to the cosmologically favoured range $0.1 < \Omega_\chi h^2 < 0.3$.

T.N.–Roszkowski–Ruiz
(JHEP0105)

Usual expansion
Large error over
sizable range of m_χ

Coannihilation

Griest–Seckel (1991)

If NLSP (χ') is nearly degenerate with LSP ($m_{\chi'} \lesssim 1.1 m_\chi$), then χ' is as abundant as χ at $T_f \approx \frac{m_\chi}{20}$.

$$\longrightarrow \left.\begin{array}{l}\sigma(\chi\chi' \to f f') \\ \sigma(\chi'\chi' \to f f')\end{array}\right\}$$ should be included.

(f, f': SM particles)

$\chi f \leftrightarrow \chi' f'$ is much faster than $\chi\chi \leftrightarrow f f'$.

$\uparrow$ $\uparrow$

Reaction rate

$$r_{\chi f} \sim n_\chi n_f \sigma_{\chi f} \sim \sigma_{\chi f} \exp\left(-\frac{m_\chi}{T}\right) \gg r_{\chi\chi} \sim n_\chi n_\chi \sigma_{\chi\chi} \sim \sigma_{\chi\chi} \exp\left(-\frac{2m_\chi}{T}\right)$$

Boltzmann eq. with coannihilation

Replacements

$$\begin{cases} n_\chi & \rightarrow \quad n = \sum_i n_i \\ \langle \sigma v \rangle & \rightarrow \quad \langle \sigma_{\rm eff}\, v \rangle = \sum_{i,j} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{\rm eq}}{n^{\rm eq}} \frac{n_j^{\rm eq}}{n^{\rm eq}} \end{cases}$$

$$\downarrow$$

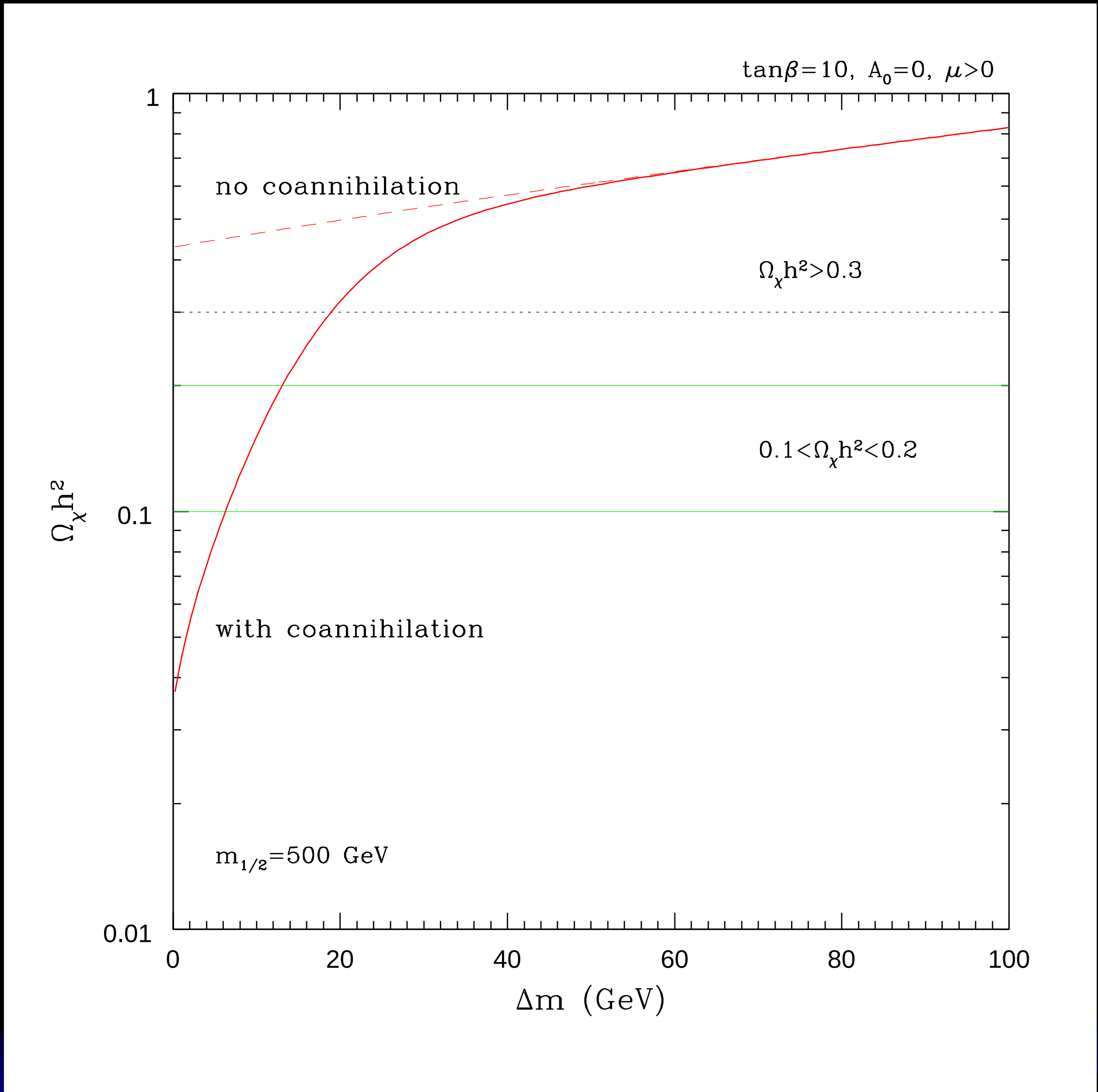
$$\frac{dn}{dt} + 3Hn = -\langle \sigma_{\rm eff}\, v \rangle (n^2 - n_{\rm eq}^2)$$

$\sigma \rightarrow \sigma_{\rm eff}$: enhanced by coann.

