2013

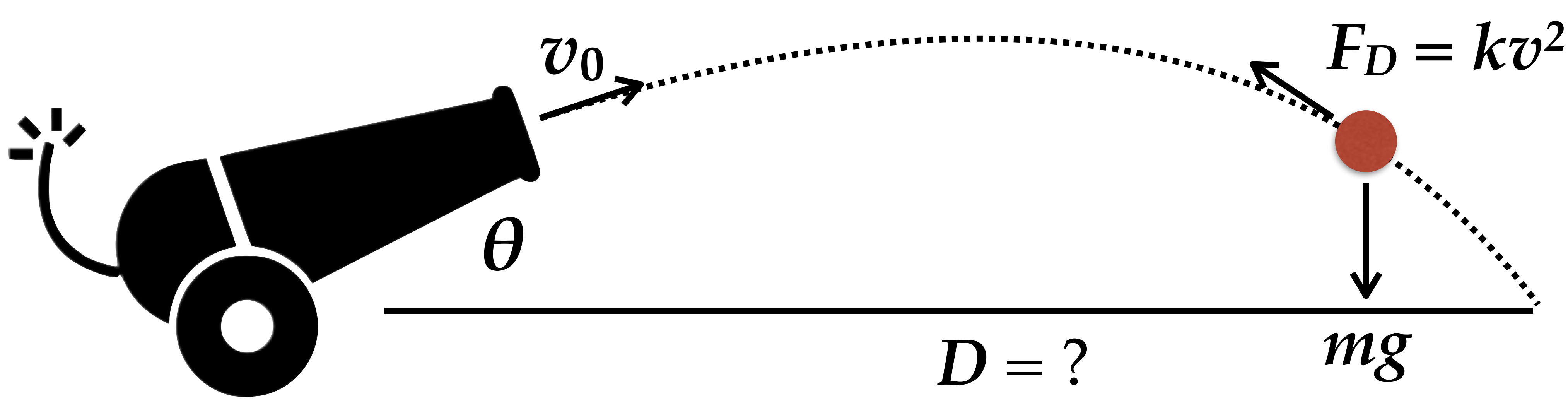
INTRODUCTION TO NUMERICAL ANALYSIS

Assignment 5

ASSIGNMENT 5-1

Cannon fire!

- Suppose we have a cannon set at angle θ and fire a iron ball of mass m with an initial speed of v_0. The iron ball has air resistance / drag force which is proportional to the square of the speed ($F_D = kv^2$) acting on it. Construct a function, which **takes θ, v_0 and k as the input arguments**, **return the flight distance of the iron ball** when it hits the ground and ignore the size of the cannon itself.



ASSIGNMENT 5-1 (CONT.)

- Here are the exact differential equations to be solved:

$$a_x = \frac{d^2x}{dt^2} = -\frac{kv^2}{m}\left(\frac{v_x}{v}\right)$$

$$a_y = \frac{d^2y}{dt^2} = -\frac{kv^2}{m}\left(\frac{v_y}{v}\right) - g$$

$$v_x = \frac{dx}{dt}$$

$$v_y = \frac{dy}{dt}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

initial condition

$$x(t=0) = 0$$

$$y(t=0) = 0$$

$$v_x(t=0) = v_0 \cos\theta$$

$$v_y(t=0) = v_0 \sin\theta$$

constants

$$m = 1 \text{ kg}$$

$$g = 9.8 \text{ m/s}^2$$

ASSIGNMENT 5-2

A multi-star system in 2D

- Suppose we have 10 stars placed in 2D space with some initial velocity. The initial position and velocity of the stars are recorded in NumPy array as following:

(m, x, y, v_x, v_y)

```
init_data = [[0.362, +0.380, -1.954, -2.364, +0.461],
             [0.168, +0.032, +0.631, +0.233, -0.608],
             [0.413, +0.280, -0.095, -0.672, -0.369],
             [0.209, -1.669, +0.116, -1.965, +0.237],
             [0.172, +0.376, +0.673, -0.370, +0.723],
             [0.322, -0.583, +0.355, -0.405, +0.831],
             [0.289, -0.619, -0.960, -0.525, -1.366],
             [0.108, +0.626, -1.931, +0.276, +1.698],
             [0.491, +0.499, +0.217, -1.237, +0.084],
             [0.325, +0.781, +1.452, -0.295, -0.827]];
```

assignment-502.py (partial)

initial condition
at t = 0
for 10 stars

ASSIGNMENT 5-2 (CONT.)

