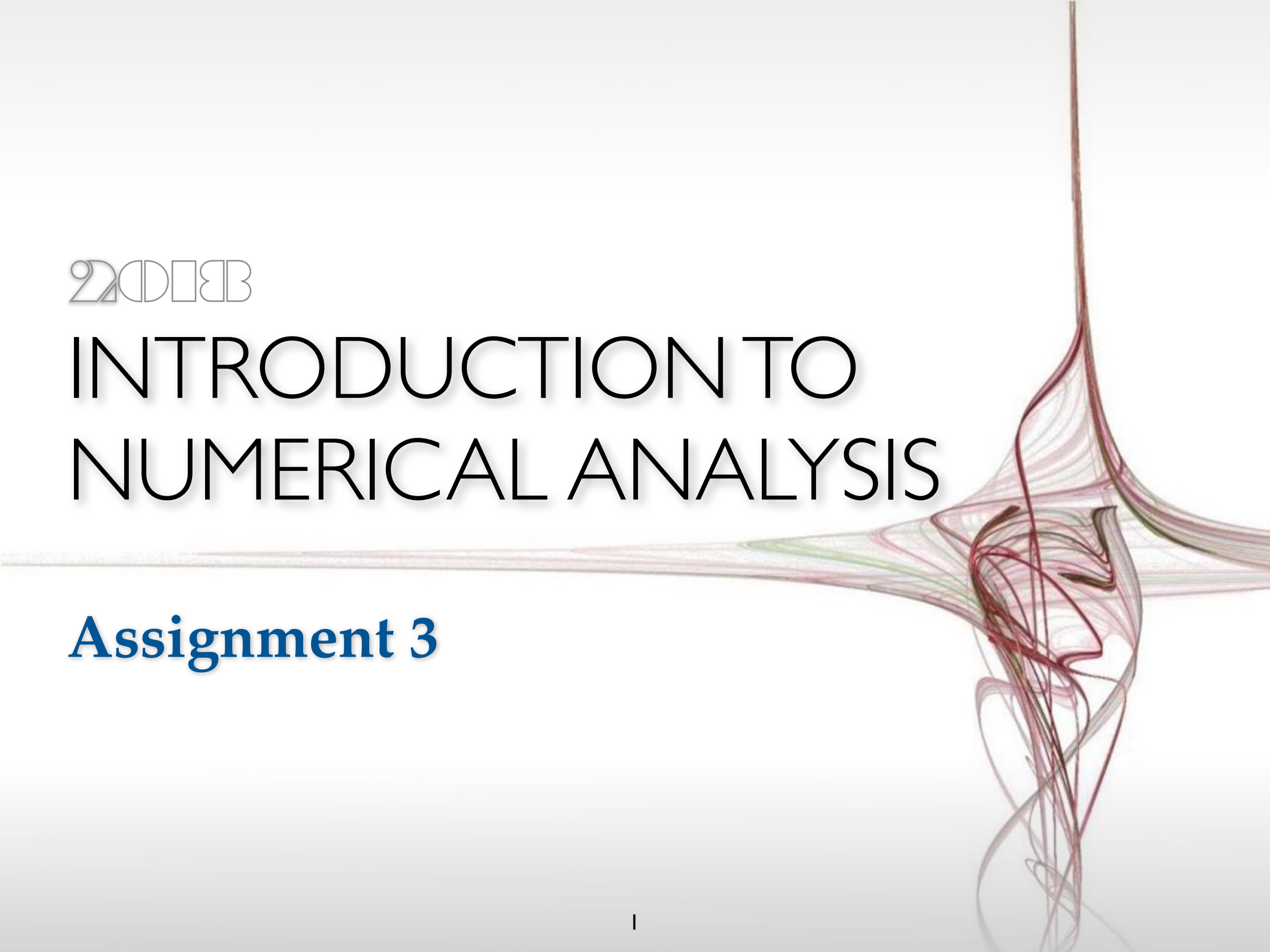
2018

INTRODUCTION TO NUMERICAL ANALYSIS



Assignment 3

ASSIGNMENT 3-1

Solving linear equations

- Construct a function which takes an **input array b** of 6 elements $[b_1,b_2,b_3,b_4,b_5,b_6]$, and output an array $x=[x_1,x_2,x_3,x_4,x_5,x_6]$ by solving the following linear equations:

$$
\begin{aligned}
x_1 + 2x_2 &= b_1 \\
x_1 + 2x_2 + 3x_3 &= b_2 \\
2x_2 + 3x_3 + 4x_4 &= b_3 \\
3x_3 + 4x_4 + 5x_5 &= b_4 \\
4x_4 + 5x_5 + 6x_6 &= b_5 \\
5x_5 + 6x_6 &= b_6
\end{aligned}
$$

ASSIGNMENT 3-2

Covariance and correlation matrix

- Construct a function which takes an **input n×n symmetric matrix** (n>3), which stores a **covariance matrix** of the following form. Please output a two-dimensional **n×n** NumPy array with the target content, which is usually called the **correlation matrix**.

Input

$$\begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{21}\sigma_2\sigma_1 & \sigma_2^2 & \rho_{23}\sigma_2\sigma_3 & \cdots & \rho_{2n}\sigma_2\sigma_n \\ \rho_{31}\sigma_3\sigma_1 & \rho_{32}\sigma_3\sigma_2 & \sigma_3^2 & \cdots & \rho_{3n}\sigma_3\sigma_n \\ \vdots & \vdots & \vdots & & \vdots \\ \rho_{n1}\sigma_n\sigma_1 & \rho_{n2}\sigma_n\sigma_2 & \rho_{n3}\sigma_n\sigma_3 & \cdots & \sigma_n^2 \end{pmatrix}$$

Output

$$\begin{pmatrix} 1 & \rho_{12} & \rho_{13} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \rho_{23} & \cdots & \rho_{2n} \\ \rho_{31} & \rho_{32} & 1 & \cdots & \rho_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ \rho_{n1} & \rho_{n2} & \rho_{n3} & \cdots & 1 \end{pmatrix}$$

ASSIGNMENT 3-3

Computing least squares

- Consider a set of 5 data points measured at the positions of **x**. The measured **y** values as well as the corresponding error matrix **Σ** are given by

```
x = np.array([1.00,2.00,3.00,4.00,5.00])

y = np.array([5.23,4.32,3.05,2.76,2.01])

Sig = np.array([[0.62,0.21,0.12,0.04,0.09],
                [0.21,0.78,0.32,0.10,0.12],
                [0.12,0.32,0.69,0.03,0.15],
                [0.04,0.10,0.03,0.95,0.20],
                [0.09,0.12,0.15,0.20,0.83]])
```

ASSIGNMENT 3-3 (CONT.)

- Suppose the data points can be modeled by a linear function of

$$f(x) = ax + b$$

and the possible values of **a** and **b** are {0.0, ±0.2, ±0.4, ±0.6, ±0.8, ±1.0}. Remark: there are 11×11 = 121 combinations of **a** and **b**.

- Construct a function to find the value of **a** and **b** with the smallest χ^2 value, e.g.

For all possible $a, b \in \{0, \pm 0.2, \pm 0.4, \pm 0.6, \pm 0.8, \pm 1.0\}$

Estimate the value of $\chi^2(a,b) = (y - f(x))\Sigma^{-1}(y - f(x))^T$

Return the a, b with smallest χ^2

For helping your debug, **χ^2(a=1,b=1) ~ 46.8095.**