- CMSSM:

$$\chi' = \widetilde{\tau}_1,\ \chi_1^\pm, \chi_2^0$$

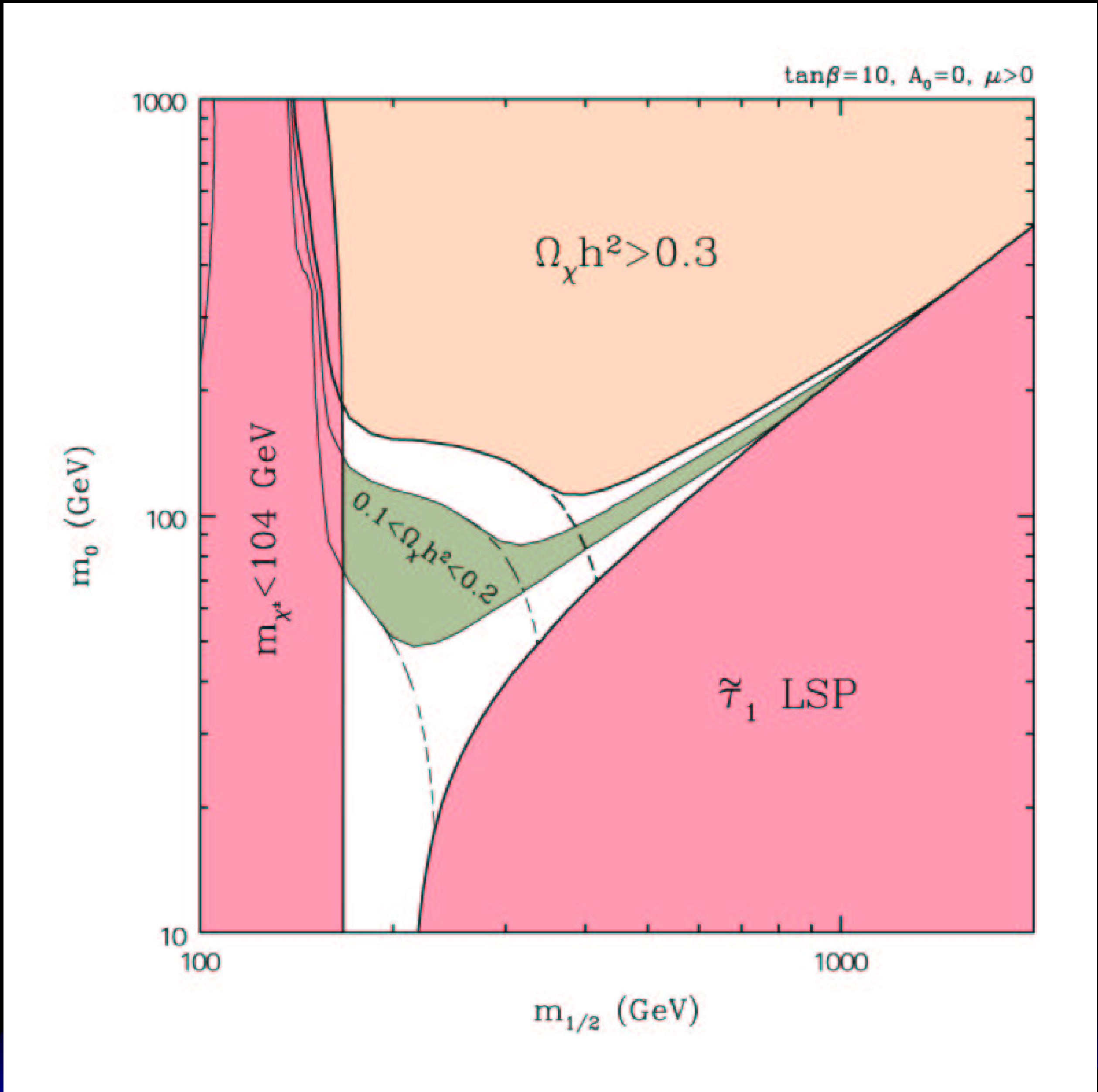
Reduction of $\Omega_\chi h^2$ via coannihilations



T.N.–Roszkowski–Ruiz (JHEP0207)

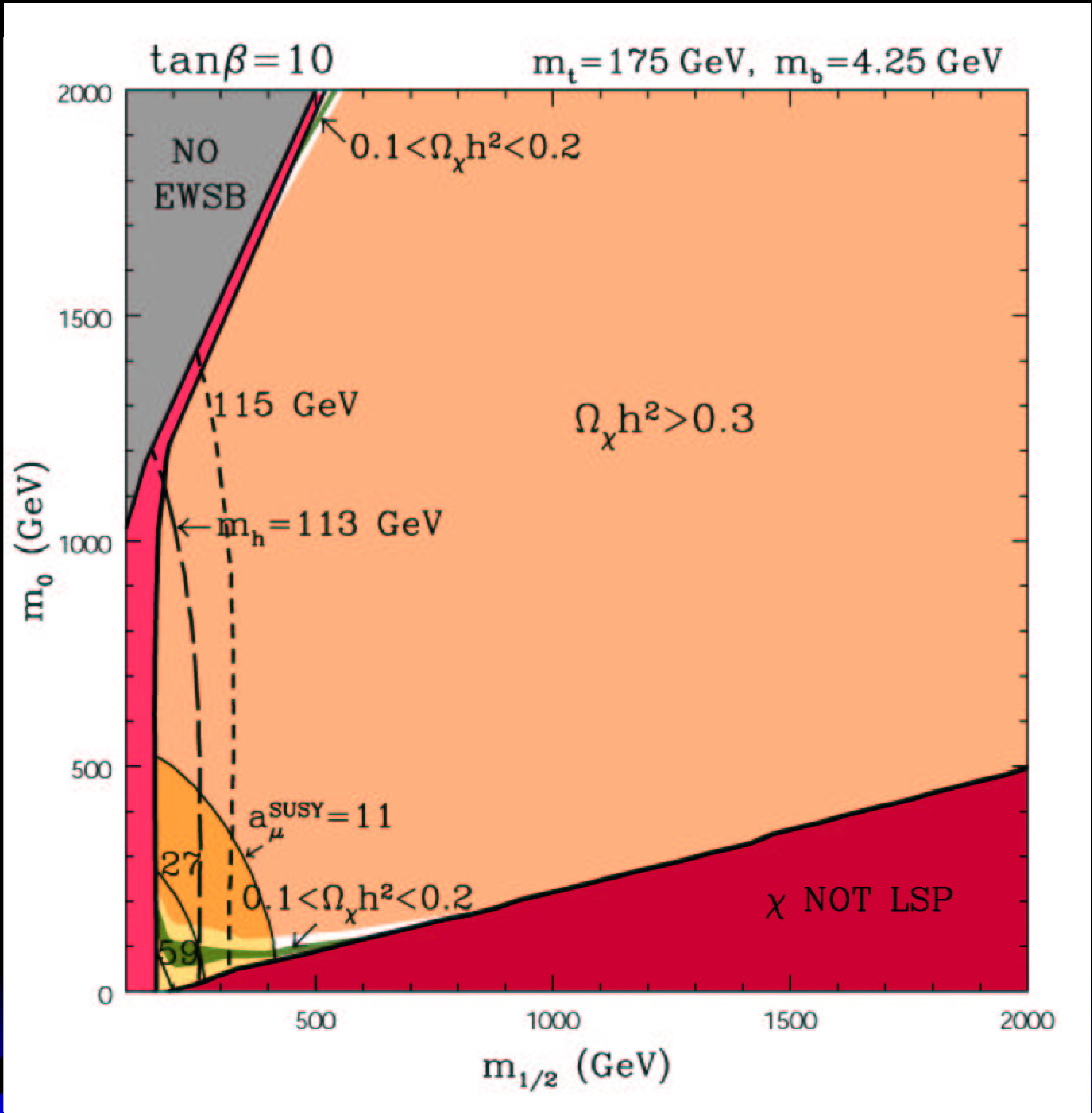
$$\Delta m = m_{\tilde{\tau}_1} - m_\chi$$