- Construct a function which **takes Δt as the only argument**, solve for the conditions of the stars after time $\boldsymbol{\Delta t}$ and **return the positions of the stars in NumPy array of shape (10,2)**.
- The exact equations for one of the stars (index $\boldsymbol{i}$) to be solved (the gravitational constant has been set to 1):

$$a_{x,i} = \frac{d^2 x_i}{dt^2} = \sum_{j \neq i} \frac{m_j}{R_{ij}^2} \cdot \left(\frac{x_j - x_i}{R_{ij}} \right)$$

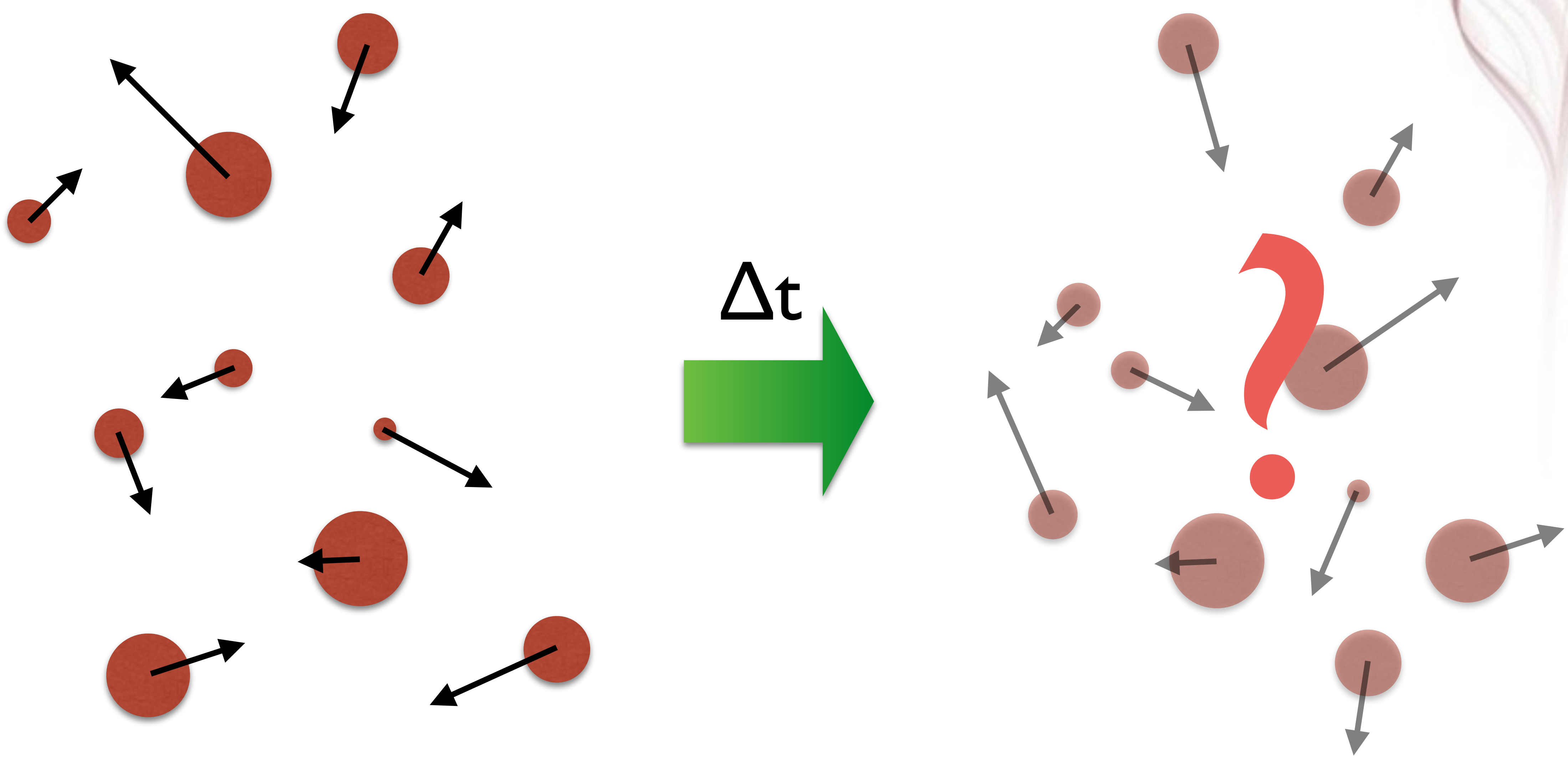
$$a_{y,i} = \frac{d^2 y_i}{dt^2} = \sum_{j \neq i} \frac{m_j}{R_{ij}^2} \cdot \left(\frac{y_j - y_i}{R_{ij}} \right)$$

