STATISTICAL ANALYSIS IN EXPERIMENTAL PARTICLE PHYSICS

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#1: BAYESIAN INTERVAL W/ POISSON + KNOWN BACKGROUND

- In our main lecture we have give the example to calculate the Bayesian intervals for the case of Poisson with known background and a uniform prior, where the posterior density can be expressed as

$$P(\mu|n) \propto \frac{(\mu+b)^n}{n!} e^{-(\mu+b)}$$

- In the case of 90% upper bound for several different # of observed events n and the known background b.

Observed	0	1	2	3
bkg = 0.0	2.30	3.89	5.32	6.68
0.5	2.30	3.51	4.84	6.18
1.0	2.30	3.27	4.44	5.71
2.0	2.30	2.99	3.88	4.93
3.0	2.30	2.84	3.52	4.36

#1: BAYESIAN INTERVAL W/ POISSON + KNOWN BACKGROUND (CONT.)

- Please practice the calculation (*you can try to modify the BayesianCalculator example code given in the main lecture, or perform the calculation by yourself*) with the following variations:
    - Replace the uniform prior to a Gaussian, with mean of 1.5 and width of 0.5. Fill the table accordingly.
    - Instead of calculating 90% UL, calculate the 68.3% central interval and fill the table accordingly (*w/ both upper/lower bounds*).

Observed	0	1	2	3
bkg = 0.0	??	??	??	??
0.5	??	??	??	??
1.0	??	??	??	??
2.0	??	??	??	??
3.0	??	??	??	??

Observed	0	1	2	3
bkg = 0.0	[??,??]	[??,??]	[??,??]	[??,??]
0.5	[??,??]	[??,??]	[??,??]	[??,??]
1.0	[??,??]	[??,??]	[??,??]	[??,??]
2.0	[??,??]	[??,??]	[??,??]	[??,??]
3.0	[??,??]	[??,??]	[??,??]	[??,??]

#2: NEYMAN CONSTRUCTION WITH BINOMINAL PDF

- Consider a typical statistical process: coin-tossing. Suppose there is a coin with unknown fraction ***p*** of showing the head. You are allowed to toss it for 10 times. Please construct the **central interval** confidence belts in 2D plane of the **# of success *n*** (*showing head in tossing*) and **true fraction *p*** with a probability β = 90%.

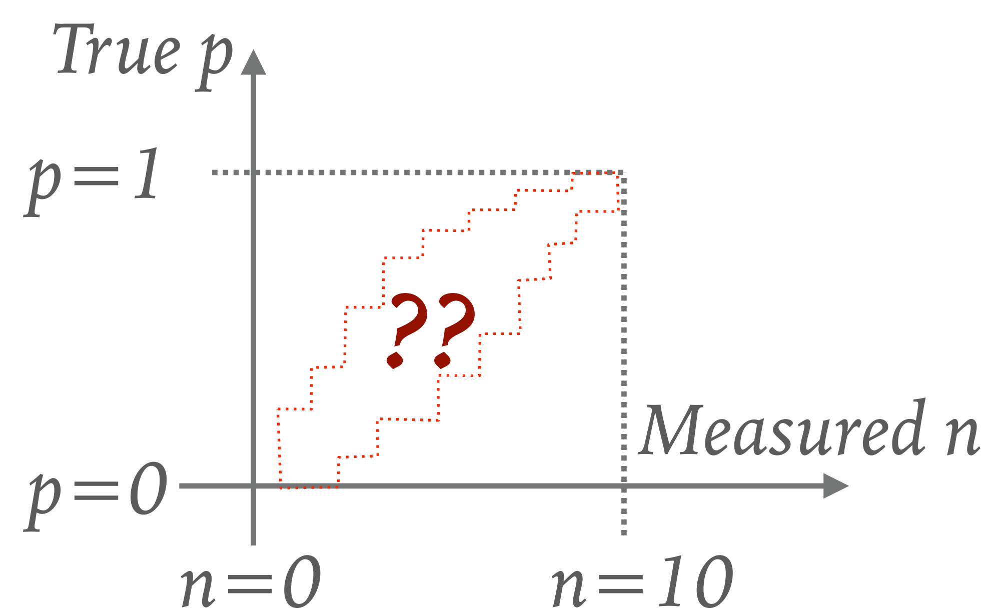
Try to draw this!