Effect of coannihilations



T.N.–Roszkowski–Ruiz (JHEP0207)

Allowed regions $(\tan\beta = 10)$



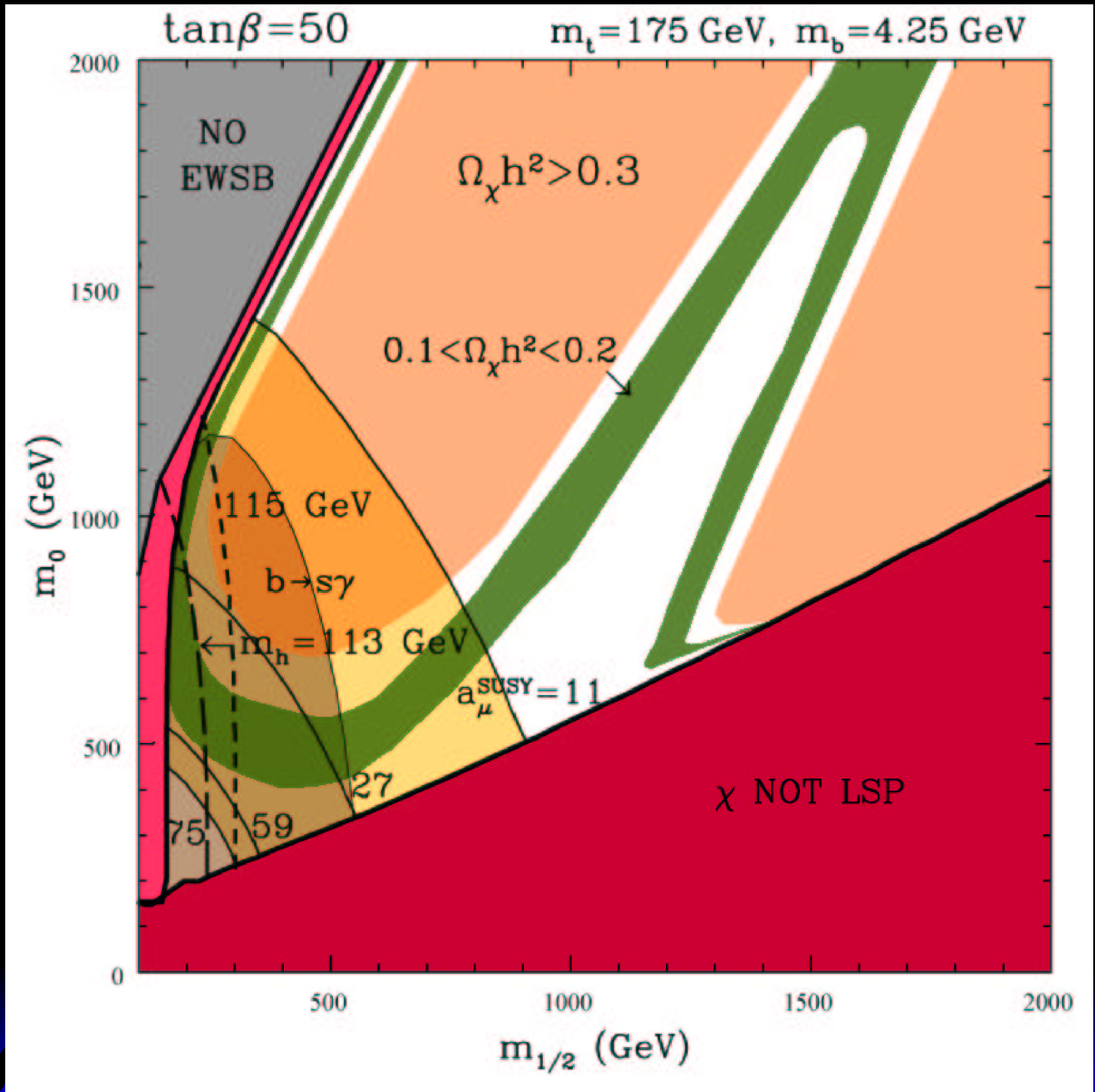
Roszkowski–Ruiz–T.N. (JHEP0108)

WMAP $\mathbf{2\sigma}$ range

$$\mathbf{0.0946 < \Omega_\chi h^2 < 0.1286}$$

$\longrightarrow$ Strong constraint
(Allowed 'line')

Allowed regions $(\tan\beta = 50)$



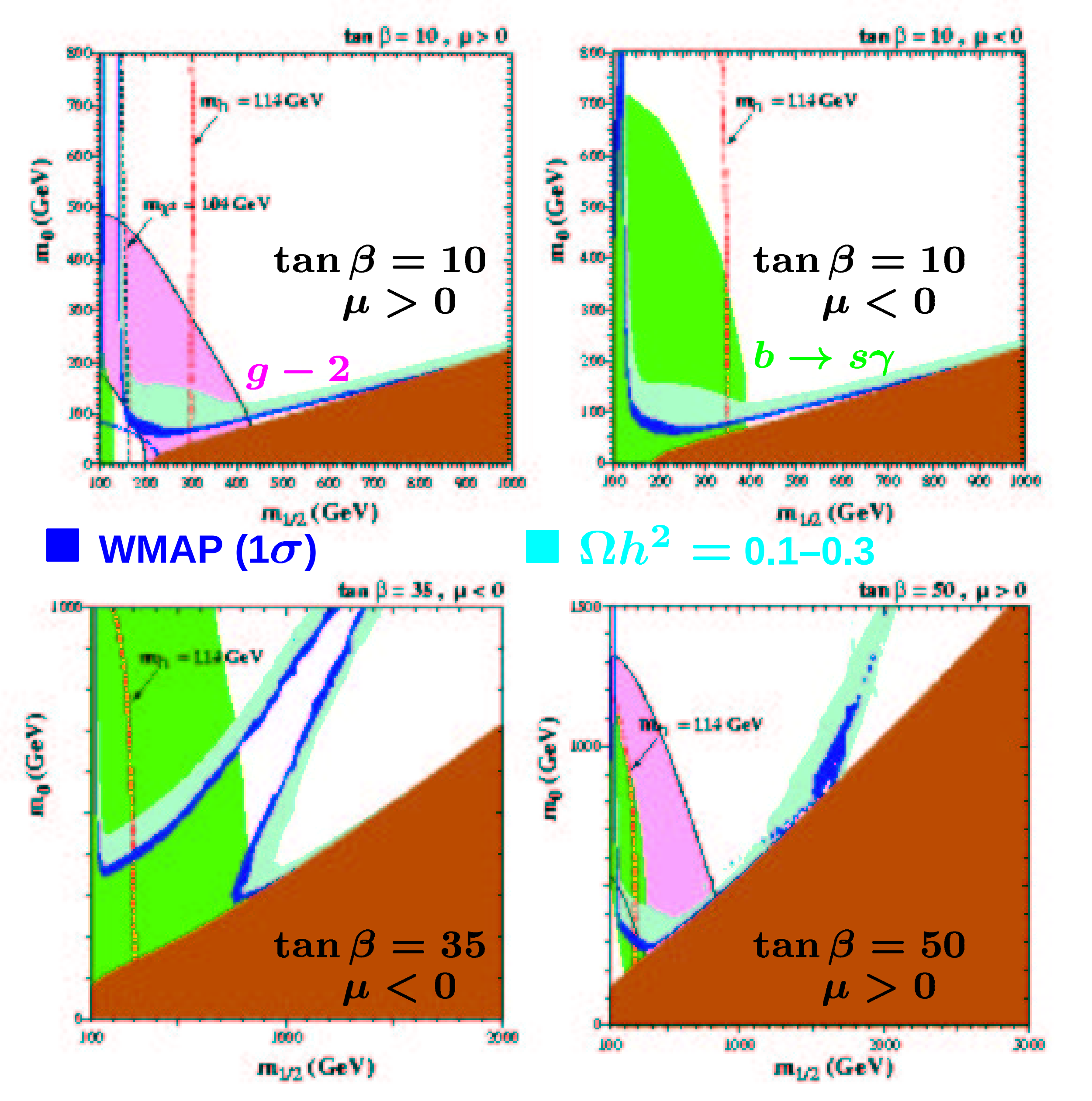
Roszkowski–Ruiz–T.N. (JHEP0108)