$$v_{x,i} = \frac{dx_i}{dt}$$

$$v_{y,i} = \frac{dy_i}{dt}$$

$$R_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

ASSIGNMENT 5-2 (CONT.)

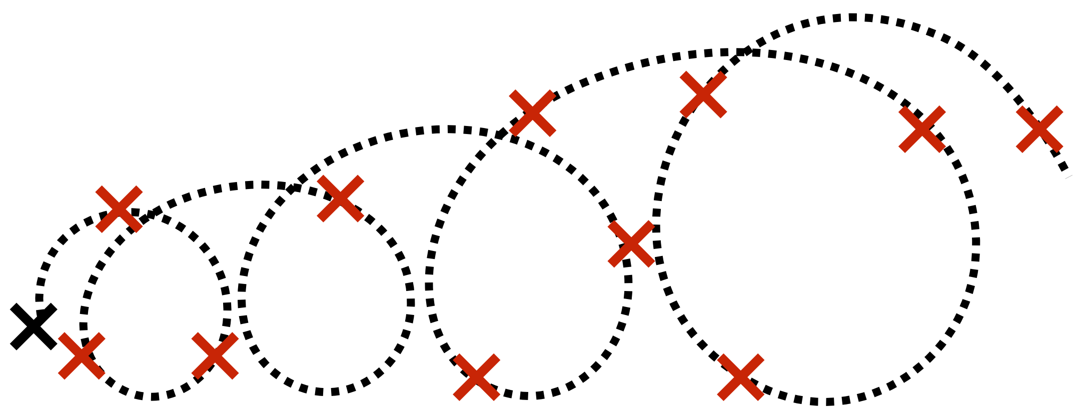


return the positions of the stars

ASSIGNMENT 5-3

Charged particle moving in magnetic and electric fields

- Consider a charged particle moving in a uniform magnetic field **B=(B_x, B_y, B_z)** and a uniform electric field **E=(E_x, E_y, E_z)**. There is a charged particle of charge ***q*=1** and mass ***m*=1** placed at the origin and at rest initially.
- The particle has both electric field and magnetic field acted on it. Construct a function which takes **two arrays of 3 elements corresponding to the vectors of B and E**. Return **a NumPy array of the shape (10,3), which records the position of the particle in 3D after 1, 2, 3,…,10 secs**.



ASSIGNMENT 5-3 (CONT.)

- Here are the exact differential equations to be solved:

$$\overrightarrow{E} = (E_x, E_y, E_z)$$

$$\overrightarrow{B} = (B_x, B_y, B_z)$$

$$\overrightarrow{x} = (x, y, z)$$

$$\overrightarrow{v} = (v_x, v_y, v_z)$$

$$\overrightarrow{a} = (a_x, a_y, a_z)$$

$$\overrightarrow{a} = \frac{d^2\overrightarrow{x}}{dt^2} = \frac{q}{m}\overrightarrow{E} + \frac{q}{m}\overrightarrow{v} \times \overrightarrow{B}$$

$$\overrightarrow{v} = \frac{d\overrightarrow{x}}{dt}$$

initial condition

$$(x, y, z) = (0, 0, 0)$$

$$(v_x, v_y, v_z) = (0, 0, 0)$$

constants

$$m = 1$$

$$q = 1$$