- Note since it is a discrete case, the coverage should be defined by

$$\sum_{n=L}^{U} P(n|N=10,p) \geq \beta$$

where the ***P*** is the binomial distribution:

$$P(n|N,p) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}$$



#3: FELDMAN-COUSINS WITH BINOMINAL PDF

- Following #2, let practice with **Feldman-Cousins likelihood ratio ordering** with exactly the same application: coin-tossing for 10 times with an unknown true fraction of p.
- Remember, the F&C method is based on the ordering of the likelihood ratio:

$$R(n|p) = \frac{P(n|N=10,p)}{P(n|N=10,p=\hat{p})}$$

where the ***P*** is still the binomial distribution. And the coverage is defined by

$$\sum_{R(n|p)>R_{\min}} P(n|N=10,p) \geq \beta$$

#3: FELDMAN-COUSINS WITH BINOMINAL PDF (CONT.)

- Let's complete the F&C table as given in the main lecture, but with the binomial PDF and for the **true p = 0.3 and p = 0.5**, e.g.

For true p = 0.3

n	P(n|p)	p̂	P(n|p̂)	R	Rank
0	**0.028**	0.0	1.000	**0.028**	
1	**0.121**	0.1	0.387	**0.312**	
2	**0.233**	0.2	0.302	**0.773**	
3	**0.267**	0.3	0.267	**1.000**	1
4					
5					
6					
7					
8					
9					
10					

Report interval: [??,??]

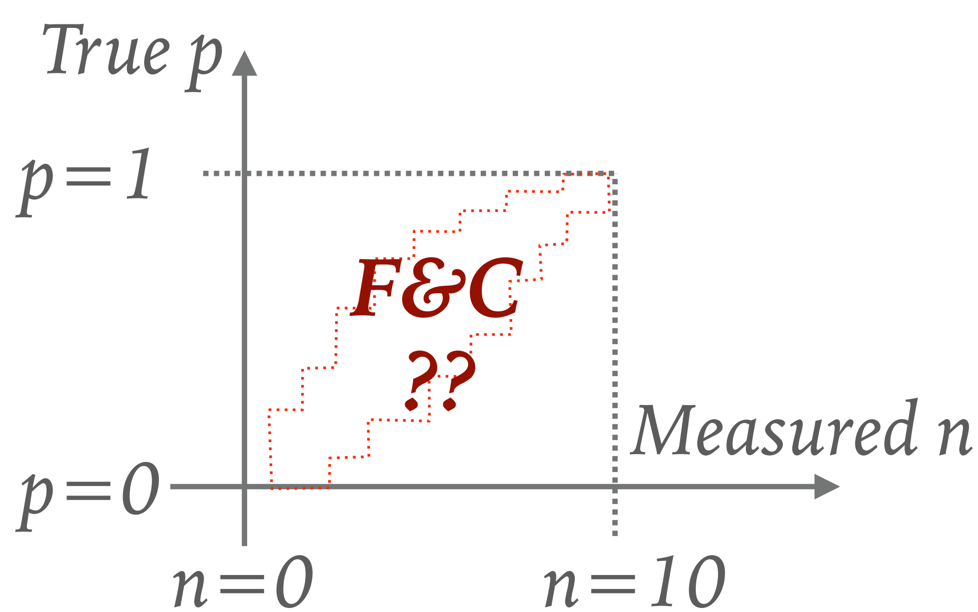
For true p = 0.5

n	P(n|p)	p̂	P(n|p̂)	R	Rank
0		0.0	1.000		
1		0.1	0.387		
2		0.2	0.302		
3		0.3	0.267		
4					
5					
6					
7					
8					
9					
10					

Report interval: [??,??]

#4: FELDMAN-COUSINS CONFIDENCE BELTS

- Following #3, what will be your full **confidence belts** if the Feldman-Cousins likelihood ratio ordering is introduced to the study? Note this can be carried out by implement your calculation for the previous exercise with scanning over the possible values of true p.



#5: REVISIT "MAGIC COIN" — HOW MANY TOSSES NEEDED?

- Suppose the coin you have is a magic coin which is expected to give only "tail" (e.g. true $p = 0$). You start to do the experiments (tossing the coin for several times) and only found "tail" until the end somehow.
- Based on F&C method, please compute the **90% upper bound on p** if you are allowed to toss the coin for 1, 2, 3, 4, … up to 10 times, and they all show "tail" indeed?
- Hint: change the total $N=10$ in the previous exercise to a different value but fixed the observed n to 0.