Allowed regions grow significantly as $\tan\beta \to 50$ (A–pole effect)

WMAP
$\to$ allowed ‘line’

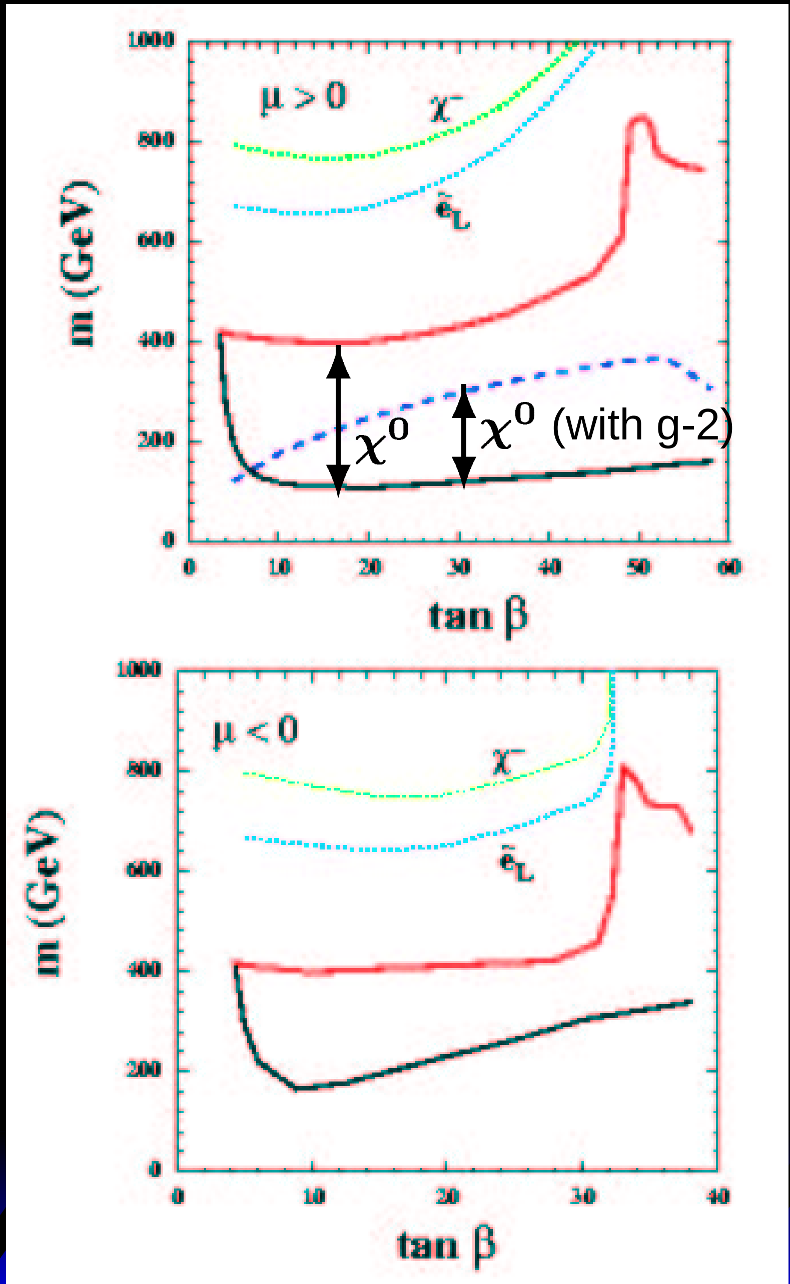
Allowed range of m_χ

Ellis–Olive–Santoso–Spanos (2003)



Mass upper bounds from WMAP

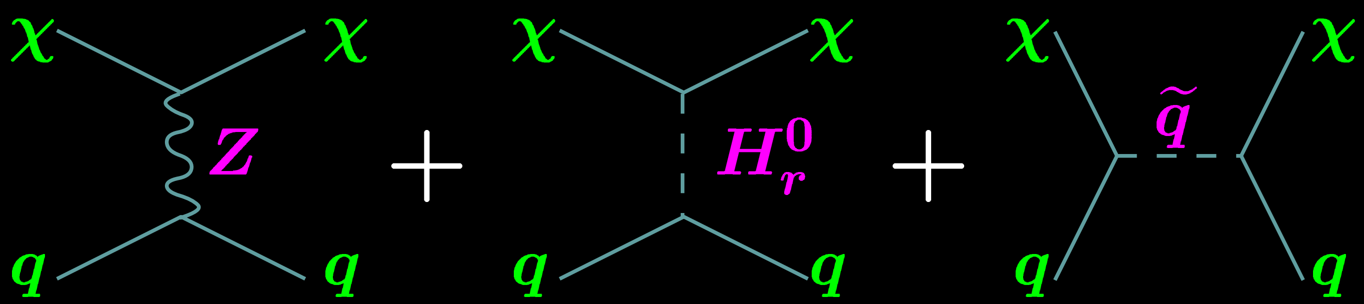
Ellis–Olive–Santoso–Spanos (2003)



$m_\chi \lesssim 500$ GeV
for $\tan\beta \lesssim 30$

Coannihilation region
$m_\chi \approx m_{\tilde{\tau}_1}$
Stau is light !
(e_R, μ_R not much heavier)

4. Direct detection of the neutralino



Goodman–Witten (1985)
Griest (1988)
Ellis–Flores (1988)
Drees–Nojiri (1993)

$$\mathcal{L}_{\text{eff}} = d_q(\bar{\chi}\gamma^\mu\gamma_5\chi)(\bar{q}\gamma_\mu\gamma_5 q) + f_q(\bar{\chi}\chi)(\bar{q}q)$$

$\Downarrow$ $\langle p|\bar{q}q|p\rangle$, $\langle p|\bar{q}\gamma_\mu\gamma_5 q|p\rangle$: input

hadronic Lagrangian

$$\mathcal{L}_{\text{eff}} = d_p(\bar{\chi}\gamma^\mu\gamma_5\chi)(\bar{p}\gamma_\mu\gamma_5 p) + f_p(\bar{\chi}\chi)(\bar{p}p)$$

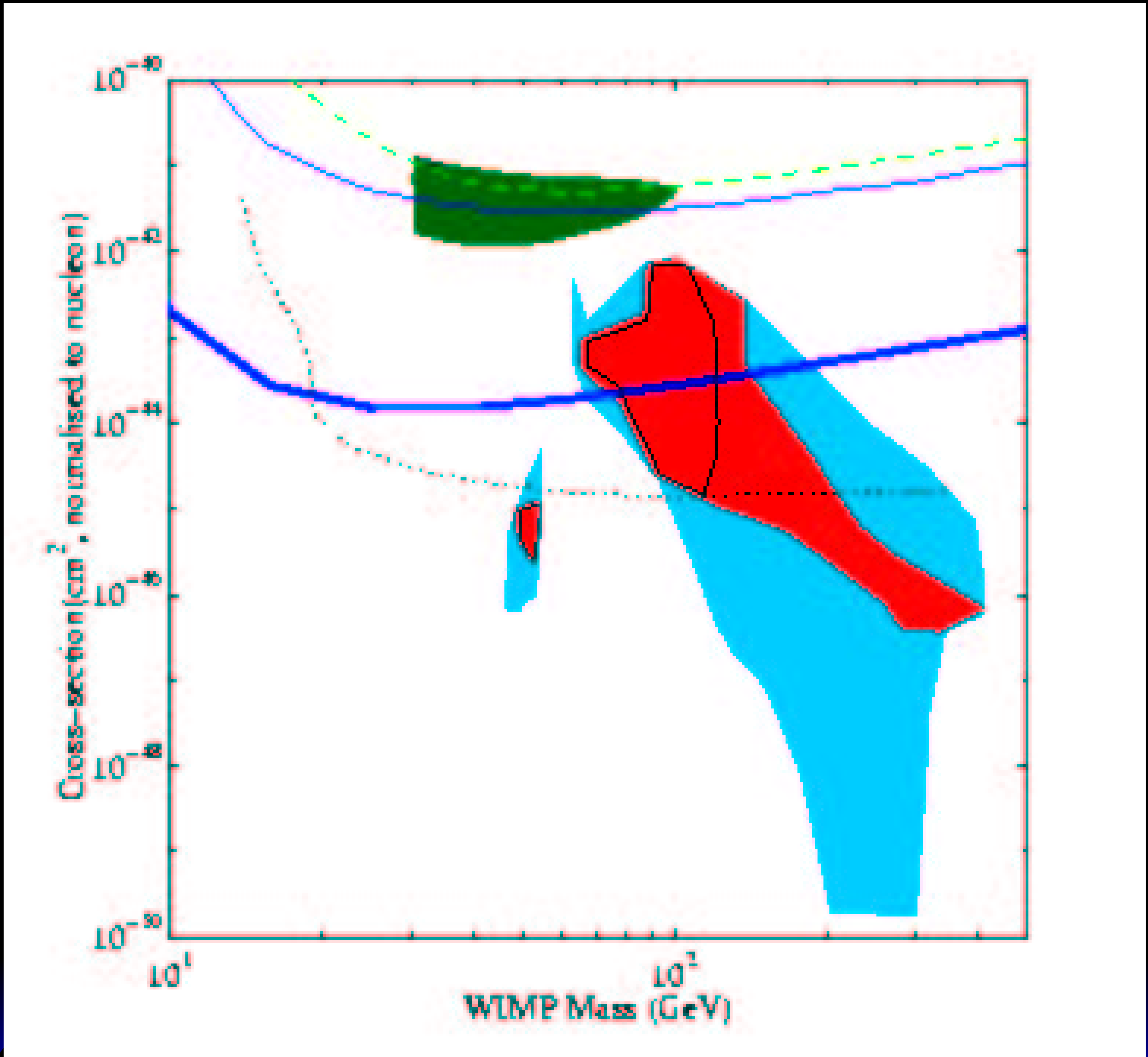
where

$$d_p = \sum_{q=u,d,s} d_q \Delta_q^{(p)}, \quad \frac{f_p}{m_p} = \sum_{q=u,d,s} f_{Tq}^{(p)} \frac{f_q}{m_q} + \frac{2}{27} f_{TG}^{(p)} \sum_{q=c,b,t} \frac{f_q}{m_q}$$

$$\longrightarrow \sigma_{\chi p}^{\text{SD}} = \frac{12}{\pi}\mu_p^2 d_p^2, \quad \sigma_{\chi p}^{\text{SI}} = \frac{4}{\pi}\mu_p^2 f_p^2, \quad \mu_p = \frac{m_\chi m_p}{m_\chi + m_p}$$

SI cross section

Mandic–Pierce–Gondolo–Murayama (2000)



$$0.052 < \Omega h^2 < 0.236$$

$$0.025 < \Omega h^2 < 1$$

$$m_0 < 1\text{TeV}$$
$$m_{1/2} < 1\text{TeV}$$
$$|A| < 3\text{TeV}$$
$$1.8 < \tan\beta < 25$$

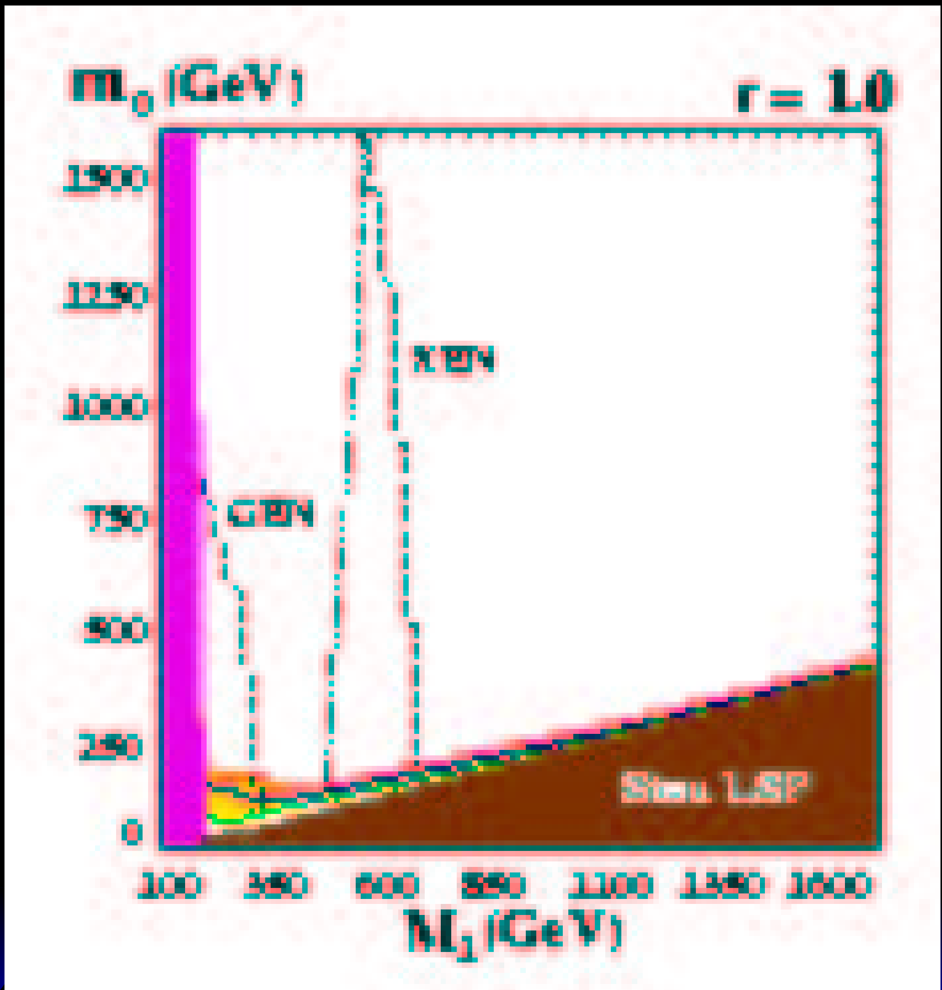
$\sigma_{SI} \approx 10^{-7}$ pb for $m_\chi \approx$ 100 GeV

$\sigma_{SI} \approx 10^{-11}$ pb for $m_\chi \approx$ 1 TeV

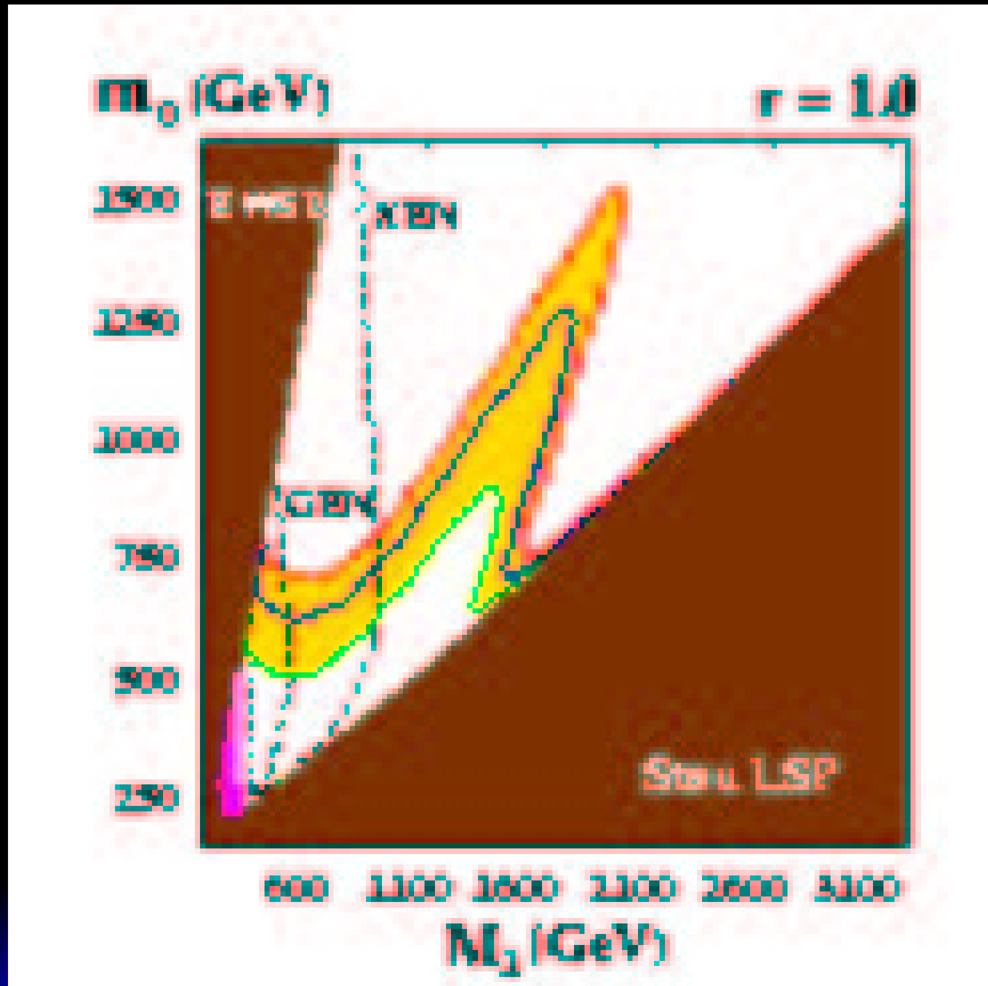
Sensitivity of future detection

Birkedal-Hansen – Nelson (2003)

$\tan\beta = 5$

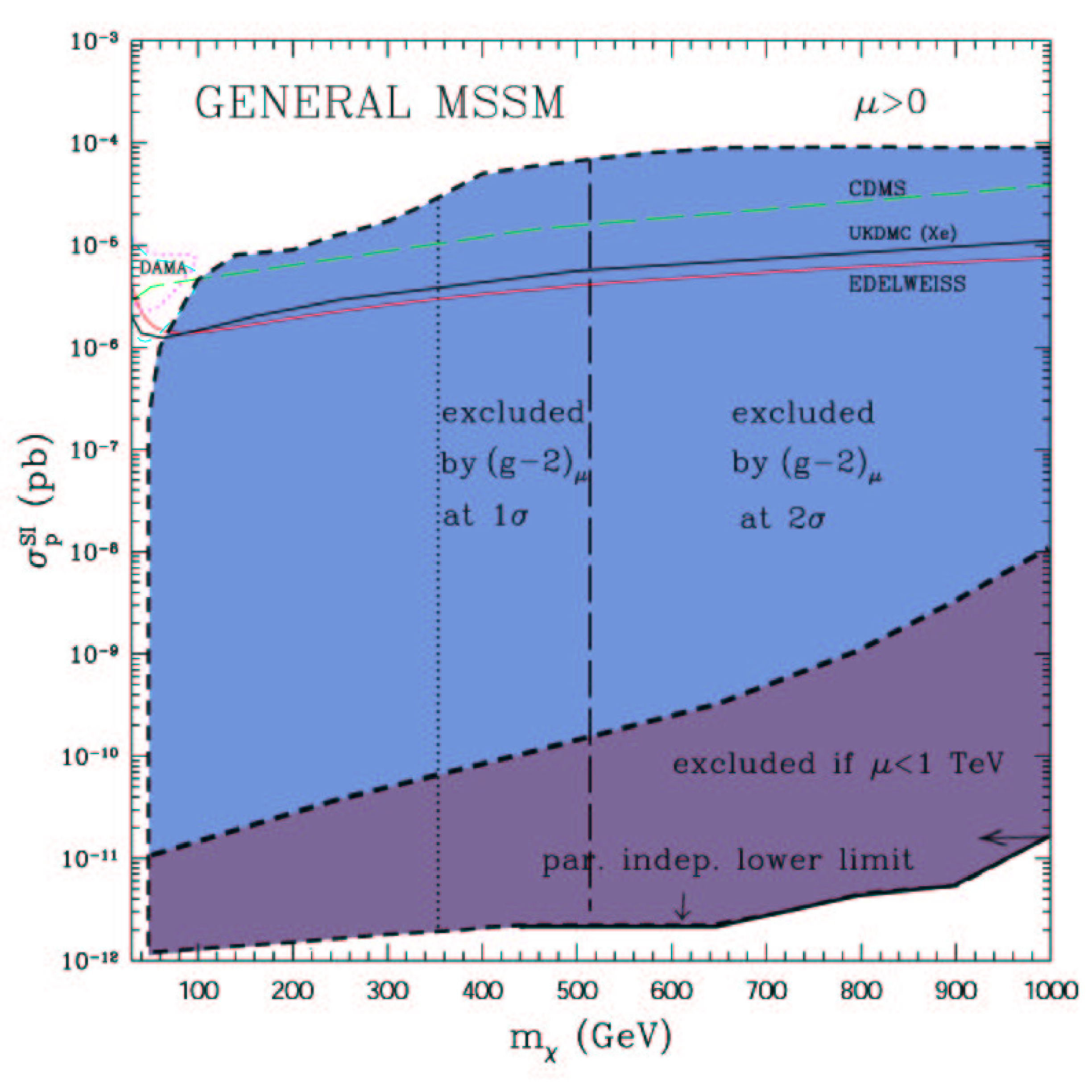


$\tan\beta = 50$



General MSSM

Kim–T.N.–Roszkowski–Ruiz (JHEP0212)



10^{-12} pb $\lesssim \sigma_{SI} \lesssim 10^{-4}$ pb
(general MSSM)

5. Effect of CP violation

- CP violating phases

  $$M_1 = |M_1| \exp(i\,\theta_{M_1}), \quad \mu = |\mu| \exp(i\,\theta_\mu)$$

  (M_2, A_f: real)

- Gaugino mass relations

  GUT-like : $|M_1| = \frac{5}{3} \tan^2 \theta_W |M_2|$

- Scalar–pseudoscalar mixing

  Pilaftsis (1998)

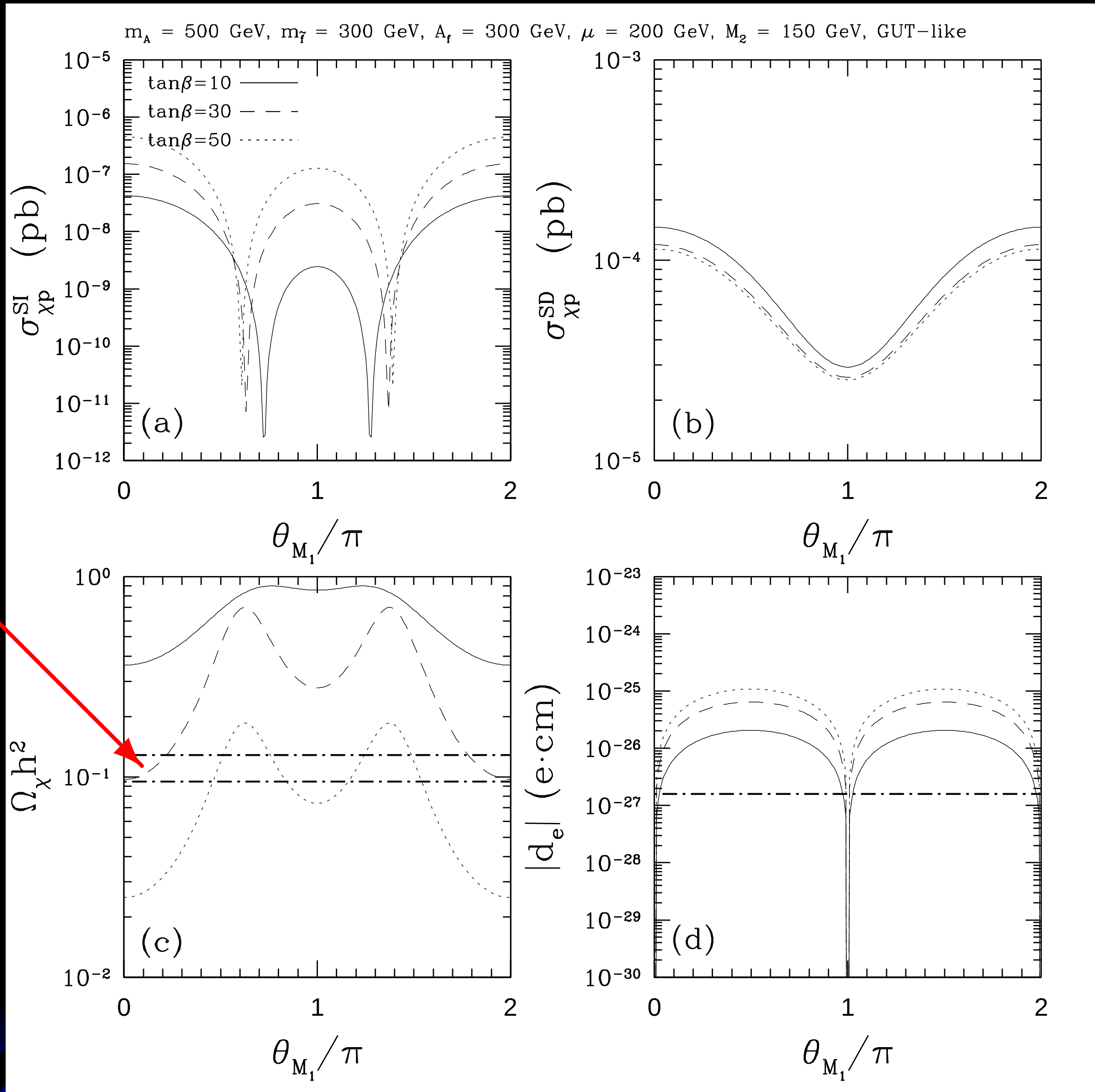
  Im($A_f\,\mu$) induces S–PS mixing at one-loop level.

  $$\begin{pmatrix} H_1^0 \\ H_2^0 \\ H_3^0 \end{pmatrix} = O_H \begin{pmatrix} \phi_1 \\ \phi_2 \\ A \end{pmatrix}$$

Bino-like LSP T.N.–Sasagawa (PRD70, 2004)

$m_A = 500\,\mathrm{GeV}$
$m_{\tilde{f}} = 300\,\mathrm{GeV}$
$A_f = 300\,\mathrm{GeV}$
$\mu = 200\,\mathrm{GeV}$
$M_2 = 150\,\mathrm{GeV}$
GUT-like

WMAP (2σ)

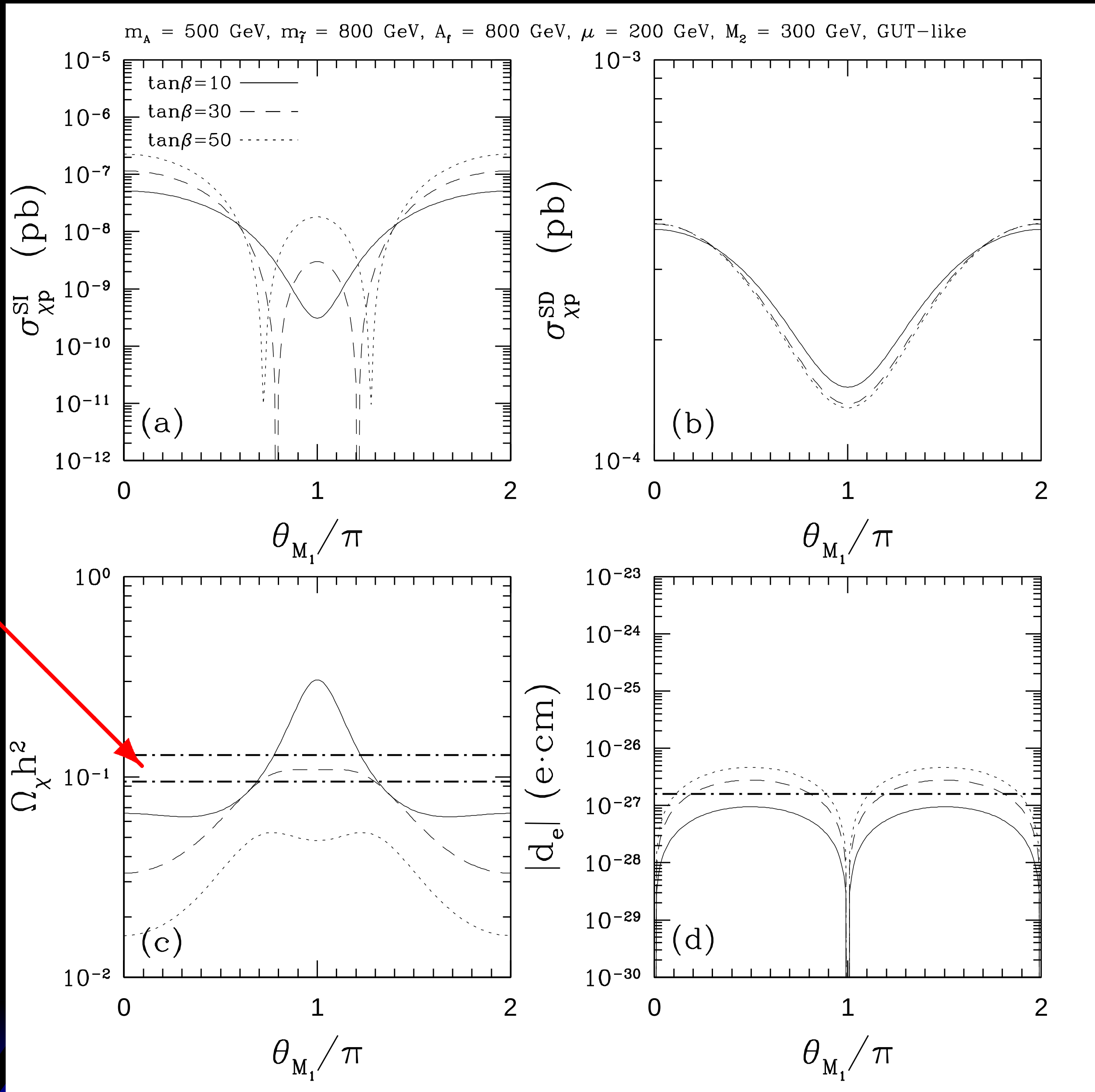


Mixed LSP

T.N.–Sasagawa (PRD70, 2004)

$m_A = 500\,\mathrm{GeV}$
$m_{\tilde{f}} = 800\,\mathrm{GeV}$
$A_f = 800\,\mathrm{GeV}$
$\mu = 200\,\mathrm{GeV}$
$M_2 = 300\,\mathrm{GeV}$
GUT-like

WMAP (2σ)



6. Conclusions

Neutralino (χ) dark matter in the CMSSM

- Relic density $\Omega_\chi h^2$ in the CMSSM
  $\longrightarrow$ CMSSM severely constrained by WMAP
- Direct detection of the neutralino
  WMAP constraint implies
  $\sigma_{\rm SI} \approx 10^{-7}$ pb for $m_\chi \approx$ 100 GeV
  $\approx 10^{-11}$ pb for $m_\chi \approx$ 1 TeV
  (gereral MSSM: 10^{-12} pb $\lesssim \sigma_{\rm SI} \lesssim 10^{-4}$ pb)
- Effect of supersymmetric CP-violating phases (θ_{M_1} , θ_μ)
  $\longrightarrow$ Strong phase dependence of $\Omega_\chi h^2$
  $\longrightarrow$ WMAP allowed region for nonvanishing CP phases